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In 2006 all ECB publications will feature a motif taken from the €5 banknote.

1 The views presented in this paper are those of the authors' and are not necessarily shared by the European Central Bank.

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Abstract:

This paper studies the implications of introducing an explicit policy objective to the management of foreign reserves at a central bank. A dynamic model is developed which links together reserves management and the exchange rate by foreign exchange interventions. The exchange rate is modelled as a mean-reverting autoregressive process incorporating a linear response to interventions. The premise is that it is the objective of the central bank to prevent undervaluation of its currency. Given this objective, the model is formulated in a one- and a multi-period setting and solved to find the optimal asset allocation. The results show that asset allocation can significantly help in achieving the desired policy objective.

Keywords: Foreign reserves management, foreign exchange intervention, exchange rate modelling, optimal asset allocation

JEL classification: G11; F31

Non technical summary

This paper develops an optimization model to study how an explicit policy objective to keep the exchange rate above a given level affects foreign reserves management at a central bank. Foreign exchange interventions form the link between reserves management and the exchange rate. The problem is to find the right balance between risk and return to provide necessary liquidity for interventions in the foreign exchange market. A risky portfolio guarantees sufficient income in the long run, but presents the risk that the assets have to be sold at unfavourable conditions when intervention needs occur. With a safe strategy, on the other hand, the accumulation of portfolio wealth is less vulnerable to interventions, but may be inadequate for intervention needs in the long run.

The exchange rate is modelled by a mean-reverting autoregressive time series which explicitly incorporates the effect of interventions. Interventions are linked to the prevailing exchange rate and to the amount of wealth available. The central bank intervenes in the market every year the exchange rate is below a lower bound set by the central bank itself – if it has enough reserves for it. In a multi-period framework, the annual intervention amounts are kept fixed at a level seen sufficient to have a signalling effect on the market.

The developed model is first studied in a single period setting. This proves to be too simplistic to reveal any significant interplay between asset allocation and the policy objective, but since the one period model can be solved analytically it allows analysing how prevailing market conditions – such as the current level of exchange rates and the risk-return properties of the risky asset – affect the optimal reserves portfolio.

A multi-period formulation is necessary for a more thorough analysis of the model at hand. The model no longer exhibits an analytical solution, and numerical methods are needed to find the optimal solution. The results show that portfolio choice has a clear impact on achieving the given policy objective of defending the currency from undervaluation. Numerical sensitivity analysis indicates that conclusions from the analytical solution of the one-period model hold also in the multi-period case.

1. Introduction

This paper examines the effects that arise from having an explicit policy objective to foreign reserves management at a central bank and studies the implications this has for the optimal reserves allocation. An optimization model is developed capturing the interplay between foreign reserve management and exchange rates through introduction of interventions. This paper provides stylised solutions for the reserves' optimal asset allocation both in a one-period and a multi-period setting.

The objective is to keep the exchange rate above a given threshold by means of intervening in the foreign exchange markets. The problem is thus to find the right balance of risk and return to provide liquidity for these operations. In the long run, investments would be made to the asset with higher expected return, but this presents the risk that in the short run positions have to be unwounded when the asset value is low and the domestic currency is under pressure.

The goal of this paper is to better understand the link between asset management and the policy requirements of reserves holdings. In practice, central banks carefully separate between monetary and exchange rate policy on the one hand and investment activities on the other. Market participants (i.e. counterparties of central banks) should not be in a position to deduce policy decisions from adjustments in the portfolios, and central banks' investment performance should not benefit from non-public information. Central banks carefully reflect these requirements in designing the governance structures for reserves management (Cardon and Coche 2004). This paper tries to shed some light on the investment problem in principle while abstracting from the practical requirements for reserves management outlined above.

Central bank interventions are an active subject of research, but most literature is focused on the effects and reasons for interventions [see, e.g., Humpage (2003) and Sarno and Taylor (2001) for extensive literature surveys]. Various reasons for interventions have been identified in the literature (Kim and Sheen (2002), Fatum et al (2003)). These comprise exchange rate manipulation (decreasing excess volatility, correcting trends) as a primary objective or secondary to a central bank's general mandate. Academics have mixed views on whether interventions have a lasting effect on FX rates. For example, Fatum et al (2003) and Dominguez (2005) report evidence supporting intervention, while Bonser-Neal et al (1998) and Humpage (1999) provide results to the contrary. In the context of this paper it is assumed that foreign exchange interventions are effective, albeit the degree of effectiveness can vary and is considered as one parameter in the model.

A two-country model with a small and a big country is considered. The perspective is on the reserves management of the small country which holds foreign reserves invested in the big country. It is assumed that interventions by the small country can in principle affect the exchange rate between the two countries. A simplified setting of a risky and a risk-free asset is used with the return of the risky asset following a normal distribution. The assets are denominated in the currency of the big country.

