EUROPEAN CENTRAL BANK

WORKING PAPER SERIES



WORKING PAPER NO. 137

EQUILIBRIUM BIDDING IN THE EUROSYSTEM'S OPEN MARKET OPERATIONS

BY ULRICH BINDSEIL

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BY ULRICH BINDSEIL*

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ISSN 1561-0810

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Abstract

Open market operations play a key role in allocating central bank funds to the banking system and thereby to steer short-term interest rates in line with the stance of monetary policy. This note presents some elements of a theory of bidding in central bank tenders in a framework such as the one of the Eurosystem. The ECB has so far used fixed rate tenders and a variant of the variable rate tender, which may be similar to a fixed rate tender depending on market circumstances. In doing so, it faced consecutively an "under-" and an "overbidding" issue. The tools developed in this note to understand the bidding behavior of banks in these operations allow revisiting these phenomena and the more general question of the optimal tender procedure and allotment policy.

JEL classifications: D84, E43, E52 Keywords: open market operations, tender procedures, central bank liquidity management

Non-technical summary

Open market operations play a key role in allocating central bank funds to banks in modern financial systems. For instance, the European Central Bank has, since the start of Stage Three, allocated per year funds worth more than EUR 3 Trillion through its regular open market operations. Even though the implementation of monetary policy by the ECB has so far worked rather smoothly, the design of the open market operations conducted by the ECB has also been questioned sometimes in relation to the phenomena of "overbidding" and "underbidding". "Overbidding" refers to extremely high bid volumes submitted to fixed rate tenders, implying, ceteris paribus, extremely low allotment ratios. "Underbidding" refers to the lack of bids in a fixed rate tender, such that the central bank cannot allot the liquidity actually needed by banks to fulfill smoothly their reserve requirements. While overbidding" still has found mainly the interest of market players and of the financial press. This note develops tools, which will allow to analyze both over- and underbidding and to draw some tentative policy conclusions.

The paper first recalls the Eurosystem's experience with over- and underbidding and presents some of the interpretations of the academic literature and the financial market press. Then, a general model of a money market and of a tender procedure in the case of a one-day maintenance period is introduced. The one-day model will be sufficient to work out the main equilibrium conditions in the absence of expectations of rate changes. At the same time, it allows to investigate how under- and overbidding may be triggered in such an environment by the liquidity policy of the central bank. While the modeling focuses mainly on the fixed rate tender (which includes the case of variable rate tenders with a minimum bid rate under rate cut expectations), the variable rate tender is included in the analysis in a very simple form to allow a comparative assessment. Among the fixed rate tenders, the 100% allotment and the discretionary allotment variant are distinguished, whereby the latter is further specified by the allotment function followed by the central bank. In terms of central

bank preferences, three alternative or complementary aspects are considered, namely the stability of interest rates within the reserve maintenance period, the smoothness of the reserve fulfillment path, and the bidding costs. After presenting the model in the framework of a one day maintenance period, the paper focuses on how rate change expectations trigger over- or underbidding, which requires the setting up of a model with two days per maintenance period.

The following main conclusions of the paper can be highlighted: Firstly, the failure of banks to coordinate their bidding perfectly and the impossibility to make a perfect use even of published autonomous factor forecasts, as well as the costs attached to bidding appear as necessary ingredients of a sensible model of the bidding behavior of banks in central bank operations. Secondly, in the absence of rate change expectations, both the variable and the discretionary fixed rate tender perform well. The 100% allotment rule has the disadvantage of implying additional interest rate volatility and noise in the reserve fulfillment path. Thirdly, it appears that in the absence of rate change expectations, in case of the use of the fixed rate tender procedure and a large free bidding potential, the liquidity management of the central bank should be neutral in the sense that, the central bank should target neutral liquidity conditions at the end of the reserve maintenance period. Fourthly, under conditions of rate change expectations, it appears that the fixed rate tender tends to have, in the model proposed here, some specific disadvantages relative to the variable rate tender. Fifthly, if discretionary fixed rate tenders are chosen under rate hike expectations, the model suggests that it is difficult to address overbidding through excess liquidity. Six, under discretionary fixed rate tenders and rate cut expectations, a small "bail-out co-efficient" will ensure that underbidding will tend to be relatively limited. In sum, it appears that the fixed rate tender is well suited to conditions of stable interest rates, but that, in the current Eurosystem framework, they may indeed cause some noise in an environment of strong rate change expectations, whereby the central bank then has only limited, if any, possibilities of stabilization through a specific liquidity policy.

In assessing these results, it should be noted that the role of open market operations to signal the monetary policy stance, and the related comparative advantages of different tender procedures, goes beyond the scope of this paper.

1. Introduction

Open market operations play a key role in allocating central bank funds to banks in modern financial systems. For instance, the European Central Bank has, since the start of Stage Three, allocated per year funds worth more than EUR 3 Trillion through its regular open market operations. The open market operations of the ECB are hence by far the largest tenders in the world ever conducted in terms of yearly total volumes, suggesting that their efficiency should be of highest interest. In its first 18 months, the ECB specified its weekly tenders, the "main refinancing operations" (MROs) as fixed rate tenders. Afterwards, they were defined as variable rate tenders with a minimum bid rate, which are actually close to fixed rate tenders if markets expect declining central bank interest rates. The tendering procedures for fixed and variable rate tenders of the Eurosystem are described precisely in ECB [2000].

Even though the implementation of monetary policy by the ECB has so far worked rather smoothly, the design of the open market operations conducted by the ECB has also been questioned sometimes by financial market participants and academic economists in relation to the phenomena of "overbidding" and "underbidding". "Overbidding" refers to extremely high bid volumes submitted to fixed rate tenders, implying, ceteris paribus, extremely low allotment ratios. This phenomenon was observed in the case of the ECB in the second half of 1999 and the first half of 2000, with bids surpassing the allotment amount by up to a factor of 100. "Underbidding" refers to the lack of bids in a fixed rate tender, such that the central bank cannot allot the liquidity actually needed by banks to fulfill smoothly their reserve requirements. This phenomenon was briefly experienced once in April 1999 and four times in 2001. While the phenomenon of overbidding has been recently analyzed relatively extensively in the academic literature, "underbidding" still has found mainly the interest of market players and of the financial press. However, as will be argued in more detail, the literature on both phenomena has lacked so far an appropriate model of the bidding behavior of banks in the Eurosystem's tenders in an environment of rate change expectations. Therefore, this note develops tools, which will allow to analyze more carefully the phenomena of over- and underbidding and to draw more subtle policy conclusions.

The illustrative evidence presented in the paper, as well as the proposed model, will focus on the case of the Eurosystem's main refinancing operations (MROs), without any explicit applications to other central banks. According to ECB [2000], MROs are "the most important open market operations conducted by the Eurosystem, playing a pivotal role in pursuing the aims of steering interest rates, managing the liquidity situation in the market, and signaling

the stance of monetary policy". Although the last of the three mentioned functions is certainly not the least important one, this paper will mainly contribute to the understanding of the first two aspects, leaving the third to complementary research.²

Although this paper focuses on the case of the Eurosystem's MROs, in principle, the results obtained should also hold for different open market operations, such as for instance outright operations, and more generally, for operational frameworks different in other respects from the one of the Eurosystem. The ECB's tenders may however be considered as an ideal object for studying multiple good auctions since they involve a high number of bidders (several hundreds), are conducted regularly over long periods of time without any institutional changes, and the aggregate data is well recorded and made publicly available by the ECB. In contrast, open market operations of other central banks are typically conducted with much fewer banks, are less standardized, and are treated as more confidential due to the small number of participants. Nevertheless, this paper will concentrate on proposing theoretical tools, and will leave it to further work to apply them to the evidence produced so far by Stage Three of EMU.

The analytical literature with regard to central bank tender procedures is very recent and so far mainly focused on the fixed rate tender in the ECB's main refinancing operations conducted between January 1999 and June 2000. No literature on the US experience exists, possibly due to the non-availability of data. The research on the ECB's fixed rate tenders was triggered by the "overbidding" phenomenon. Bindseil and Mercier [1999] provide a simple model of the ECB's fixed rate tenders and distinguish different cases with regard to the requirements of collateral to cover tender bids or allotments. Nautz and Oechsler [1999] model specifically the overbidding phenomenon experienced in the fixed rate tenders. Breitung and Nautz [2000] revisit this question using individual bidder data from Germany. Ayuso and Repullo [2000], [2001] also focus on the overbidding phenomenon and argue that the ECB had an asymmetric preference function under which it systematically tended to provide too little liquidity, causing the overbidding. Finally, Valimaki [2001] presents an equilibrium model of the determination in the interbank market for overnight liquidity when the central bank uses fixed rate tenders.³ The two papers that come nearest to the present one are Ayuso and Repullo [2000] and Valimaki [2001]. The main difference between these papers and the present one lies in the role of assumptions regarding the cost of bidding and the accuracy at which the banks can match liquidity needs through their bids. It is argued here that costly bidding and the noise in the bid amount relative to liquidity needs are

² With regard to the signalling power of alternative tender procedures, it appears that the fixed rate tenders are likely to be preferable compared to pure variable rate tenders. In adopting a variable rate tender with minimum bid rate, the ECB combined the signalling capacity of the fixed rate tender with its resistency against overbidding. See also Issing et al. [2001, 120].

essential elements of a model of bidding behavior. Finally, the exact specification of the sequence of events differs between the different models.

A further line of literature of relevance for the present paper is the one describing the specific economic environment of central bank open market operations, i.e. especially the factors affecting the value of the good that is auctioned (reserves with the central bank). The demand and supply factors affecting the scarcity of the good in question, reserves of banks held with the central bank, are analyzed for instance by Hamilton [1998] for the US and Bindseil and Seitz [2001], Hartmann, Manna and Manzanares [2001] or Wuertz [2001] for the euro area. More theoretical issues relevant for the market for reserves with the central bank are discussed, among many others by Hamilton [1996], Bartolini, Bertola and Prati [1998], and Bindseil [2001]. From, this literature, the following crucial aspects of the market for reserves may be highlighted: (1) In case of a high degree of efficiency, sufficiently high reserve requirements and averaging, the overnight interest rates expected at any moment in time for the remainder of the maintenance period corresponds to the present overnight rate, i.e. the overnight interest rate follows a martingale. (2) The overnight interest rate at any moment in time should correspond to a weighted average of the standing facility rates, whereby the weights reflect the probabilities of an aggregate shortness or excess of reserves compared to reserve requirements at the end of the reserve maintenance period. (3) Hence, expectations with regard to standing facility rate changes and liquidity until the end of the reserve maintenance period will determine at any moment the overnight rate.

The rest of the paper is organized as follows: Section 2 briefly recalls the Eurosystem's experience with over- and underbidding and presents some of the interpretations of the literature. Section 3 introduces the general model of a money market and of a tender procedure in the case of a one day maintenance period. The one-day model will be sufficient to work out the main equilibrium conditions in the absence of expectations of rate changes. At the same time, it allows to investigate how under- and overbidding may be triggered in such an environment by the liquidity policy of the central bank. Section 4 focuses on how rate change expectations trigger over- or underbidding. This requires the setting up of a model with two days per maintenance period. Finally, section 5 concludes on what tender procedure and allotment policy the central bank should choose under different environments and preferences and comes back to the basic question of the nature of under- and overbidding.