In a one-period setting the model can be solved analytically. This solution provides useful insight into the problem despite the simplistic setting, but the asset allocation decision is seen to help only marginally with the policy objective. However, this conclusion is to be expected since the problem is intertemporal in nature and thus calls for a solution within a multi-stage model setting. In this setup, numerical solution methods are needed since the model can no longer be solver analytically. With simulation techniques it is seen that asset allocation has a major effect on the exchange rate. More specifically it is found that the optimal asset allocation can substantially reduce the probability of adverse exchange rate levels. Given the specific assumptions made in this study a rather high share of the risky asset is found to best serve the needs of the central bank.

The paper is organized as follows. Section 2 introduces the objective function and the different constraints used together with the time-series model for the exchange rate. In section 3 a one-period model is considered which is generalized to multi-period setting in section 4. Section 5 concludes.

2. Model Specification

The exchange rate, the price of the foreign currency in terms of the home currency, at time t is denoted by e_t and is assumed to have a time-invariant natural level P.[3] A level of ρP ($\rho<1$) is seen to be the lowest exchange rate acceptable and the central bank's objective is to prevent the exchange rate from falling below this barrier.[4] Reflecting this view the objective function is defined as minimizing the probability of exchange rate falling below ρP. This approach is relatively simple to work with numerically, and can be solved analytically in a single stage framework under some general assumptions. The aim is to make the occurrence of exchange rates under a certain barrier as rare as possible, without caring about the actual foreign exchange rates when this happens. This resembles a Value-at-Risk approach where only the probability of the tail is of interest – not the tail distribution.

In an intertemporal setting with T periods the probability of falling below the critical barrier is taken into account for every period. Time preferences are introduced by a time preference factor δ. In this general setting the objective function is given by:

$$\min_{\alpha} \sum_{t=1}^{T} \delta^t \, prob(e_t < \rho P). \tag{1}$$

When δ is less than 1, more emphasis is placed on the near future. As δ equals one, all periods become equally important. With δ greater than 1 more weight is placed in the distant future.

Interventions are seen as necessary and exogenous events, from the perspective of foreign reserves management, triggered when the exchange rate falls below a pre-defined level. Reserves managers can

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[3] The natural level of the exchange rate can be determined by using the purchasing power parity (PPP) or the uncovered interest rate parity (UIP), for example.

[4] The model could be generalized to a situation where a corridor for the exchange rate is defended, as in ERM2. This would, however, require the introduction of a home currency portfolio in addition to the foreign reserves to make interventions possible to the other direction. For simplicity, this study considers only the case where a lower boundary is defended.

minimise the probability of an exchange rate shortfall by choosing the weight of the risky asset α which affects growth rate and volatility of wealth accumulation over time. Higher wealth translates into a higher number of credible interventions.

The exchange rate at time t is assumed to respond to interventions by an amount determined by an efficiency parameter γ and the (fixed) volume of foreign reserves used for intervention at time t, denoted by I_t. Foreign exchange rates are assumed to exhibit mean-reversion around the fundamental value P. Adding these properties to an AR(1)-process leads to the following model for exchange rates

$$e_t = (1-b)P + be_{t-1} + \gamma I_t + \varepsilon_t = P + b(e_{t-1} - P) + \gamma I_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_e^2), \quad t = 1,2,...,T,$$

where $1\text{-}b$ determines the magnitude of mean-reversion. Interventions are assumed to take place at time t and have an immediate effect on the exchange rate. The introduction of interventions causes the distribution of the exchange rate to be right-skewed.

The intervention policy represents the view that interventions have a signalling effect on the currency markets. In this case, the amount of intervention will always be a constant proportion of initial wealth. The central bank observes the level of exchange rates without intervention at time t and immediately intervenes when necessary, if enough foreign reserves are available. Accordingly, intervention volumes are defined as:

$$I_t = \begin{cases} cW_0, \, if \; (1-b)P + be_{t-1} + \varepsilon_t < \rho P \; and \; W_t > cW_0 \\ 0, \, otherwise \end{cases},$$

where c represents the proportion of initial wealth used for interventions. In this case the volume of reserves limits the number of interventions that can be carried out. The term cW_0 can be considered as an intervention volume that gives a credible signal to the market.

The risk-free one-period return r^f is assumed to be constant over time. The one-period return of the risky asset, denoted by r_t, is assumed to be normally and independently distributed over time with time-invariant mean μ_r and volatility σ_r. The exchange rate error term and the risky return are for simplicity assumed to be independent (a joint normal distribution can also be used, see the appendix for details). The amount of wealth left after intervention at time $t\text{-}1$ is invested according to the chosen asset allocation. Wealth before a possible intervention at time t is thus:

$$W_t = (1 + r^f + \alpha(r_t - r^f))(W_{t-1} - I_{t-1}), \quad r_t \sim N(\mu_r, \sigma_r^2), \quad t = 1,2,...,T.$$

3. One-period case

This section formulates and solves the problem in a one-period setting. The main motivation for this simplified framework is that the problem can be solved analytically. Despite the simplicity of this approach the solution shows nontrivial dependencies between parameters and the optimal asset allocation. A closer inspection reveals that with any reasonable parameter values the actual impact of asset allocation on the exchange rate remains marginal.

In the one-period setting the central bank has an initial wealth W_0 of which a proportion α is invested in the risky asset and a proportion $1-\alpha$ in the risk-free asset. After one time step the risky return r_1 and the innovation ε_1 become known, after which an intervention I_1 is conducted if necessary. In this case the parameter δ is not needed and the model becomes

$$\min_{\alpha} \; prob\,(e_1 < \rho P)$$
$$\text{subject to}$$
$$W_1 = (1 + r^f + \alpha(r_1 - r^f))W_0, \quad r_1 \sim N(\mu_r, \sigma_r^2),$$
$$e_1 = (1-b)P + be_0 + \gamma I_1 + \varepsilon_1, \quad \varepsilon_1 \sim N(0, \sigma_e^2),$$
$$I_1 = \begin{cases} cW_1, \, if \; (1-b)P + be_0 + \varepsilon_1 < \rho P \\ 0, \, otherwise \end{cases}.$$

No interventions are assumed to be done at time 0.

Since the amount of final wealth W_1-I_1 does not enter the objective function, it can be assumed that the full amount of accumulated wealth at time 1 will be used to intervene. In fact, this would always be optimal if the amount of intervention was chosen freely.

Accordingly, the model is written as:

$$\min_{\alpha} \; prob\left((1-b)P + be_0 + \gamma\left(1 + r_1^f + \alpha(r_1 - r_1^f)\right)W_0 + \varepsilon_1 < \rho P\right)$$

To obtain an analytical solution for this model a distribution needs to be assumed for the stochastic factors r_1 and ε_1. They are assumed to be independently normally distributed with means μ and 0 and volatilities σ_r and σ_e, respectively. The independence assumption can be relaxed by assuming a bivariate normal distribution for r_1 and ε_1, and the initial wealth can be assumed to be 1 by scaling γ appropriately.

The optimal solution in the general case of a joint normal distribution is derived in Appendix 1. Assuming zero correlation, $\sigma_{r\varepsilon}=0$, leads to the following optimal solution

$$\overline{\alpha} = \gamma^{-1}\sigma_e^2 \tfrac{\mu_r - r^f}{\sigma_r^2}\left(P(1-\rho) + b(e_0 - P) + (1 + r^f)\gamma\right)^{-1}. \tag{2}$$

To analyze this solution it is assumed that all parameters are strictly positive and fulfil the relations:

$$\rho, b < 1,$$
$$\mu_r > r^f,$$
$$e_0 \geq \rho P.$$

These natural assumptions ensure that the optimal α is always positive. This can be seen by adding and subtracting $P\rho b$ to the denominator and rearranging the terms

$$\begin{aligned} \overline{\alpha} &= \gamma^{-1}\sigma_e^2 \tfrac{\mu_r - r^f}{\sigma_r^2}\left(P(1-\rho) + be_0 - P(1-\rho)b - P\rho b + (1 + r^f)\gamma\right)^{-1} \\ &= \gamma^{-1}\sigma_e^2 \tfrac{\mu_r - r^f}{\sigma_r^2}\left(P(1-\rho)(1-b) + b(e_0 - P\rho) + (1 + r^f)\gamma\right)^{-1} > 0. \end{aligned} \tag{3}$$

All of these terms are positive which makes it easy to identify the effect of each parameter.

The first conclusion is that the optimal weight of the risky asset increases with the variance of the exchange rate. A natural interpretation for this is that, holding gamma constant, higher expected wealth is required to limit the left tail of a wider distribution of the exchange rate. For higher expected return more money needs to be invested in the risky asset.

The third term, excess return divided by its variance, represents the attractiveness of the risky asset. The more attractive the risky asset is the more the central bank is willing to invest in it.

The effectiveness of interventions, γ, only occurs in the denominator and thus decreases investment in the risky asset. This is natural as the more impact interventions have the less money is needed to counter adverse movements in the exchange rate.

A higher critical level of the exchange rate ρP increases the volume of interventions necessary to rebalance the exchange rate. Hence the greater share of the risky asset in the optimal asset allocation.

The effect of the AR-coefficient b depends on the sign of e_0-P. This is seen from equation (2). If the initial exchange rate equals the desired level P, then the value of b has no effect on the optimal α. This is due to the fact that if the exchange rate is already in its mean then the amount of mean-reversion has no effect to the value at time 1.

If $e_0>P$ then stronger mean-reversion means a higher probability of a low exchange rate in the next period. This is reflected in the solution by the fact that greater mean-reversion increases the amount of risky asset holdings in the optimal portfolio.

Finally, in case $e_0<P$, greater mean-reversion decreases the probability of exchange rate undervaluation at time *1* and hence reduces the need of risk-taking in the portfolio.