³ Nyborg and Strebulaev [2001] treat a different fixed rate tender topic, namely the possibility of bidders to corner the market through specific bidding strategies.

2. The Eurosystem's experience with (quasi-) fixed rate tenders and interpretations given in the literature

As will be justified in depth in this paper, variable rate tenders with minimum bid rate, such as applied by the ECB from the end of June 2000 onwards, are, in the Eurosystem framework, quasi-equivalent to fixed rate tenders if interest rate cut expectations prevail (as also explained in ECB [2001b, 57]). In this section, the experience of the ECB with its (quasi) fixed rate tenders (i.e. including the variable rate tender with minimum bid rate under rate cut expectations) is summarized briefly, and some assessments by financial market newspapers and academics is provided.

2.1. The overbidding episode

The overbidding episode has been described on various occasions in the literature. The ECB itself summarized its experience after announcing the switch to the variable rate tender (ECB [2000, 37]) as follows:

"As highlighted in the press release published after the meeting of the Governing Council on 8 June [2000], the switch to variable rate tenders was a response to the severe overbidding which had developed in the context of the fixed rate tender procedure. In the last two main refinancing operations executed prior to the switch to the variable rate tender, the allotment ratio was below 1%. The strong rise in bids in the first half of 2000 was due to the fact that, during most of that period, there were market expectations of interest rate hikes and short term money market rates were significantly above the main refinancing rate. This made it attractive for banks to bid for large amounts of liquidity from the central bank."

Chart 1 draws the allotment ratios in fixed rate tenders and the spread between the fixed tender rate and the 2 weeks market deposit rate. While the precise dynamic interaction between the two series is complex, the following patterns clearly emerge: Firstly, allotment ratios tend to fall in periods of a high positive differential between the market and the tender rate. Secondly, allotment ratios appeared to remain fairly free of a trend in periods in which the spreads were clearly below 5 basis point. Finally, when the allotment ratio had fallen to rather low levels due to a high interest rate spread over a longer period, a fall of the spread to moderate level does not lead to unchanged bidding, but to a sudden rebound of the allotment ratio. This could be observed regularly after interest rate hike decisions were taken (namely on 10 November 1999, on 9 February, 22 March, and 15 June 2000).



Chart 1 Allotment ratios (left hand scale, bar chart) and the EONIA--MRO spread (right hand scale, line chart) during the use of the fixed rate tender procedure by the ECB

Apparently, banks remained hesitant to fully exploit the arbitrage opportunities, and to increase their bids really aggressively at once. A closer look at the overbidding time series suggests that both an element of adaptive behavior, and some aversion against excessive bidding, reflecting the perceived costs of overbidding, explains best the observed dynamics.

In the quotation given above, the ECB did not further spell out why the very low allotment ratios were regarded as a problem. However, it seems clear that tendering with extreme overbidding has to be regarded as a *special type of allocation of funds through queuing, instead of an allocation through a pure price mechanism.* Queuing is known to be a less efficient allocation mechanism, compared to the price mechanism, since it works through the using up of resources in the form of transaction costs. Queuing always occurs when a price is kept at a level which is below the market value of the good sold, at least for the quantity that is offered. The queuing equilibrium is characterized by a marginal condition under which the marginal cost of queuing exactly fills the gap between the fixed price at which the good is offered and the market price.

The queuing costs implied by overbidding are special if compared to classical cases of queuing in two respects: first, the relevant queuing cost function seems to be unstable, i.e. over time, bidders can lower their costs of overbidding through certain investments. Secondly, the queuing costs take to a large degree the form of risk taking, which is less tangible than other costs. How do we have to imagine the nature and dynamics of overbidding costs? At the start, i.e. with moderate overbidding, costs of bidding should be negligible as long as banks own enough cheap, unused collateral (e.g. non-Jumbo "Pfandbriefe" for which no repo market exists, or loans – see ECB [2001a]) to cover their bid in case they would obtain the full allotment. If they bid for more, they may envisage to use in

case of full allotment, more expensive, so-called "general" collateral for which other uses exist (e.g. Bunds, Jumbo-Pfandbriefe). If they bid even beyond that, they could envisage to get the collateral in the market after the allotment decision is made public and before the settlement of the operation, which however certainly implies further cost. However, in addition, banks may start to find their bidding risky since full allotment would imply that they receive *so much cash* that they could have problems to place it in the market due to credit limits. Finally, if the overbidding becomes extreme, banks are well aware that they would be unable to get enough collateral in the market in time⁴, and that they are hence taking a speculative stance of which it can even be doubted that it is legally sound. Risk managers in banks typically attach a high cost to such strategies, and it is usual in banks that a cost is attached to risk taking, such that the incentives of risk takers are adequately influenced.

One may summarize that the marginal cost of bidding should increase with the extent of overbidding. This will be modeled in the present paper in the simplest way possible, namely by assuming that bidding is free up to a certain amount, but then exhibits increasing marginal costs. This will be sufficient to derive relatively simple and intuitive behavioral equilibria between the central bank and the market place. A dynamic adaptation of the cost of bidding curve is not modeled explicitly, but is an obvious extension within the proposed model.

Overbidding has found noticeable interest by academics. The following papers on the topic should be mentioned. The first authors assessing the overbidding experience of the ECB were Nautz and Oechsler, who came in October 1999 to the conclusion that (p.18-19) "the auction rules are flawed since they encourage banks to increasingly exaggerate their demand for reserves... Considering the vanishing quota the ECB's repo auctions are about to become a farce... in view of these problems our suggestion for the ECB would be to employ price discriminating variable rate tenders...". Erhard [2000] provides experimental evidence which is supposed to show that the ECB's fixed rate tender system unavoidably leads to overbidding and that "even accommodate policy cannot prevent increasing exaggeration in the bids", hence confirming the result of Nautz and Oechsler [1999]. Ayuso and Repullo [2000], [2001] concentrate on demonstrating that the ECB had an asymmetric objective function, which made it provide systematically too little liquidity, hence creating the overbidding problem. Ayuso and Repullo [2000] and Nautz and Oechsler [1999] both share the view that allotment ratios in fixed rate tenders will either be indeterminate (or drawn from a continuum of equilibria) or will tend to infinity (or to a limit). Nautz and Oechsler [1999] make use of adaptive expectations building to reconcile the indeterminacy with the evidence. Martinez Resano [2001] argues that "a perverse set of circumstances caused the extreme overbidding dynamics: first, strong interest rate raise expectations; second, destabilising

⁴ Indeed, in the hot overbidding phase in the spring of 2000, banks bid several times more than the total amount of eligible collateral for Eurosystem operations.

liquidity shocks at the start of maintenance periods; and finally, the structural characteristics of fixed rate tenders." Nautz and Breitung [2001] take a more moderate approach by admitting, on the basis of an analysis of individual bidding data, that rate hike expectations should also have played a role in triggering overbidding.

The current paper will come to rather different conclusions with regard to the nature of dynamic overbidding, namely that this phenomenon should normally (i.e. in the case of a central bank with a reasonable liquidity policy) be linked to rate hike expectations, and that indeed this alone was the reason for the dynamic overbidding in the case of the Eurosystem. In so far, the analysis will confirm the ECB's position with regard to the sources of overbidding (see also ECB [2001b, 52]).

2.2 The underbidding experience

As summarized in the following table, underbidding in MROs has occurred so far five times in the euro area.

Table 1. The five cases of underbloding (all amounts in billion of euro)				
(a) Date of	(b) Bid	(c) Allotment volume	(d) Reserve	(e) Net recourse to the
settlement of	volume (=	that would have allowed	fulfillment deficit	marginal lending
the MRO	actual	a smooth fulfillment of	that accumulated	facility before end of
	allotment)	reserve requirements	until next MRO*	reserve maintenance
				period
07/04/99	67	84	113	11
14/02/01	65	88	145	72
11/04/01	25	53	232	61
10/10/01	60	79	118	25
07/11/01	38	66	168	-3

Table 1: The five cases of underbidding (all amounts in billion of euro)

* Excluding the liquidity effect of any recourse to standing facilities

The shortfall of bids relative to the neutral allotment amount (the difference between column b and c) varied between EUR 28 billion (11 April and 7 November 2001) and EUR 17 billion (7 April 1999). These shortfalls implied the accumulation of a deficit in the fulfillment of reserve requirements of up to EUR 232 billion (column d; assuming that no recourse to standing facility would have taken place). The costs in terms of net recourse to the marginal lending facility before the end of the reserve maintenance period (column e) varied substantially according to whether or not the ECB decided to increase the allotment amounts in the subsequent tenders to allow banks to catch up with the fulfillment of their required reserves before the end of the reserve maintenance period on the 23rd of each month. While the ECB fully rescued the market in November 2001, the "bail out" ⁵ was especially limited in

⁵ The use of the term "bail-out" in the present paper should of course be distinguished from the one made in other contexts, such as the "no-bail-out" clause in the Maastricht treaty, or the bailing out of an insolvent credit institution.

February and April, when the "bail out coefficient", which may be defined as 1-(e)/(d), was only 50% and 74%, respectively.

The underbidding episodes have been discussed by the financial market press, but less so far by academics. The following excerpts provide an overview of market opinions:

"The publication of the tender result [of 10 April 2001] came as a bad surprise for money market participants. Despite the fact that traders had expected underbidding beforehand, the extent of it triggered growing astonishment, and later on panic demand for funds.... Traders of banks who had submitted bids to the tender suggested that the ECB should remain tough, as in February, and should not rescue the market through a quick tender. This would be the only way to teach speculators an orderly bidding behavior. Traders that had remained absent from the tender expressed their dissatisfaction regarding the current regime in the money market. Since the minimum bid rate would avoid the possibility to submit bids at low rates, similar incidents would happen again and again in the coming weeks." (Boersenzeitung of 11 April 2001, p.2, own translation)

The more critical line of the traders that lost money is adopted in the following comment:

"The ECB and many money market traders blame speculation for the repeated disaster: the liquidity managers of banks should ask for what they need at the auctions, instead of speculating for a rate cut. Such accusations are based on a moral argument, and are hence not appropriate in a market context, besides the fact that they are not even correct... In a phase of rate cut expectations, there is no equilibrium in which there are regularly enough bids. The problem will hence return again and again, as long as rate cut expectations remain... The steering problems could be solved in the most elegant way by getting rid of the minimum bid rate. Another approach would be to hold auctions more regularly. Professor Axel Ockenfels, specialist on auction design, sees the problem in the lack of knowledge of the bidders regarding the bidding behavior of their colleagues. The bankers have to submit bids, which can no longer be corrected. Often, it is then revealed that the bids were based on wrong expectations regarding the behavior of the others." (Financial Times Deutschland, by N. Haering, April 2001 "Im Zickzack durch den Geldmarkt", own translation).

Apparently, there are two schools of interpretation in the market: one that feels that the central bank can cope with the phenomenon of underbidding by choosing the appropriate liquidity management strategy (i.e. mainly not to bail-out the market after underbidding occurred), while the others feel that a minimum bid rate, or, equivalently a fixed rate tender, does not allow any reasonable equilibrium in an environment of rate cut expectations. The model exposed in section 4 will allow to revisit this question and to provide a conditional answer. After several months of rate cut expectations without renewed underbidding, the underbidding case of 9 October 2001 was again commented on the same lines. The Boersenzeitung had noted in the morning of 9 October that under the prevailing circumstances, underbidding was likely, although "also this time, one should not expect the ECB to help market participants out of the trouble, if liquidity would then be short". On 10 October, the Boersenzeitung commented:

"There we have the mess: apparently assuming that the ECB would cut rates next Thursday, many banks have stayed away from the ECB's main refinancing operation... The market reacted quickly to the news: overnight money increased by 30 basis points... The only rescue would be an ECB quick tender, but since the central bank did neither come in February nor April to help the market, there was little hope among money market dealers... Even if the ECB would lower the minimum bid rate by 25 basis points, the effect on the money markets could be minor, if liquidity is missing."