To assess the gains from choosing the optimal fraction of the risky asset, the objective value using a risk-free portfolio is compared to the value achieved by the use of the optimal portfolio. For illustrative purposes the following parameter values were used:[5]

$$r^f = 0.05,\ \mu_r = 0.1,\ \sigma_r = 0.15,\ \sigma_e = 0.1,\ \gamma = 0.1,\ b = 0.9,\ P = 1,\ e_0 = 0.9\ and\ \rho = 0.8.$$

The period length is assumed to be one year.

The derivation of the optimal solution provides the following formula for the objective function (see the appendix)

$$prob(e_1 - \rho P < 0) = \Phi\left(-\frac{be_0 + P(1-\rho-b) + \gamma W_0\left(1+r^f+\alpha(\mu_r - r^f)\right)}{\sqrt{\alpha^2\gamma^2W_0^2\sigma_r^2+\sigma_e^2}}\right).$$

Evaluating this formula for different values of α produces the graph in Figure 1.

[5] Other reasonable parameter values were tried out as well producing similar results with only a little gain from asset allocation

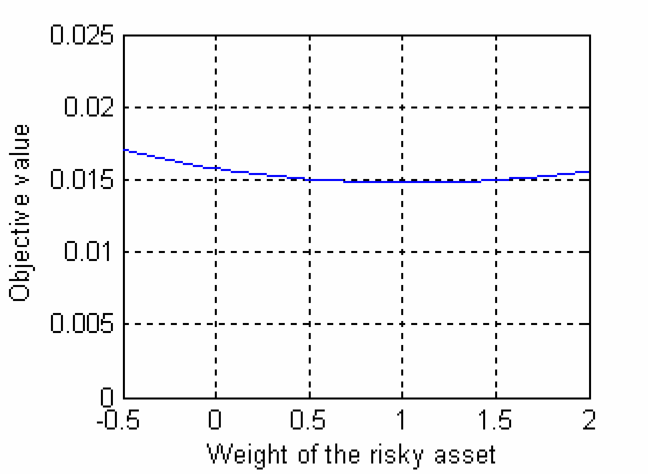


Figure 1 Objective function for the one-stage problem

The solution is 1.03 which can also be directly calculated using the analytical formula (2). This exercise shows that the gains from choosing the optimal portfolio, with $\alpha = 1.03$, are limited: the relative decrease in the optimum value obtained with the optimal asset allocation is 6.3% when compared to investing solely in the risk-free asset.

4. Multi-period case

The analysis in previous sections shows that in a one-period setting, with reasonable parameter values, asset allocation decision only marginally helps in fulfilling the policy objective. To further examine the impact, this section extends the problem to a multi-period setting. In the intertemporal case, an analytical solution can no longer be found; instead the importance of asset allocation is studied with simulation methods.

As discussed in Section 2, interventions are assumed to be a constant proportion of initial wealth. In contrast to the one-period model, where more wealth translated into larger interventions, sufficient wealth guarantees that interventions can be carried out when necessary. Using the specifications introduced in Section 2, the analytical model is in the multi-period case:

$$\min_{\alpha} \sum_{t=1}^{T} \delta^t \, prob\,(e_t < \rho P)$$

subject to

$$W_t = (1 + r^f + \alpha(r_t - r^f))(W_{t-1} - I_{t-1}), \quad r_t \sim N(\mu_r, \sigma_r^{\ 2}), \quad t = 1,2,\ldots,T,$$

$$e_t = (1-b)P + be_{t-1} + \gamma I_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_e^{\ 2}), \quad t = 1,2,\ldots,T,$$

$$I_t = \begin{cases} cW_0,\ if\ (1-b)P + be_{t-1} + \varepsilon_t < \rho P\ and\ W_t > cW_0 \\ 0,\ otherwise \end{cases}, \quad t = 1,2,\ldots,T.$$

The problem is simulated using the same parameter values, except for the higher value of γ, as in Section 3 for the one-period case:

$$r^f = 0.05,\ \mu_r = 0.1,\ \sigma_r = 0.15,\ \sigma_e = 0.1,\ \gamma = 0.3,\ b = 0.9,\ P = 1,\ e_0 = 0.9\ and\ \rho = 0.8.$$

Proportion of wealth used for interventions, c, is assumed to be 30%. Together with the effectiveness of 0.3 this means that interventions shift the exchange rate up by 0.9 standard deviations. This is roughly equal to the "average" effect of interventions in the one-period case where all wealth was, for analytical tractability, assumed to be used.

The period length is again assumed to be one year and the investment horizon is 30 years. In addition, $\delta=1$ which means that the central bank is indifferent with respect to the times the exchange rate falls below the threshold value.

Thirty-year paths with one year periods are simulated for the risky return and the exchange rate error term. The simulation is done using Monte Carlo sampling with 100 000 paths. The objective is evaluated against the simulated sample paths with the value of α fixed across time and ranging from 0 to 2 with an interval of 0.025. To confirm that the found optimum is indeed a global one, the evaluations were also done on a larger scale with α ranging from -50 to 50. The results are shown in Figure 2.

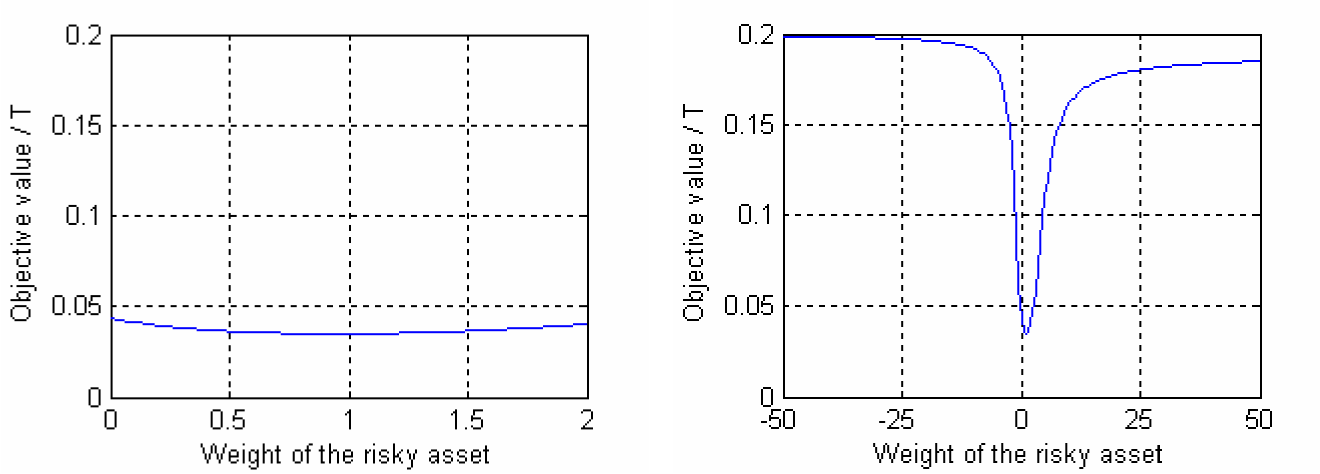
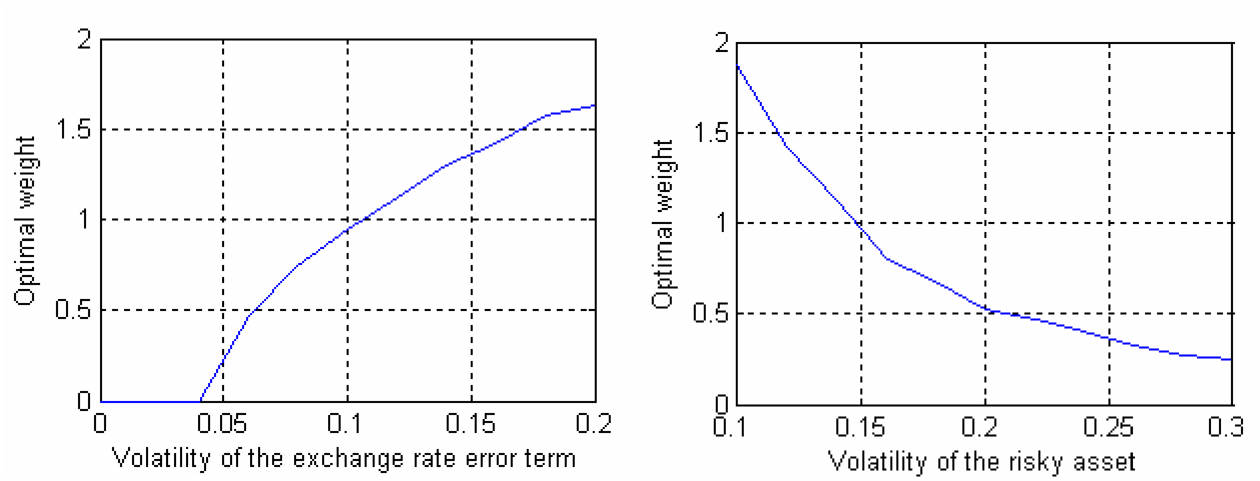


Figure 2 The simulated objective function in the multi-period case. The figure on the left shows the objective values with risky asset weights close to the optimum of 0.95, and the right figure shows the same objective function on a larger scale. Y-axis values represent the average probability of exchange rate falling below ρP.

The objective is divided by the number of periods to ease comparison with the single period case. This has the interpretation of an average probability for the exchange rate falling below the barrier. The optimal weight $\alpha=0.95$. In contrast to the one-period setting, this example shows significant gain from choosing the optimal instead of the risk-free portfolio: the objective value decreases by 19.6 %.

This case is further analyzed by examining the sensitivity of the optimal α with respect to different parameter values. The same 100 000 scenarios as above are used for the stochastic terms. More scenarios would be necessary to eliminate numerical inaccuracy which is apparent in some the figures. However, 100 000 sample paths are enough to reveal the general pattern of dependency.