Two elements are especially noteworthy from this last comment: first, the ECB had build up a reputation to not bail out banks in case of insufficient bidding, keeping the overnight rate high. Secondly, market players noted that the effect of tight liquidity on rates would outweigh the rate cut even if the latter would actually be implemented. As Table 1 above indicated, the ECB however accommodated the effects of the underbidding in November 2001. This was clear to the market after the allotment decision in the next MRO, i.e. on 13 November 2001, and it is again worth looking at the Boersenzeitung's reporting of market views on that day:

"Many traders had expected, that the ECB would not fully remove the accumulated lack of liquidity, in order to express its dissatisfaction with the previous week's "bidder strike" of the banks. Markets were correspondingly relieved... However, some traders also expressed criticism regarding the high liquidity injection: with its U-turn, the ECB rewarded especially those banks which were, through their absence from the bidding in the previous week's tender, responsible for the miserable liquidity conditions."

Also reflecting the articles quoted above, an important feature of the model proposed will be the problem of bidders, mentioned in the first two articles quoted, to make effective use of the information on liquidity needs and to coordinate their bidding behavior. The following tentative explanations may be given for the lack of ability to bring the aggregate bids in line with liquidity needs. First, money market traders usually act in a way that can be regarded as mix of optimization and rule driven, bounded rationality type of behavior. When the total amount of bids submitted by others matters for the own optimal bid, which is usually the case in central bank tenders, then the equilibrium is rather complex to model. In highly complex environments, the combination of optimization behavior and bounded rationality may lead to a rather unpredictable result.⁶ Secondly, the individual bank, when deciding on its bid amount, will estimate its liquidity needs on the basis of the payment flows that are known to occur within the bank (e.g. issuance of bank paper, estimate of customer related flows

⁶ Experimental evidence generally confirms the existence of co-ordination failures even in relatively simple environments. The issue of co-ordination failure in co-ordination games is discussed for instance in Van Huyk et al. [1990] who argue, along with other authors, that experimental evidence confirms the likelihood of inefficient outcomes in complex dynamic situations even if deviating from the co-ordination equilibrium is not a useful strategy. The experimental evidence presented by Erhard [2000] regarding a game reproducing elements of the Eurosystem's fixed rate tenders confirms that the evolution of aggregate bidding behaviour was rather different in different experiments, suggesting that

to/from other banks, estimation of banknotes withdrawn by customers), and on the basis of the autonomous factor forecast of the central bank. The signals sent by the two sources are correlated to some extent, but not perfectly. Indeed, it is not perfectly known in how far the individual estimates are related each time with the central bank forecast, e.g. whether a high published forecast is due to a high anticipated circulation of banknotes, which is also anticipated by the bank internally, and whether there is hence no additional information in the central bank forecast, or whether the high forecast of the central bank is due to something else, such as a special operation of a central bank with a Government, which is not known by any bank from its internal projections. The signal extraction errors related to this phenomenon will be correlated across banks, and hence, the resulting noise in the bidding will not be reduced when the share of each bidder decreases.

3. A stylized model of aggregate bidding behavior: a one-day maintenance period without rate change expectations

In this section, a simple model of a money market and a central bank tender operation will be presented which will allow to discuss, under the assumption of absence of rate change expectations, the equilibrium aggregate bidding behavior of banks. The model used here is derived from the model used in Bindseil [2001]. However, the signal extraction problem with regard to autonomous factors and central bank liquidity targets is made trivial as it is now assumed that liquidity forecasts are always published. However, it is instead assumed that there is some uncertainty in the liquidity needs that just cannot be taken into account by banks when they submit their bids for the reasons exposed in the previous section. This uncertainty is assumed to be resolved with the publication of the tender allotment amount. The modeled reserve maintenance period consists in this section of only one day. The following sequence of 6 events within the reserve maintenance period is assumed: (1) The reserve maintenance period begins with the opening of the settlement accounts of banks with the central bank. At the moment of the opening of the accounts, the funds held on the current accounts are still determined by the previous maintenance period's open market operation. However, all outstanding open market operations mature on the same day. (2) The open market operation takes place. The banks submit their bids, and the central bank takes its allotment decision on the basis of its forecast of liquidity needs and possibly its liquidity target. The allotment amount may be restricted by the available bids. The allotment decision is made public. The operation is settled. (3) The interbank market session takes place and a market clearing overnight rate i_1 is determined. (4) The realization of the autonomous liquidity factor shock takes place. (5) Finally, the banks take recourse to

random elements play an important role and hence make total bid amounts to some degree unpredictable.

standing facilities to cover any liquidity imbalance. This recourse is purely mechanic, i.e. it fills the gap between the counterparties reserves and reserve requirements (set to be zero). The model assumes a perfect interbank market and homogenous banks, such that either all banks will have recourse to the marginal lending facility, or all will have recourse to the deposit facility, but there is no simultaneous recourse to both facilities by different banks. (6) The reserve maintenance period ends. The sequence of events is also summarized in the following chart.

Chart 2: the sequence of events in the one day maintenance period case:



Reserve requirements and the demand for excess reserves are zero, such that the banks will target zero balances on their account on the points in time relevant for the fulfillment of reserve requirements. We assume that there is only one such a point in time in the reserve maintenance period, which is its very end (such that our model may be interpreted as a oneday maintenance period model). The autonomous liquidity factors are assumed, for the sake of simplicity, to be white noise with a structural constant, i.e. $a = A + \mathcal{E} + \eta$, with $A \in \mathfrak{R}_{>0}$ a constant and \mathcal{E}, η being normal distributed random variables with expected value zero and variances $\sigma_{\varepsilon}^2, \sigma_{\eta}^2$. It is assumed that the structural liquidity deficit of the banking system, A, is large, such that the probability that a < 0 is negligible. The central bank is assumed to have, as the market, no prior information on η . However, it perfectly anticipates ϵ , which is also revealed to the market right together with the allotment decision. Note that ϵ is designed in a way that it could also be interpreted as capturing shocks stemming from the imperfect co-ordination of bidding. Apart from the open market operation, there is one other type of monetary policy instrument available, namely standing facilities to which banks can have unlimited access, however at a penalty rate. For the sake of simplicity, the rate of the deposit facility is set to zero and the rate of the marginal lending facility is set to one, such that overnight rates always fluctuate in the unity space. With regard to the open market operation, it is assumed that in case of a fixed rate tender, the tender rate corresponds always to the mid point of the corridor, i.e. r = 0.5.⁷ It is assumed that when deciding on the allotment volume in its open market operation, the central bank takes into account its

autonomous factor forecasts and the liquidity surplus or deficit it would like to see at the end of the maintenance period. This liquidity target is denoted in the following by γ . Counterparties are assumed to know this target. Formally, the intended open market operation volume is $A + \varepsilon + \gamma$. The actual allotment volume corresponds to the intended one if the total volume of bids, D, allows this, i.e. $D > A + \varepsilon + \gamma \Rightarrow m = A + \varepsilon + \gamma$. Otherwise, it corresponds to the bids, i.e. m = D. Alternatively, the central bank may adopt the so-called "100% allotment rule" where it simply always allots what the banks request for. Finally, the central bank may offer a variable rate tender. It is assumed that under a variable rate tender, there will always be sufficient bids, at least at low rates, such as to avoid the underbidding problem. In fact, this is the major property associated to variable rate tenders in the present paper.⁸ Marginal rates are assumed to follow market rates, whereby the central bank is assumed to care primarily about quantities in its allotment decisions, in the same way as it does in its fixed rate tender. The following table summarizes all the strategies assumed to be available to the central bank.

Table 2: tender procedures and allotment strategies in a one-day maintenance period

	change expectations
100% allotment (FRT)	m = D
Discretionary fixed rate tender	$m = \min(A + \varepsilon + \gamma, D)$
Variable rate tender (purely quantity oriented)	$m = A + \mathcal{E} + \gamma$

The **bidders** are assumed to be homogenous in all respects. It is further assumed, in line with the case of the euro area, that there is a high number of bidders, such that bidders operate under full competition. When submitting bids, two types of costs could potentially play a role: **costs of submitting bids**, and costs of obtaining an allotment. In the previous section, the assumption of positive bidding costs was motivated. Costs of allotment are obviously linked to the cost of collateral. A distinction between both types of costs is certainly required if one attempts to calibrate a model of the bidding behavior with the Eurosystem data. However, for the sake of shortness, allotment costs will be ignored here and only a rather simple functional form, which is in line with the considerations made in the previous section, will be assumed for the bidding costs. Denote by $C(D):[0,\infty[\rightarrow [0,\infty[$ the cost of bidding that the banking system faces when intending to submit a total bid of D. It is assumed that C(D)=0 for $D \le \widetilde{D} \in \Re_{\ge 0}$ and C(D) > 0 for $D > \widetilde{D}$ with $\partial C/\partial D > 0; \partial^2 C/\partial^2 D > 0$. Hence, the marginal cost of bidding are zero until a bid volume

⁷ An asymmetric position of the tender within the corridor could also be analysed in the present model. However, this issue is omitted here for the sake of shortness.

of \tilde{D} is reached, afterwards, they are positive and increasing. Note that the latter property should follow from the decline of allotment ratios if the bidding costs are for instance related to the potential costs of obtaining collateral. We will refer to \tilde{D} as the free bidding potential. In the case of the Eurosystem which does not require counterparties to cover bids with collateral and which accepts a wide range of eligible collateral (needed at the moment of settlement), \tilde{D} should be clearly above refinancing needs.

The **welfare analysis** will focus exclusively on the following two types of welfare costs. Firstly, the total bidding costs are one constituent to welfare costs. Secondly, stochastic deviation of the overnight interest rate from their expected value is regarded as costly. The loss functions of the central bank (which may be composed of both factors or not) are summarized in the following table.

Table 3: Two central bank loss functions in a one day reserve maintenance period		
Deviation of market overnight rate from mid point of corridor set by standing facilities	$L_1 = E(i - E(i))^2$	
Total cost of bidding	$L_2 = C(D)$	

Note that the interest rate related loss function focuses only on the volatility of overnight rates, and not on the bias of the overnight rate relative to the tender rate *r*. This may be motivated as follows: if the central bank would dislike any bias, it could simply correct for the bias by shifting the corridor set by standing facilities and the tender rate correspondingly. Both the choice of the tender procedure, and in case of the discretionary procedures, the size of the parameter γ will affect the loss function of the central bank. One may add to this short list the decision of the central bank to invest or not in the quality of its autonomous factor forecasts, i.e. to reduce through investment the size of the variance of the unpredictable autonomous factor shock σ_{η}^2 . However, this will not be further investigated here.

3.1 The fixed rate tender with 100% allotment

The following proposition distinguishes between two cases regarding the size of the free bidding potential. The case of a large free bidding potential corresponds more or less to the one of the Eurosystem: there are huge amounts of collateral outstanding, and the bids themselves do not need to be covered by collateral. The second case of a limited bidding potential may correspond to cases of central banks which impose that bids are fully covered by collateral (the case of the Bundesbank before 1999) and collateral is not too abundant.

⁸ This assumption seems to be well in line with experience, see annex.

Proposition 1: In the case of the 100% fixed rate tender, in the absence of rate change expectations, and if bidding is not costly in the domain of bids covering the expected liquidity ($\tilde{D} > A$), banks will exactly bid their expected liquidity needs, $D^*=A$ and the market interest rate expected by the market at the moment of the allotment decision will be the mid point of the corridor E(i) = r. In the case of the 100% fixed rate tender, in the absence of rate change expectations, and if bidding is costly in the sense of $\tilde{D} < A$, banks will tend to bid less than their liquidity needs, $D^*<A$ and the expected market interest rate will be above the mid point of the corridor E(i) > r.

It is obvious that $D > \widetilde{D}$ will not be an equilibrium since then the cost of obtaining funds is higher than the expected value of funds. The equilibrium condition of intended bidding may thus be written:

$$0.5 + \frac{\partial C}{\partial D} \stackrel{!}{=} E\left[\left(1 - \Phi\left(\frac{D^* - A - \varepsilon}{\sigma_{\eta}}\right)\right)\right] = \int_{-\infty}^{\infty} \left(1 - \Phi\left(\frac{D^* - A - \varepsilon}{1 - \sigma_{\eta}}\right)\right) f_{\varepsilon}(x) dx$$

The left hand side represents the marginal costs of obtaining funds, which are equal to the tender rate r (=0.5) plus the marginal bidding costs. The right hand side reflects the expected value of funds, which corresponds, for risk-neutral banks, to the probability of the banking sector of an aggregate recourse to the marginal lending facility.⁹ The function $\Phi()$ refers to the cumulative standard normal distribution. Denote by $h_1(D^*)$ the left hand side and by $h_2(D^*)$ the right hand side of the equation above. Consider first the case $\tilde{D} > A$ (large free bidding potential) This is a continuous function $h_2(D^*):[0,\infty[\rightarrow[\phi,0[$ (with $\phi > 0.5)$) which falls monotonously. Hence, there must be a unique equilibrium value D^* . Since $h_2(A) = 0.5$, the unique equilibrium value is $D^*=A$ and hence $E(i) = r \cdot 1^0$ Now consider the case $\tilde{D} < A$ (costly bidding). We distinguish two sub-cases (1) $D^* \leq \tilde{D}$. Then, the result $D^*<A$ is obvious. (2) In the case $D^* > \tilde{D}$, note that $h_1(D^*):[0,\infty[\rightarrow[0.5,\infty[$ is monotonously increasing and $h_2(D^*):[0,\infty[\rightarrow[\phi,0[$ (with $\phi > 0.5)$) is monotonously decreasing, implying that there is a unique equilibrium bidding volume D*. Since $h_1(A) > 0.5$ and $h_2(A) = 0.5$, $D^*<A$ and hence E(i) > 0.5.

The ε -shocks will of course have an impact on the effective overnight rate in the market session after the bid and allotment amounts have become known. The interest rate volatility will increase with the variance of ε . In terms of the two loss functions exposed above, we may hence conclude that, in the case of fixed rate tender and 100% allotments under a large free bidding potential, L1>0, L2=0, while in the case of a large free bidding potential, L1>0,

⁹ See for instance Bindseil [2001].

L2>0, i.e. interest rates will always be volatile under the 100% approach, while bidding costs will be only relevant in the case of a limited free bidding potential

3.2. Discretionary fixed rate tender

Now, the existence of ε -shocks (shocks relating to the non-ability of the banks to submit aggregate bids identical to liquidity needs, as known by the central bank, even if those are published), together with the possibility of the central bank to allot less than the banks have bid for, will introduce an asymmetry. If a negative ε -shock is experienced, than the central bank will tend to eliminate this shock through a correspondingly lower allotment (and an allotment ratio below 1). If there is in contrast a positive shock such that the effective bid is below the liquidity needs, then the central bank cannot help and market rates will soar beyond the tender rate. This asymmetry will tend to make banks bid more than they would under the 100% fixed rate tender. The crucial parameter of the central bank under this procedure will be its liquidity target γ in its allotment function $m = A + \varepsilon + \gamma$. We again distinguish between different cases regarding the free bidding potential. In the following, the term "underbidding" will be used precisely to describe a situation in which $D < A + \mathcal{E} + \gamma$, i.e. the bid is below the amount that the central bank would like to allot according to its allotment function. Again, we will have to distinguish between two cases regarding the free bidding potential. First, we will assume a very large free bidding potential, " $\tilde{D} >> A$ ", such that the likelihood of bids being below the actual liquidity needs in case that banks bid for their entire free potential will be negligible. Leaving aside intermediate cases, it will alternatively be assumed that $\widetilde{D} = 0$, i.e. that bidding is costly from the first unit of bids on. Furthermore, depending on the choice of the liquidity target γ , three cases can be distinguished. Those are reviewed in the following one by one.

γ =0, i.e. the central bank is neutral

If the central bank aims at neutral liquidity conditions (i.e. $m=A+\varepsilon$) and the costs of submitting bids become relevant only far above the actual liquidity needs, i.e. $\tilde{D} >> A$, then it will be possible to achieve both a perfect steering of overnight rates and zero bidding costs.

Proposition 2: In the discretionary fixed rate tender with neutral liquidity policy and large free bidding potential, i.e. $\gamma = 0$ and $\tilde{D} >> A$, banks will submit exactly their free bidding potential, $D^* = \tilde{D}$, and the interest rate will exactly correspond to the tender rate without any volatility. In the case of the discretionary fixed rate tender with neutral liquidity policy and no free bidding potential, i.e. $\gamma = 0$ and $\tilde{D} = 0$, banks will either bid more, at, or

¹⁰ See also lemma 1 in Bindseil [2001].

less than their actual liquidity needs depending on the volatility of σ_{ϵ}^2 and the cost of bidding function C(D).

Consider first the case of a large free bidding potential ($\tilde{D} >> A$). First, it has to be shown that banks will not bid less than \tilde{D} . The cost of bidding are r. The value of funds will be either r (in case banks bid $A + \varepsilon$ or more) or higher (if banks bid less). Hence, we obtain the following equilibrium condition for the optimal bidding volume:

$$r + \partial C / \partial D \stackrel{!}{=} E \left[r(P[\varepsilon \le D - A]) + \left(1 - \Phi \left(\frac{D - A - (\varepsilon | \varepsilon > D - A)}{\sigma_{\eta}} \right) \right) P[\varepsilon > D - A] \right]$$

$$\Leftrightarrow r = r \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) + \int_{D - A - \gamma}^{\infty} \left(\left(1 - \Phi \left(\frac{D - A - x}{\sigma_{\eta}} \right) \right) \frac{\phi \left(\frac{x}{\sigma_{\varepsilon}} \right)}{\left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right)} \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) dx \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) dx \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) dx \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) dx \right) dx \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) dx \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) dx \right) dx \right) dx \left(1 - \Phi \left(\frac{D - A}{\sigma_{\varepsilon}} \right) dx \right) dx$$

The left hand side represents the marginal cost of obtaining funds in the tender. The right hand side is composed of two elements. The first is the expected value of funds in case no underbidding has occurred. The second is the expected value of funds if underbidding occurred. The interest rate in case underbidding occurred will obviously depend on the extent of overbidding, which becomes immediately known after the allotment. This allows then calculating a new fair value of funds after the underbidding has become known. The explicit form of the equation writing out the expectations operator reflects e.g. Theorem 20.1 of Greene [1997, 949]. Denote by $h_2(D^*)$ the right hand side of the equation. This is a continuous function $h_2(D^*):[0,\infty[\rightarrow[\phi,0.5[$ with $\phi>0.5$ which falls monotonously. Since it falls monotonously towards 0.5 without ever reaching that value, there is no reason for the banks to stop before \widetilde{D} . Now, it has to be shown that bidding should not go beyond \widetilde{D} . When reaching \widetilde{D} , bidding costs come suddenly into play. It has been assumed that $\widetilde{D} >> A$ such that $P[\widetilde{D} < A + \varepsilon]$ is always so small that further reducing it by increasing further the bid D should never outweigh the marginal cost of additional bidding. Consider now the case of no free bidding potential ($\widetilde{D}=0$). It is easy to show, by providing examples, that all cases (D > A, D = A, D < A) can indeed occur. The equilibrium bidding condition (see above) reveals that if the marginal cost of bidding increases sufficiently slowly, then for $\sigma_{\epsilon}^2 > 0$ the bidding will be large. If in contrast $\sigma_{\epsilon}^2 = 0$ and the bidding costs increase rapidly, then, there will be a unique equilibrium value of D with D*<A. Finally, it can be shown easily that $orall \sigma_{arepsilon}^2$, there exists a bidding cost function fulfilling the assumed properties such that D*>A. The bidding will be ceteris paribus the higher, the higher σ_{ε}^2 and the lower the bidding cost. Example: Assume that the cost of bidding function is: $C(D) = D^2/1000$. Then we obtain for the parameter values depicted above D*=6.41. If the cost of bidding function is however: $C(D) = D^2/100$, then, the bid amount will be 5.16 Finally, if the cost of bidding function is $C(D) = D^2/10$, then the bid will only amount to 2.35.¹¹

In the case of a large free bidding potential, the banks will tend to "overbid" naturally in the domain $D \leq \tilde{D}$ in order to make the likelihood of underbidding close to zero. This allows the volatility of overnight rates to become negligible, while avoiding any bidding costs. Hence, a perfect situation is obtained. The condition for this perfect world is obviously the existence of ample free bidding potential. It would be unreasonable if the central bank would in this case have an aversion against the relatively low average allotment ratio (\tilde{D}/A) resulting in this case since this low allotment ratio does not reflect any social costs while at the same time ensuring that the central bank can, through its discretionary choice of the allotment ratio, avoid interest rate volatility. In contrast, in the case of no free bidding potential, a perfect solution cannot be reached.

γ >0, i.e. the central bank tends to provide surplus liquidity.

We will concentrate here on the case with large free bidding potential (the case of no free bidding potential is rather similar). If the central bank is able to allot the excess funds, market rates would be pushed below the tender rate, such as to make successful bidders loose money. This loss will have to be compensated by occurrence, at least from time to time, of underbidding, in the case of which the value of funds will be higher then the tender rate. The higher γ , the lower the intended bid D will hence be, and the bid will finally tend to the 100% allotment solution, when γ becomes very large, where $D^*=A$. The formal equilibrium condition for bidding will be:

$$r = E \begin{bmatrix} \left(1 - \Phi\left(\frac{\gamma}{\sigma_{\eta}}\right)\right) \left(P[\varepsilon \leq D - A - \gamma]\right) \\ + \left(1 - \Phi\left(\frac{D - A - (\varepsilon \mid \varepsilon > D - A - \gamma)}{\sigma_{\eta}}\right)\right) P[\varepsilon > D - A - \gamma] \end{bmatrix}$$

The first term on the right hand side of the equation represents the interest rate if the liquidity shock ϵ is sufficiently small such that there are more bids than the intended allotment

¹¹ In the latter case, the interest rate will of course not remain in the mid of the corridor set by standing facilities. Hence, the central bank would have to shift the corridor accordingly to achieve that the

amount $(A + \varepsilon + \gamma)$. The second term represents the interest rate if there are less bids than the central bank would like to allot. In this case, the precise deficit of bids relative to the original allotment aim of the central bank is relevant.