First, the dependence of the optimal risky asset weight on the exchange rate volatility is studied. The optimal value is solved numerically in the same way as before for σ_e ranging from 0 to 0.2 with steps of 0.02. The results are shown in Figure 3.

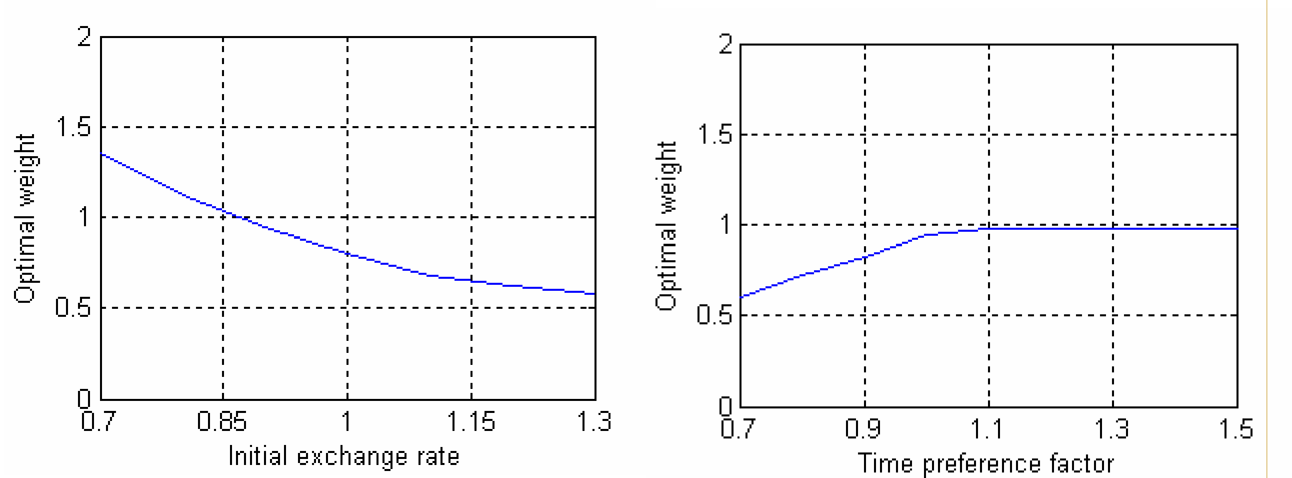


Figures 3 and 4 Dependence of optimal α on the volatility of the exchange rate error term (left) and the volatility of the risky asset (right)

The simulation shows that the need for risk-taking increases with the volatility of the exchange rate error term. The same conclusion was reached in the one-period case. Higher volatility of exchange rate increases both the occurrence of interventions and the magnitude of deviations from the critical barrier. Thus, higher expected wealth is required to hold the exchange rate above ρP. The sensitivity of the optimal solution seems to be slightly decreasing as a function of σ_e.

The sensitivity of the optimal solution on the risky asset volatility is analyzed for values of σ_r between 0.1 and 0.3 with steps of 0.02. The results are shown in Figure 4. The results show a clear negative dependence on risky asset volatility as expected - greater variability in the return of the risky asset makes it more undesirable. Sensitivity of the optimal solution seems to decrease as σ_r grows.

To assess the dependence of optimal α on the initial exchange rate the same analysis is performed with the values 0.7, 0.8, …, 1.3 for e_0. Figure 6 shows the results. The optimal risky asset weight decreases as a function of the initial exchange rate in a nearly linear manner. Higher initial exchange rate makes interventions less frequent and thus a smaller expected level of wealth is sufficient.



Figures 6 and 7 Dependence of the optimal α on the initial exchange rate (left) and on the time preference factor δ (right)

Figure 7 shows the results for the time preference factor δ with values 0.7, 0.8, …, 1.5. When $\delta<1$ the near future is emphasized. In this case, the optimal share of the risky asset decreases somewhat as δ decreases. This is due to the fact that if only the next couple of years are of interest, the expected growth of wealth is not relevant, since initial wealth alone is sufficient for three interventions.

When the value of δ is over 1 the distant future gets a bigger weight and the optimal weight of the risky asset increases slowly. Interestingly with values of δ over 1.1 the solution remains unchanged at slightly below 1. A likely explanation for this is that as δ increases the relative weight of the terminal probability increases strongly, and thus the solution converges to the solution of a problem where only the exchange rate at T=30 is of interest. Already at δ=1.5 with a 30-year horizon the weight of the terminal probability is half of all the other weights combined.

As a last test, the sensitivity of the solution to different time horizons is analyzed. Figure 8 shows the results of a simulation where the value of T ranged between 10 and 90 with an interval of 10 years. The same set of paths was used for the first 30 years. The results show an increasing trend in the optimal asset weight with respect to time horizon. The optimal weight increases faster for the first 30 years and then seems to be slowly converging towards a value slightly above 1. When the time horizon gets longer the near future starts to weigh less as all years are equally important in the objective function. To guarantee sufficient wealth in the long run, more weight should be given to the risky asset.