In the following, we will repeatedly illustrate the results obtained by an example. In this example, we will consistently assume the following parameter values: A = 5, \tilde{D} = 20, σ_{ε}^2 = 1, σ_{η}^2 = 1. For these parameter values, the following chart 3 depicts the relationship between the liquidity target of the central bank and the bid amount that will be submitted by banks:



Bidding costs will in any case be zero in this environment. The variance of the interest rate is the second parameter of interest. The variance of the interest rate for a given pair $(\gamma, D^*(\gamma))$ is given by:

$$\begin{aligned} &Var_{\gamma,D^*(\gamma)}(i) == E(i-E(i))^2 = E(i-r)^2 = \int_{-\infty}^{\infty} (i(\varepsilon)-r)^2 f_{\varepsilon}(x)dx \\ &= \int_{-\infty}^{D^*(\gamma)-A-\gamma} \left(\frac{1}{2} - \Phi\left(\frac{\gamma}{\sigma_{\eta}}\right)\right)^2 f_{\varepsilon}(x)dx + \int_{D^*(\gamma)-A-\gamma}^{\infty} \left(\frac{1}{2} - \Phi\left(\frac{D-A-x}{\sigma_{\eta}}\right)\right)^2 f_{\varepsilon}(x)dx \end{aligned}$$

The following chart illustrates that the variance of the overnight rate increases monotonously with the size of the positive liquidity target, converging to the variance that would be observed under the 100% allotment ratio rule:

average overnight rate corresponds to its target rate.



The following proposition, which is easy to prove, summarises the results.

Proposition 3. In the case of discretionary tenders and a large free bidding potential, the central bank can achieve any average allotment ratio it wishes between \tilde{D}/A and 1 by adequately choosing a target liquidity surplus $\gamma > 0$. If $\gamma = 0$, the allotment ratio will be \tilde{D}/A and the variance of overnight rates will reach a minimum. If $\gamma \rightarrow \infty$, the average allotment ratio tends to the one of the 100% rule, namely D/A = 1 and the variance of overnight rates will be at a maximum, i.e. at the value of the 100% rule. In between, the allotment ratio decreases and the interest rate variance increases monotonously.

Hence, a loose liquidity policy is normally *inferior* to neutrality in discretionary fixed rate tenders. The same results hold in the case of no free bidding potential: there are little merits in such a policy since it will only increase the volatility of the overnight rates.

γ <0, i.e. the central bank is systematically tight.

In this case, the interbank market rate will tend to be systematically higher than the tender rate. The gap between the two rates can be reconciled with the idea of a bidding equilibrium only with the help of overbidding and its implied costs.

Proposition 4. In the case of a large free bidding potential ($\tilde{D} >> A$), tightness ($\gamma < 0$) is systematically inferior to neutrality ($\gamma = 0$). In case of a limited free bidding potential, (e.g. $\tilde{D} = 0$), tightness can be superior to neutrality if the central bank attaches a high weight to the stability of interest rates.

The following equilibrium condition will determine the bidding behaviour and hence the equilibrium allotment ratio:

$$r + \frac{\partial C}{\partial D} = \left(1 - \Phi\left(\frac{\gamma}{\sigma_{\eta}}\right)\right)$$

In this case, we obtain zero losses on the side of interest rate volatility, but bidding costs will be positive. Hence, a negative liquidity target is always inferior to a neutral liquidity policy in the case of high free bidding potential. However, a tight allotment policy may have its merits in the case of no free bidding potential. Consider the example specified above with $C(D) = D^2 / 100$ and hence a bidding volume under neutrality of 5.16. With this bidding volume, it will happen relatively often that the liquidity needs including ε will be above the bids, and overnight rates will hence be above the mid point of the corridor. The volatility of rates (around a higher level within the corridor) could indeed be lowered by tightening the liquidity policy. For instance, a liquidity target $\gamma = -0.5$ would imply a bid of 5.66 and thus a correspondingly lower frequency of underbidding and lower volatility of overnight rates. Depending on the weights of the loss functions in the central bank's preferences, it is hence possible that tightness is superior.

Overall, one may conclude that the optimal approach of the central bank in case of discretionary fixed rate tenders, the absence of rate change expectations and a large bidding potential \tilde{D} is to be neutral and to allow allotment ratios to move up to A/\tilde{D} . Then, the likelihood of underbidding and the volatility of overnight rates is practically zero and no bidding costs emerge. In case of a limited bidding potential, the central bank, when choosing its liquidity target γ , may face a trade-off. Depending on bidding costs, it cannot be excluded that a tightening of the policy can lower the likelihood of underbidding significantly and therefore lower the volatility of overnight rates. This however comes at the price of higher bidding costs. Depending on the central bank's preferences, it cannot be excluded that it will prefer some degree of tightness. The allotment policy of the Deutsche Bundesbank may have been a good example for such a bias to tightness. Indeed, in the Bundesbank regime, banks tended to end the reserve maintenance period with recourse to the marginal lending facility. The Bundesbank required bids to be fully collateralised, while allotment ratios were in the last years of Stage Two around 20%, such that bidding was likely to be costly. The latter is also confirmed by the fact that in the fixed rate tender regime, the market rates were typically around 10 basis points above the tender rate. The policy of the Bundesbank may therefore be understood as aiming to keep the market tight to ensure always a sufficient amount of overbidding. The implied bidding costs were apparently accepted.

3.3 The variable rate tender

As mentioned beforehand, it is assumed here, also on the basis of empirical evidence summarised in an annex, that variable rate tenders ensure that there will always be sufficient

bids since bidders can react to low expected market rates by submitting bids at lower rates, instead of reducing their bid volume. Of course, it is not claimed here that variable rate tenders do not diverge from fixed rate tenders in many other aspects. For instance, fixed rate tenders (or variable rate tenders with minimum bid rate) are likely to be superior in terms of signalling the stance of monetary policy. It will hence not be useful to conclude from the present analysis regarding the general relative merits of fixed and variable rate tenders. This paper will nevertheless allow us to assess how important the advantage of the variable rate tender to avoid over- or underbidding is under different circumstances. It is assumed that the central bank continues to follow a quantity oriented allotment policy, i.e. its allotment function does not depend on the rates at which bids are submitted. If the central bank tends to be tight, bid rates will move upward with the implied higher value of funds, and the reverse in case the central bank is loose. Generally, marginal tender rates would move with expected market rates in equilibrium. The total amount of bidding should be rather independent from the liquidity target and the level of rates. Hence, the central bank can focus on achieving stable market rates around the mid point of the corridor by choosing the adequate liquidity supply. In so far, the variable rate tender should also allow to bring both types of assumed loss functions close to zero.

4. Introducing rate change expectations: the two days reserve

maintenance period case

4.1 Introduction

The previous section demonstrated that in case of stable official interest rates rates, central banks of which the counterparties have a large free bidding potential should be able to achieve, for instance with the discretionary fixed rate tender procedure and a neutral liquidity policy, major operational goals, namely stable overnight rates and an efficient tender procedure without excessive bidding costs. Unfortunately, these results will no longer necessarily hold under rate change expectations. To be able to model those, the one-day maintenance period needs to be replaced by one with two days. Regarding the bidding costs, it will now be generally assumed that $\tilde{D} >> 2A$, i.e. that the free bidding potential is so large that if banks intend to bid the free bidding potential in the first tender, then the likelihood that the bids are not sufficient to cover the liquidity needs on both days of the reserve maintenance period are negligible. This assumption will allow us to encounter bidding costs only in the case of acute overbidding. As argued beforehand, this assumption should be well in line with the case of the Eurosystem. The analysis can be restricted to the case of a reserve maintenance period with only two market sessions, one allotment decision, and one autonomous liquidity factor shock occurring at the very end of the reserve maintenance period. This case provides for a sufficient micro-cosmos of the major

phenomena of relevance. Tender operations before the last tender operation of the maintenance period are indeed mainly dependent on what happens in the last tender of the reserve maintenance period, since this is the one in which the marginal demand and supply of funds are matched. In previous tenders, demand and supply are not marginal, and hence relevant only in so far as they reveal anything about the end of the reserve maintenance period marginal conditions. However, to be able to analyze the effect of rate change expectations on the bidding behavior, we need at least one tender before the potential rate change decision, and hence we end up with two tenders. Something similar holds for autonomous factor shocks occurring earlier in the reserve maintenance period. All autonomous factor shocks before the last allotment decision in the reserve maintenance period should normally be neutralized through the following allotment decision and hence before the end of the maintenance period, when they would become relevant (Bindseil and Seitz [2001, 27] provide evidence for this in the case of the ECB). Autonomous factor shocks after the last allotment decision, but before the last market session, move the market in a rather mechanistic way. An autonomous factor shock after the last trading session is needed in any case for a sensible model to avoid that the overnight rate in the last market session is not adsorbed by a standing facility rate.





The following **sequence of 9 events** is assumed for the modeled reserve maintenance period: (1) The reserve maintenance period begins similarly to the one-day model. However, interest rate change expectations are now present in the market. Money market players share homogeneous expectations. The size of a possible interest rate change, λ , is given, whereby it can be either negative or positive. The probabilities of a rate change are mutually exclusive in the sense that there are either rate cut, or rate hike expectations, *but never both at once*. Hence, *P* will indicate the probability of a rate change. For the rate cut and rate hike expectations, we introduce a series of dummy variables for the sake of a simple notation, namely d^{he} , d^{ce} , d^{c} , d^{h} which take the value 1 in case that hike expectations are present in the morning of the first day; that rate cut expectations are present in the morning of the first day; that rate cut expectations are present in the morning of the first day.

day, that a cut has effectively taken place at the end of the first day, that a hike has effectively taken place at the end of the first day, respectively. Note that $d^c = 1 \Rightarrow d^{ce} = 1$, $d^h = 1 \Rightarrow d^{he} = 1$, $d^{ce} = 1 \Rightarrow d^{he} = 0$, etc. (2) The first open market operation takes place (as in the one-day model). (3) The first market session takes place and a market clearing overnight rate i_1 is determined. (4) The Central Bank Council takes a decision with regard to the interest rate of standing facilities at the end of the reserve maintenance period. In the case of a fixed rate tender, the fixed tender rate is always shifted in parallel to the standing facility rates.¹² (5) The first day of the reserve maintenance period ends, which means that reserve holdings at that moment in time are relevant for the fulfillment of reserve requirements over the reserve maintenance period. (6) The second open market operation takes place. (7) The second market session takes place and a market clearing overnight rate i_2 is determined. (8) The realization of the autonomous liquidity factor shock takes place. (9) Finally, the banks take recourse to standing facilities to cover the liquidity imbalance. The reserve maintenance period ends with the second reserve fulfillment point in time.

The following table displays for each of the three tender procedures already analyzed in the previous section intuitive allotment rules. The model will then be solved as follows: For a given tender procedure and allotment strategy of the central bank, the banking sector solves collectively its optimization problem under perfect competition to arrive at some bid volume under which the expected cost of obtaining funds equals the expected value of funds in the inter-bank market. Then, the analysis turns to the choice of the central bank regarding the tender procedure and allotment rule, and works out the optimal parameter values of the allotment rules, as far as relevant, assuming different possible loss functions of the central bank. Finally, the outcomes of the different procedures, for their respective optimal specification, are compared. This is done separately for rate hike and loss expectations, since the central bank can of course switch from one procedure to the other when expectations change. Note that the following alternative procedures and tender rules are not exhaustive in the sense that some possible combinations of the different procedures with specific allotment rules are not presented for the sake of shortness. Furthermore, for the same reasons, other alternative tender procedures as for instance the variable rate tender with the minimum bid rate, or the fixed rate tender with a minimum allotment ratio, are not discussed, even if the proposed model is perfectly appropriate to do so.¹³

¹² Note that since we have set the rate of the deposit facility to zero, any reduction of this rate implies that it becomes negative. Even though somewhat counterintuitive, this does not pose any problem in the model. The model could of course also be restated with a corridor set by standing facilities at a higher level.

¹³ Specifically in the case of the minimum bid rate variable rate tender and the minimum allotment ratio fixed rate tender, it can be shown easily that these specifications are quasi-equivalent to those specifications explicitly analysed.

Table 3: Procedures and strategies available to the central bank. ¹	14

	Allotment functions	
(1) 100% allotment	$m_1 = D_1$	
Fixed rate tender	$m_2 = D_2$	
(2) Discretionary FRT	$m_1 = \min(A + \varepsilon_1, D_1)$	
With partial bail-out	$m_2 = \min(A + \varepsilon_2 + \alpha(A + \varepsilon_1 - m_1), D_2)$	
(3) Variable rate tender – contingent liquidity target	$m_1 = A + \varepsilon_1 + \pi^{he} d^{he}$	
(purely quantity oriented)	$m_2 = A + \varepsilon_2 + \pi^{ce} d^{ce} + \pi^c d^c + \pi^h d^h$	

Consider the three procedures one by one:

- (1) The 100% allotment rule has, by definition, the simplest form: allotment amounts correspond to bids.
- (2) In contrast, the discretionary fixed rate tender may contain as many additional functional terms as the central bank can imagine. For the sake of shortness, the analysis here is limited to one intuitive functional form. It foresees that, in principle, the central bank always allots the expected same day liquidity need. However, in case of underbidding on day one, it is willing to partially bail-out the banks on the second day, i.e. to provide them more liquidity to allow them to catch up at least partially with their fulfillment of reserve requirements. The allotment decisions on the second day may compensate hence a share $0 < \alpha < 1$ of the liquidity shortage that occurred on the previous day in relation to underbidding. A central bank with $\alpha = 0$ has "no mercy" with the banks that have underbid, while a central bank with $\alpha = 1$ goes for "full bail-out".
- (3) The variable rate tender, which we assume to be purely quantity oriented, may contain some contingent terms in the allotment rule. Specifically, contingent liquidity components relating to rate hike/cut expectations or actual rate hikes/cuts are included, whereby the analysis is limited to either one or the other of those (considering both at once represents a simple extension). The case of underbidding is not relevant since it is assumed that under this procedure, banks will always bid sufficiently at low rates, i.e. if there are rate cut expectations, they will simply bid at those rates which correspond to the expected market rate on day 2.

The tender procedures and allotment strategies will be analyzed in relation to the following four possible components of loss functions:

¹⁴ Note that these three cases also roughly cover the case of a variable rate tender with minimum bid rate. This tender should be equivalent to a discretionary fixed rate tender if the market leans towards rate cut expectations or the central bank tends to be loose (q>0), and it should be equivalent to a variable rate tender if the market expects a rate increase or the central bank tends to be tight (q<0).

Deviation of the market rate from the prevailing mid point of the corridor	$L_0 = E(i_1 - r_1)^2 + E(i_2 - r_2)^2$
Intra-maintenance period volatility of the overnight rate	$L_1 = E(i_1 - i_2)^2$
Deviation from a smooth fulfillment of reserve requirements	$L_2 = E(m_1 - A)^2 + E(m_2 - A)^2$
Cost of bidding	$L_3 = E(C(D_1)) + E(C(D_2))$

Table 4: possible loss functions of the central bank in the 2-days maintenance period case

The loss function L_0 could also be entitled as the "operational target loss function", assuming that decisions of rate changes are at the same time decisions to change the operational target accordingly. However, it is easy to show that no allotment rule or tender procedure exists that can bring this loss function down to zero. Indeed, in an environment where $E(r + \lambda d^h) > r$, achieving the aim $i_1 = r$ and $i_2 = r + \lambda d^h$ would imply $E(i_2) > i_1$ which contradicts the martingale property. A similar argument holds under rate cut expectations. One may shift the level of rates on both days up or down, but one can never engineer an expected change of market rates within the maintenance period. The central bank should hence abstain from any attempts to achieve, through liquidity management, a minimization of deviations of market rates from prevailing tender rates in case of rate change expectations. However, the choice of the tender procedure could still reduce the part of the volatility that is due to ε -shocks. Nevertheless, the loss function L_0 will not be pursued further. In any case, one may conclude that the concept of an explicit operational target rate may have something ambiguous and possibly misleading in the case of an operational framework such as the one of the Eurosystem.

The loss function L_1 reflects the idea that within the reserve maintenance period, rates should be stable. The loss function L_2 focuses on the smoothness of total bank reserves over time. Reserves play an important role as buffers against all kinds of aggregate and individual liquidity shocks. Hence, an allotment rule that leads to very low reserves on some days may be viewed as detrimental to the stability and smoothness of the interbank market and the payment system. Note that an externality may be at stake here: while it could be beneficial for single banks to profit from arbitrage opportunities by front- or back-loading strongly their reserve fulfillment, this strategy may, if practiced by all, be welfare-reducing for the banking system as a whole. If this is indeed the case, the central bank should integrate a corresponding element in its loss function. Finally, the proposed loss function L_3 looks at bidding costs, which will only be relevant in the case of overbidding (rate hike expectations and discretionary tenders). It will become clear in the course of this section that under rate hike expectations, ε -shocks will only constitute a problem under the 100% rule, but not in the discretionary fixed rate tender. However, ε -shocks leading to underbidding will constitute a general problem in the case of rate cute expectations, i.e. also in the discretionary tender case. Hence, one of the major questions to be investigated will be how the implied underbidding problems can be controlled. Now, the different allotment rules and their implications on the possible loss functions of the central bank will be considered one by one, starting from the simplest case.

4.2 The 100% allotment rule

4.2.1 The 100% allotment rule under rate hike expectations

Note that bidding costs can be ignored in this case since we took the assumption $\tilde{D} >> 2A$. Banks will tend to cover their entire expected liquidity needs for the reserve maintenance period in the first refinancing operation, since funds can only become more expensive in the second tender, or stay as expensive as they are. Consider first for a moment the case without ε -shocks.

Proposition 5: In the absence of ε -shocks, i.e. if $\sigma_{\varepsilon}^2 = 0$ ("if banks can perfectly co-ordinate their bids to submit an aggregate bid corresponding to the central bank's estimated liquidity needs"), banks will bid under the 100% allotment rule and under rate hike expectations already in the first tender of the maintenance period more than the liquidity needs for the entire reserve maintenance period.

In this case, the equilibrium condition for bidding will be:

$$r \stackrel{!}{=} i_1 = E(i_2) \Leftrightarrow r = r + P\lambda + \left(1 - \Phi\left(\frac{D_1 - 2A}{\sigma_\eta}\right)\right) - r$$
$$\Leftrightarrow D_1 = \Phi^{-1}(1 - r - P\lambda)\sigma_\eta^2 + 2A$$

Hence, without ε -shocks, $D_1 > 2A$ to compensate for rate change expectations.

The ε -shocks make the calculus in the first tender somewhat more complicated. Bidding in the first tender less than in the previous case creates a kind of buffer against positive liquidity shocks which would otherwise imply a liquidity surplus already after the first day of the reserve maintenance period.¹⁵

Proposition 6: In the presence of ε -shocks, i.e. if $\sigma_{\varepsilon}^2 > 0$ ("if banks cannot perfectly co-ordinate their bids to submit an aggregate bid corresponding to the

¹⁵ This effect is somewhat related to the effect described by Perez-Quiros and Mendizabal [2001].

central bank's estimated liquidity needs"), banks will bid less than in the case that $\sigma_{\epsilon}^2 = 0$.

The larger the bid, the lower will be the probability that the banking system will have to go again to the tender at the possibly higher price, but the higher will also be the probability that banks will have more than 2A of reserves already on the first day of the maintenance period. In case of a sufficiently large negative ε , banks have incentives to go again to the central bank's open market operation on the second day. This is the case if ε is such that: $\varepsilon_1 > D_1 - 2A$, i.e. if the banking system ends surprisingly in an expected liquidity deficit after the first day of the reserve maintenance period. In that case, the marginal value of funds will be $r + \lambda d^h$. After the tender decision has been announced, the interbank rate thus takes one of the following values:

If
$$\varepsilon_1 > D_1 - 2A$$
: $i_2 = r + \lambda d^h$; If $\varepsilon_1 \le D_1 - 2A$: $i_2 = \left(1 - \Phi\left(\frac{D_1 - 2A - \varepsilon_1}{\sigma_\eta}\right)\right) + \lambda d^h$

Therefore, the equilibrium condition for the bidding has to be restated as follows:

$$\begin{aligned} r \stackrel{!}{=} i_{1} &= E(i_{2}) \Leftrightarrow r = P\lambda + rP(\varepsilon_{1} > D_{1} - 2A) + E\left(1 - \Phi\left(\frac{D_{1} - 2A - (\varepsilon_{1} \mid \varepsilon_{1} < D_{1} - 2A)}{\sigma_{\eta}}\right)\right) P(\varepsilon_{1} < D_{1} - 2A) \\ \Leftrightarrow r &= P\lambda + r\left(1 - \Phi\left(\frac{D_{1} - 2A}{\sigma_{\varepsilon}}\right)\right) + \left(\int_{-\infty}^{D_{1} - 2A} \left(1 - \Phi\left(\frac{D_{1} - 2A - \varepsilon_{1}}{\sigma_{\eta}}\right)\right) \frac{\phi\left(\frac{x}{\sigma_{\varepsilon}}\right)}{\Phi\left(\frac{D_{1} - 2A}{\sigma_{\varepsilon}}\right)} dx\right) \Phi\left(\frac{D_{1} - A}{\sigma_{\varepsilon}}\right) \end{aligned}$$

Note that $P(\varepsilon_1 < D_1 - 2A)$ increases with the intended bid amount, while the term E() decreases monotonously with D_1 , implying that the right hand side of the equation monotonously decreases with the bid and that one hence obtains a unique equilibrium value of bids in which bids are below the equilibrium value in the absence of ε -shocks.

As has been verified by calculus, this equilibrium amount of bids will fall monotonously when σ_{ε}^2 increases. The intuition behind this result is as follows: the higher σ_{ε}^2 , the more likely it is that the bidders will be exposed to a liquidity absorbing shock such that they will use the option to go to the second day's tender. Hence, the "insurance" effect of the option to go to the second day's tender. Hence, the "insurance" effect of the option to go to the second day's tender becomes more important. On the other side, the likelihood of liquidity *injecting* shocks also increases when σ_{ε}^2 increases, however without an option that limits the costs of these events. Hence, it becomes more and more important with an increasing σ_{ε}^2 to react preemptively by bidding lower amounts.