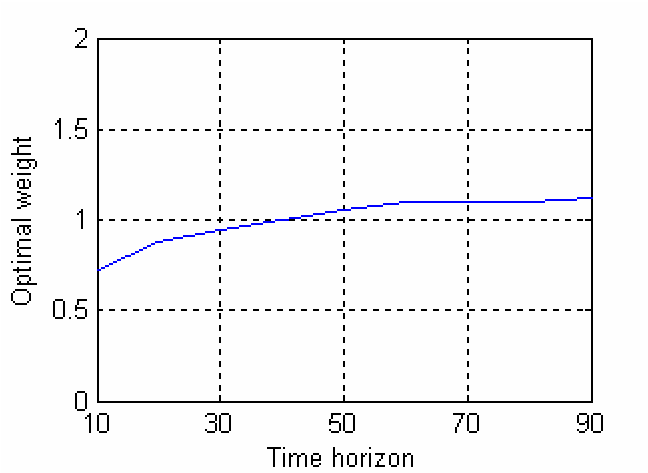


Figure 8 Dependence of optimal α on the time horizon T

5. Conclusion

This study presented a stylised model for central bank's foreign reserves management incorporating foreign exchange interventions explicitly. The analysis highlights the importance of the interplay between intervention policy and asset allocation decision.

A one-period formulation of the problem was first studied. The main motivation to analyze this simplified case is that it can be solved analytically. From the explicit solution it was concluded that the main determinants of asset allocation are the initial level of exchange rate relative to the exogenously determined equilibrium level, volatility of the exchange rate and the excess return and volatility of the risky asset.

The analytical solution gives valuable insight into the determinants of the asset allocation decision, but in the one-period problem the asset allocation was found to help only marginally in achieving the policy objective. In the more general multi-period setting an optimal asset allocation was found to substantially reduce the probability of adverse exchange rate levels. Given the specific assumptions made in this study we find a rather high share of the risky asset.

In future research, it might be useful to extend the analysis in a multi-asset setting with dynamic asset allocations (using e.g. stochastic optimization). Other useful extensions could be to model the size of the foreign exchange market and tie the effectiveness of the interventions to this total turnover. Another generalization could be to build models for the two countries' consumer price indices and tie the mean-reversion level of the exchange rate dynamically to the implied purchasing power parity. One interesting extension to the model could be the use of time-varying volatility, using a GARCH specification for example, since there is evidence that interventions also affect exchange rate volatility, not just the level.

The adequate amount of liquidity is a hard problem in the management of foreign reserves. An interesting subject for future research would be to study this question with the model presented in this paper by replacing the risky and risk-free assets by a liquid and an illiquid asset. Then the liquidity premium would be lost if the illiquid asset has to be sold before maturity to answer intervention needs. Non-linear transaction costs e.g. in a stochastic programming model could possibly be used to accomplish this.

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Appendix: Derivation of the optimal solution for the 1-stage problem

This annex derives the optimal solution for the 1-stage model described in Section 3. The error term in the exchange rate and the risky asset return are assumed to be jointly normally distributed. In the derivation, the risky asset return is denoted without the subscript 1 to simplify notation.

The problem is to minimize the following probability

$$prob(e_1 - \rho P < 0)$$

$$= prob((1-b)P + be_0 + \varepsilon_1 + \gamma A_1 - \rho P < 0)$$

$$= prob((1-b)P + be_0 + \varepsilon_1 + \gamma(1 + r^f + \alpha(r - r^f))W_0 - \rho P < 0)$$

$$= prob(\varepsilon_1 + r\alpha\gamma W_0 + P(1-\rho-b) + W_0\gamma\left(r^f - \alpha r^f + 1\right) + be_0 < 0)$$

If $\begin{pmatrix} r_1 \\ \varepsilon_1 \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_r \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_r^2 & \sigma_{re} \\ \sigma_{re} & \sigma_e^2 \end{pmatrix}\right)$ are jointly normal, then

$$e_1 - \rho P \;= \begin{pmatrix} \alpha\gamma W_0 & 1 \end{pmatrix}\begin{pmatrix} r \\ \varepsilon_1 \end{pmatrix} - \alpha\gamma W_0 r^f + P(1-\rho) + b(e_0 - P) + \gamma W_0(1 + r^f) \sim N(\mu, \sigma^2),$$

where the mean value is given by

$$\mu = \alpha\gamma W_0(\mu_r - r^f) + P(1-\rho) + b(e_0 - P) + \gamma W_0\left(1 + r^f\right)$$

and variance equals

$$\sigma^2 = \begin{pmatrix} \alpha\gamma W_0 & 1 \end{pmatrix}\begin{pmatrix} \sigma_r^2 & \sigma_{re} \\ \sigma_{re} & \sigma_e^2 \end{pmatrix}\begin{pmatrix} \alpha\gamma W_0 \\ 1 \end{pmatrix} = \alpha^2\gamma^2 W_0^2\sigma_r^2 + 2\alpha\gamma W_0\sigma_{re} + \sigma_e^2.$$