The loss functions under this approach take the following values. L1: The intra-maintenance period volatility of rates will be relatively large. Both ε -shocks will normally impact on the interest rates. Furthermore, the interest rate decision will always move interest rates on its own. L2: The non-smoothness of the reserve requirement path is at a maximum under this allotment rule, since little or no bids are normally submitted in the second tender and, hence, no or little reserves are held on the second day. L3: As we assumed $\widetilde{D} >> 2A$, no bidding costs will become relevant. The search for the optimum values of parameters of the allotment function is obsolete here since there are no such parameters in the 100% allotment rule case.

4.2.2 The 100% allotment rule under rate cut expectations

The equilibrium bidding in this case if described by the following proposition.

Proposition 7: Under the 100% allotment rule and rate cut expectations, banks will only bid in the second tender of the reserve maintenance period, whereby they will bid in this tender exactly their expected liquidity needs: $D_1 = 0, D_2 = 2A$.

Banks will never submit any bids in the first tender, since funds can only stay as cheap as they are or become cheaper. Market rates in the first maintenance period will correspond to the expected mid point of the corridor on the second day of the reserve maintenance period, $i_1 = r - P\lambda$, which is below the tender rate, such that any bidding would constitute a loss. On the second day, bidding will be such that market rates correspond to the new bidding rates. This is ensured if the bid covers exactly the expected liquidity need:

$$i_2 \stackrel{!}{=} r + d^c \lambda \Leftrightarrow E \left(1 - \Phi \left(\frac{D_2 - 2A - \varepsilon}{\sigma_\eta} \right) \right) = 0 \Leftrightarrow D_2 = 2A \quad \blacksquare$$

The three loss functions will behave as follows in this case: L1: The rate change decision will necessarily change market rates between the two periods. Furthermore, the ε -shock on day 2 will be relevant. L2: Again, the non-smoothness will be at a maximum under the 100% allotment rule. L3: Again, the bidding costs are zero by the assumption that $\tilde{D} >> 2A$. Some difference between the cases of rate hike and rate cut expectations is noteworthy. Hence, when giving a comparative assessment of the 100% allotment rule, it will always be necessary to distinguish between the two cases.

4.3 Discretionary FRT allotment with partial bail-out

4.3.1 Discretionary FRTs with partial bail-out under rate hike expectations

Starting from the allotment rule displayed in table 3, the following proposition summarizes the bidding equilibrium in this case:

Proposition 8: Under discretionary fixed rate tenders and rate hike expectations, the central bank may allot the liquidity needs on each day of the maintenance period, i.e. $m_1 = A + \varepsilon_1$; $m_2 = A + \varepsilon_2$. On day 1, market interest rates will correspond to the expected end of maintenance period mid point of the corridor, while on day 2, they will correspond to the actual end of maintenance period mid point of the corridor, i.e. $i_1 = r + P\lambda$, $i_2 = r + d^h\lambda$. The gap between the tender and the market rate on the first day, $P\lambda$, will have to be counterbalanced by bidding costs.

The proposition is obvious. First, it should be noted that the bail-out question is not relevant here, since there will always be sufficient bids in the first tender (remember the assumption $\dot{D} >> 2A$). The central bank can hence always allot the liquidity needs it expects in the first tender. In the second tender, the central bank is then exactly in the same situation as it was in the one-day maintenance period analyzed in the previous section. There, it was shown that banks will bid their free bidding potential, and therefore, again, the central bank will always be able to allot the liquidity needs. As can also be derived from the one-day case, the overnight rate on the second day in the present two days model will correspond to the midpoint of the then prevailing corridor. The overnight rate on the first day corresponds to the expected overnight rate on the second day. However, then, a difference $P\lambda$ prevails between the tender rate and the market interest rate on the first day. To reconcile the two rates in a competitive equilibrium, bidding costs have to come into the play: to obtain funds in the first tender operations, banks will overbid until their cost of bidding will have made them indifferent between obtaining funds in the market and obtaining them from the central bank. The following marginal condition has to be fulfilled: $\partial C/\partial D = P(hike)\lambda$ i.e. the marginal cost of bidding has to correspond to the expected rate hike. The actual welfare costs of overbidding will hence depend on the convexity of the bidding cost curve.

Regarding the loss functions, the following may be observed: L1: Market interest rates will necessarily move due to the rate decision. However, *ɛ*-shocks will not play any role. Hence, the volatility of interest rates within the reserve maintenance period will be lower than under the 100% fixed rate tender. L2: The path of reserve fulfillment is fully smooth under this approach, i.e. the value of this loss function is zero. L3: In terms of implied bidding costs, this
allotment strategy performs poorly. Indeed, this is the main weakness of the fixed rate tender under rate hike expectations.

4.3.2 Discretionary FRTs with partial bail-out under rate cut expectations

Now, banks will be tempted to underbid in the first tender, and the central bank has to choose on the second day the "bail out" co-efficient α . The following proposition summarizes some elements of the bidding equilibrium in this case.

Proposition 9: Under rate cut expectations and discretionary fixed rate tenders, banks will always, i.e. even in case of a zero bail out, bid less than A on the first day, while they will bid their free bidding potential on the second day: $\forall \alpha \in [0,1]: D_1 < A; D_2 = \tilde{D} >> 2A$.

Assume for a moment a zero bail-out ($\alpha = 0$). Banks will then still tend to underbid to some extent in the first tender, since if they would receive the full amount A in the first tender, they would enter the second day under neutral liquidity conditions. However, since there is no reason to underbid on the second day, the market rate on day 2 would be $i_2 = r - d^c \lambda$, and hence the market rate on day 1 should be $r - P\lambda$, which is below the price of funds in the tender. The sufficient availability of bids on the second day follows from the assumption that $\tilde{D} >> 2A$ and that banks enter the second day never in a situation of an accumulated surplus. The allotment rule can therefore be simplified to: $m_1 = \min(D_1, A + \varepsilon_1)$; $m_2 = A + \varepsilon_2 + \alpha(A + \varepsilon_1 - m_1)$

As long as the bail-out is incomplete, the tendency to underbid on the first day will imply an expected lack of liquidity on the second day, and hence a corresponding liquidity-implied upwards drift of the overnight rate. The equilibrium condition determining the equilibrium bid amount on day 1 will be $r = E(i_1) = E(i_2)$, which is equivalent to:

$$r = -P\lambda + rP(\varepsilon_1 < D_1 - A) + E\left(1 - \Phi\left(\frac{-(1 - \alpha)(A - D_1 + (\varepsilon_1 | \varepsilon_1 > D_1 - A))}{\sigma_\eta}\right)\right) P(\varepsilon_1 > D_1 - A)$$

The last term on the right hand side of the equation represents the (more likely) case that the bidders accumulated some deficit on the first day. This will be the case if $\mathcal{E}_1 > D_1 - A$. Then, the interest rate will be determined by the scarce liquidity made available by the central bank. The previous term represents the case in which the liquidity shock ε is so surprisingly liquidity injecting that the bidders will come to the second tender for less than A. In this case, the interest rate on day 2 would be simply the tender rate on day 2. The following chart describes the evolution of the bidding behavior as a function of the bail-out coefficient for our

standard example. Unsurprisingly, it appears that the higher the bail-out, the more incentives are provided to banks to underbid in the first tender. Note that in practice, bids of course cannot be negative. Denote by $\tilde{\alpha}$ the critical value of the bail-out coefficient such that $\alpha > \tilde{\alpha} \Rightarrow D_1 = 0$. Obviously, the "tender rate = market rate" equilibrium condition can not be fulfilled if the bail-out parameter is above this critical value and hence $\alpha > \tilde{\alpha} \Rightarrow r > E(i_1)$. To become more general, recall that above, the (realistic) assumption was taken that $\Phi(A) - r > \lambda$, i.e. that the interest rate effect of one full day of liquidity missing is always larger than the effect of a potential rate change. This implies that when α =0, banks will never bid a zero amount on the first day. The following proposition suggests that the properties of the equilibrium depend on the size of the bail-out coefficient, while it is possible to distinguish two cases.

Proposition 10: Under the discretionary fixed rate tender with partial bail-out and rate cut expectations, for given $P\lambda$, there exists a critical value of the bail-out parameter $\widetilde{\alpha}$ with $1 > \widetilde{\alpha} > 0$ such that $\alpha = \widetilde{\alpha} \Rightarrow D_1 = 0, i_1 = r; \alpha > \widetilde{\alpha} \Rightarrow D_1 = 0, i_1 < r; \alpha < \widetilde{\alpha} \Rightarrow D_1 > 0, i_1 = r$.

First, note that $B_1 > 0 \Rightarrow E(i_1) = r$, since $E(i_1) > r$ is excluded in case of rate cut expectations and the proposed allotment rule, while $E(i_1) < r \Rightarrow B_1 = 0$ since funds could then be obtained cheaper in the money market. In the case of positive bidding, the bidding has hence to be such that $r = E(i_1) = E(i_2)$. Setting the collectively intended bid amount to zero in the equation at the end allows obtaining $\tilde{\alpha}$. Obviously, increasing further the bail-out coefficient can no longer affect the bidding behavior on day 1, but interest rates will decrease monotonously with $\alpha > \tilde{\alpha}$ since the total amount of reserves made available to the banking system will monotonously increase. Further decreasing the bail-out coefficient would, if day 1 bidding would remain unchanged at zero, push interest rates raising above r. This however makes bidding on day 1 attractive, and bids will hence increase such as to restore $r = E(i_1) = E(i_2)$.

The following chart displays the relationship between the bail-out co-efficient and the bidding and expected interest rate on day 1, $E(i_1(\alpha))$ for the concrete example chosen above.



Chart 6: Bid amount and expected interest

With a bail-out coefficient $\alpha = 0$, the bids in the first tender will amount to 4.17 (against a daily liquidity need of 5). The critical value of the bail-out coefficient when bids reach zero amounts to $\tilde{\alpha} = 0.936$. Up to this value, the expected market interest rates will remain at the level of r. When further increasing the bail-out coefficient, expected market rates will fall up to the level of fully anticipating the probability weighted interest rate cut, i.e. to 0.375 in our example. Turning again to the implied values of the loss functions, the following may be observed: L1: Both the interest rate decision, and the ε -shocks will impact on the intramaintenance period volatility of market rates. The relevance of the latter will depend to some extent on the bail-out coefficient. A bail-out coefficient of one has the advantage that ε -shocks would have no impact. L2: From the point of view of the stabilization of the reserve fulfillment path, a bail-out coefficient of zero should be chosen, since this minimizes underbidding. However, still, the reserve fulfillment path will be destabilized to some extent. L3: Bidding costs will not be an issue in this case. One can conclude that the optimal choice of the bail-out coefficient depends on the weights given by the central bank to the different elements of its loss function.

4.4 The variable rate tender

4.4.1 The variable rate tender under rate <u>hike</u> expectations with a contingent liquidity target

i) A liquidity target contingent on hike expectations

In contrast to the discretionary fixed rate tender, it is now assumed (see also annex 1) that there will always be enough bids, such that the central bank can in both tenders allot any amount it wishes to. The allotment rule hence does not need to contain the bid amounts: $m_1 = A + \varepsilon_1 + \pi^{he} d^{he}$; $m_2 = A + \varepsilon_2$. The following proposition summarizes the outcome in this case.