Thus

$$prob(e_1 - \rho P < 0) = \Phi(-\tfrac{\mu}{\sigma}) = \Phi\left(-\frac{\alpha\gamma W_0\mu_r + be_0 + P(1-\rho-b) + \gamma W_0\left(r^f - \alpha r^f + 1\right)}{\sqrt{\alpha^2\gamma^2 W_0^2\sigma_r^2 + 2\alpha\gamma W_0\sigma_{re} + \sigma_e^2}}\right) = \Phi\left(g(\alpha)\right),$$

where the argument of the cumulative distribution function has been denoted by *g(α)*. Φ is strictly increasing and is thus minimized when its argument *g* is minimized. To find the minimum, a derivative of *g* with respect to α is taken, which has to equal zero at a minimum

$$g'(a) = \frac{\gamma W_0}{\left(\alpha^2\gamma^2 W_0^2\sigma_r^2 + 2\alpha\gamma W_0\sigma_{re} + \sigma_e^2\right)^{\frac{3}{2}}}\Big[\sigma_{re}\left(P(1-\rho) + b(e_0 - P) + \gamma(1 + r^f - \alpha(\mu_r - r^f))W_0\right) -$$

$$\sigma_e^2(\mu_r - r^f) + \alpha\sigma_r^2\gamma\left(P(1-\rho) + b(e_0 - P) + \gamma(1 + r^f)W_0\right)W_0\Big]. \qquad (*)$$

This expression is zero when the expression in the brackets vanishes, i.e. when α equals

$$\overline{\alpha} = \frac{\sigma_e^2(\mu_r - r^f) - \sigma_{re}\left(P(1-\rho) + b(e_0 - P) + (1 + r^f)\gamma W_0\right)}{-(\mu_r - r^f)\gamma W_0\sigma_{re} + \sigma_r^2 W_0\gamma\left(P(1-\rho) + b(e_0 - P) + (1 + r^f)\gamma W_0\right)} \;= -(\gamma W_0)^{-1}\frac{\sigma_e^2(\mu_r - r^f) - \sigma_{re}\left(P(1-\rho) + b(e_0 - P) + (1 + r^f)\gamma W_0\right)}{\sigma_{re}(\mu_r - r^f) - \sigma_r^2\left(P(1-\rho) + b(e_0 - P) + (1 + r^f)\gamma W_0\right)},$$

assuming that the denominator

$$\sigma_{re}(\mu_r - r^f) - \sigma_r^2\left(P(1-\rho) + b(e_0 - P) + \gamma(1 + r^f)W_0\right) \qquad (**)$$

is nonzero. This holds when $\sigma_{re} \leq 0$ and $e_0 \geq \rho P$, which can be seen by adding and subtracting $P\rho b$ from the latter term to get

$$\sigma_{re}(\mu_r - r^f) - \sigma_r^2\left(P(1-\rho) + be_0 - P(1-\rho)b - P\rho b + \gamma(1+r^f)W_0\right)$$
$$= \sigma_{re}(\mu_r - r^f) - \sigma_r^2\left(P(1-\rho)(1-b) + b(e_0 - P\rho) + \gamma(1+r^f)W_0\right)$$

which is always negative because $b<1$ and $\rho<1$.

To verify that the found extremum is actually a minimum, we collect the terms containing α together from the expression (*) for the derivative:

$$\alpha\left(-\sigma_{re}\gamma(\mu_r - r^f)W_0 + \sigma_r^2\gamma\left(P(1-\rho) + b(e_0 - P) + \gamma(1+r^f)W_0\right)W_0\right).$$

The multiplier of α equals minus the expression (**) which was under the assumptions $\sigma_{re} \leq 0$ and $e_0 \geq \rho P$ proved to be negative. What has been shown is that

$$g'(\alpha) = f(\alpha)(c_1\alpha + c_2)$$

where c_1 and c_2 are constants (i.e. do not depend on α), c_1 is positive and f is positive for all α. In addition,

$$g'(\overline{\alpha}) = 0 \Leftrightarrow c_1\overline{\alpha} + c_2 = 0.$$

To show that an extremum is a global minimum, it is enough to prove that $g'(\overline{\alpha} - \Delta) < 0$ and $g'(\overline{\alpha} + \Delta) > 0$ for all $\Delta > 0$. But this is immediate:

$$g'(\overline{\alpha} + \Delta) = f(\overline{\alpha} + \Delta)(c_1(\overline{\alpha} + \Delta) + c_2) = f(\overline{\alpha} + \Delta)c_1\Delta > 0,$$

$$g'(\overline{\alpha} - \Delta) = f(\overline{\alpha} - \Delta)(c_1(\overline{\alpha} - \Delta) + c_2) = -f(\overline{\alpha} - \Delta)c_1\Delta < 0.$$

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