Proposition 11: Under variable rate tenders and rate hike expectations, a central bank following a quantity oriented allotment policy with a term contingent on rate hike expectations may fully or partially prevent the interest rate from anticipating the rate hike. However, it cannot prevent that the rate decision will have an effect λ on market rates (i.e. that the potential rate decision implies intra-maintenance period volatility of interest rates).

The interest rate on the first and second day will be:

$$i_1 = P\lambda + \left(1 - \Phi\left(\frac{\pi^{he}}{\sigma_{\eta}}\right)\right); \quad i_2 = d^h\lambda + \left(1 - \Phi\left(\frac{\pi^{he}}{\sigma_{\eta}}\right)\right)$$

Obviously, the central bank can thus, by choosing π^{he} , achieve that $i_1 = 0.5$. However, the equations also reveal that the effect of the rate hike decision on the market rate will necessarily be λ . This, together with the martingale property in this case implies that $i_{2h} = r + (1 - P(hike))\lambda$ and $i_{2 \to h} = r - P(hike)\lambda$.

Note that the equilibrium is somewhat related, but not identical to the one occurring under the 100% rule: the difference is that in the 100% rule, the ε -shocks remain relevant. At the same time, the outcome of the normal fixed rate tender is identical to the present one for $\pi^{he} = 0$. Consider now again the different loss functions one by one. L1: The ε -shocks do not have any impact here, but the rate decision still moves the market. L2: A perfect smoothness of the reserve fulfillment path can be achieved. L3: No bidding costs occur.

ii) A liquidity target contingent on the actual occurrence of an interest rate hike.

The allotment rule takes here the form: $m_1 = A + \varepsilon_1$; $m_2 = A + \varepsilon_2 + \pi^h d^h$. The following proposition suggests that under this allotment rule, the central bank can fully stabilize interest rates within the reserve maintenance period.

Proposition 12: Under variable rate tenders and rate hike expectations, a central bank following a quantity oriented allotment policy with a term contingent on the occurrence of a rate hike may fully or partially prevent the interest rate from anticipating the rate hike. In addition, it can achieve a perfect stability of interest rates within the reserve maintenance period.

The interest rates on the first and second day will be:

$$i_1 = P\left(\lambda + 1 - \Phi\left(\frac{\pi^h}{\sigma_\eta}\right)\right) + (1 - P)0.5 \qquad i_2 = d^{-h}0.5 + d^h\left(\lambda + 1 - \Phi\left(\frac{\pi^h}{\sigma_\eta}\right)\right)$$

The central bank can thus, again, achieve that $i_1 = 0.5$ by choosing accurately π^h . By the martingale property and the market rate equations above, this contingent liquidity injection at the same time leads to $i_2 = 0.5$.

The central bank may hence achieve zero losses with regard to all three loss functions considered.

4.4.2 The variable rate tender under rate cut expectations

This case is identical to the case of variable rate tenders under rate hike expectations. Indeed, a general symmetry between the two cases holds for variable rate tenders.

5. Conclusions

On the basis of a simple model of the bidding of banks in central bank open market operations, some important aspects of the question of the optimal tender procedure and optimal allotment policy of the central bank and in particular the debates on "over"- and "under-" bidding were revisited with a focus on the bidding equilibrium under rational expectations, i.e. under a full anticipation of the central bank's allotment strategy. The cases of a one-day and a two days reserve maintenance period were distinguished, the latter allowing to incorporate rate change expectations. The one-day maintenance period case was analyzed to focus especially on the precise role of bidding costs for the bidding equilibrium. While the modeling focused mainly on the fixed rate tender (which included the case of variable rate tenders with minimum bid rate under rate cut expectations), the variable rate tender was included in the analysis in a very simple form to allow a comparative assessment. Among the fixed rate tenders, the 100% allotment and the discretionary allotment variant were distinguished, whereby the latter was further specified by the allotment function followed by the central bank. In terms of central bank preferences, three alternative or complementary aspects were considered, namely the stability of interest rates within the reserve maintenance period, the smoothness of the reserve fulfillment path, and the bidding costs. In drawing policy conclusions from the results exposed below with regard to the choice of the tender procedure, it should however be recalled that one of the three main aims of the Eurosystem's MROs, namely the signaling of the monetary policy stance, has not been analyzed in depth in the present paper. Without going to details, it appears likely that the fixed rate tender and the variable rate tender procedure with minimum bid rate go in this respect beyond what the pure variable rate tender can perform. This being said, the conclusions of the paper may be summarized along the following seven aspects.

Firstly, the failure of banks to coordinate their bidding perfectly and the impossibility to make a perfect use even of published autonomous factor forecasts, as well as the costs attached to bidding appeared as necessary ingredients of a sensible model of the bidding behavior of banks in central bank operations. These elements have so far been partially neglected in the literature.

Secondly, in the absence of rate change expectations, both the variable and the discretionary fixed rate tender perform well. The 100% allotment rule has the disadvantage of implying additional interest rate volatility and noise in the reserve fulfillment path. If the discretionary fixed rate is seen to have advantages over the variable rate tender which are outside the scope of the model presented in this paper, then the fixed rate tender would be the adequate choice under conditions of stable interest rates. Indeed, in the case of the euro area, as long as no rate change expectations prevailed, the fixed rate tender regime worked smoothly. A certain preference of market participants for this tender procedure, as long as it does not cause problems relating to rate change expectations, is indeed sometimes reported (e.g. being simpler than variable rate tenders for less informed banks, sending an additional monetary policy signal, etc.). The 100% allotment rule has never been proposed forcefully as a viable alternative to the discretionary fixed rate tenders by market participants, who seem to assign a high value to the ability of the central bank to act as coordinating agent by setting the allotment amount.

Thirdly, it appears that in the absence of rate change expectations, in case of the use of the fixed rate tender procedure and a large free bidding potential, the liquidity management of the central bank should be neutral in the sense that, the central bank should target neutral liquidity conditions at the end of the reserve maintenance period. In case there is no large free bidding potential, this conclusion is no longer generally valid, since a tighter liquidity management may contribute to reduce the risks of underbidding and hence the volatility of interest rates. The fact that the ECB, which does not impose bids to be covered by collateral, tends to follow a neutral allotment policy, while the Bundesbank, who required such a coverage, tended to be tight, appears to be in line with this insight.

Fourthly, under conditions of rate change expectations, it appears that the fixed rate tender tends to have, in the model proposed here, some specific disadvantages relative to the variable rate tender. The discretionary fixed rate tender has a weakness especially under rate hike expectations, namely overbidding and its associated costs. Under rate cut expectations, the discretionary fixed rate tender invites to underbid to some extent and hence to create some associated instability of overnight rates. At the same time, the reserve fulfillment path tends to be destabilized. The 100% fixed rate tender tends to destabilize the reserve fulfillment path to a maximum both under rate cut and under rate hike expectations, and also creates interest rate volatility associated to the imperfect co-ordination of bidding and use of autonomous factor forecasts.

Fifthly, if discretionary fixed rate tenders are chosen under rate hike expectations, the model suggests that it is difficult to "fight" against overbidding through excess liquidity. Indeed, the approach of the ECB in the second half of 1999 to provide relatively systematically excess liquidity at the end of the reserve maintenance periods did not stop overbidding, but contributed to volatility of overnight rates and allotment ratios. The fact that allotment ratios always rose steeply in the last tender of the maintenance period, however without, in the relevant period reaching 100%, as the present model would have predicted, illustrated that the market learns relatively slowly. Of course, the ECB had not announced its policy, and it therefore seems natural that banks needed time to extract it.

Six, under discretionary fixed rate tenders and rate cut expectations, a small bail-out coefficient will ensure that underbidding will tend to be relatively limited. Only in case the central bank has a strong aversion against interest rate volatility, it may consider a full bailout and thereby eliminate the interest rate shocks due to the imperfect coordination of bidding and use of autonomous factor forecasts, which seems however unlikely to be applicable in practice. In so far, one may conclude that both parties in the debate reported in the Boersenzeitung (see section 2) are to a certain extent right: underbidding will happen again, but the central bank has, by choosing a low bail-out coefficient, the possibility to make it less frequent. Some additional noise will in any case remain relative to a switch to the variable rate tender.

Finally, one may come back to the academic debate presented in section 2 on whether the fixed rate tender is generally a badly specified procedure to which overbidding is inherent (Nautz and Oechsler [1999], Erhard [2000]) or whether it was just the ECB which steered liquidity in a too tight way and thereby triggered the overbidding problem (Ayuso and Repullo [2000], [2001]). In the light of the model presented in this paper and the interpretation of the evidence it suggests, both interpretations seem to be misleading. Instead, the conclusion should be that fixed rate tenders are well suited to conditions of stable interest rates, but that they may indeed cause some noise in an environment of strong rate change expectations, whereby the central bank then has only limited, if any, possibilities of stabilization through a specific liquidity policy.

References

Ayuso, J. and Repullo, R. [2000], "A Model of the Open Market Operations of the European Central Bank", CEPR Discussion Paper No. 2605.

Ayuso, J. and Repullo, R. [2001], "Why did banks overbid? An empirical model of the fixed rate tenders of the European Central Bank", memo.

Bartolini, L., Bertola, G. and Prati, A. [2000], "Banks' Reserve Management, Transaction Costs and the Timing of Federal Reserve Intervention", Working Paper, *Federal Reserve Bank of New York*, May 2000.

Bindseil, Ulrich [2001] "Central bank forecasts of autonomous factors: quality, publication, and the control of the overnight rate", *ECB Working Paper Series, No. 70.*

Bindseil, U. and Mercier, P. [1999], "The single monetary policy and some aspects of its implementation", in: *SNF Aarbook 1999*, O. Thogersen (ed.), Bergen, Norway: Fagbokforlaget.

Bindseil, U. and Seitz, F. [2001], "The supply and demand for Eurosystem deposits – the first 18 months", *ECB Working Paper Series No. 44.*

Breitung, J. and Nautz, D. [2001], "The empirical performance of the ECB's repo auctions: Evidence from aggregate and individual bidding data", memo.

ECB [2000] "The single monetary policy in stage three – general documentation on ESCB monetary policy instruments and procedures", September 2000.

ECB [2001a] "The collateral framework of the Eurosystem", Monthly bulletin of the ECB, May 2001.

ECB [2001b] "Bidding behaviour of counterparties in the Eurosystem's regular open market operations", Monthly bulletin of the ECB, October 2001.

Ehrhart, Karl-Martin [2000], "Repo Auctions: Experimental investigation of the fixed rate tender", Memo.

Greene, William H. [1997], *Econometric Analysis*, Third Edition, Prentice-hall, Upper Saddle river, New Jersey.

Hamilton, J.D. [1996], "The daily market for federal funds," *Journal of Political Economy*, 104, 26-56.

Hamilton, J.D. [1998], "The supply and demand for Federal Reserve Deposits," *Carnegie-Rochester Conference Series on Public Policy*, 49, 1-44.

Hartmann, P, M. Manna, and A. Manzanares [2001], "The Microstructure of the Euro Money Market", ECB working paper series, No. 80.

Issing, O., V. Gaspar, I. Angeloni and O. Tristani [2001], *Monetary policy in the euro area*, Cambridge University Press.

Martinez Resano, Jose Ramon [2001], "Revisiting the overbidding problem in the light of liquidity and the EONIA swap market information", memo.

Nautz, Dieter [1995], "Optimal Bidding in multi-unit auctions with many bidders", *Economics Letters*, 48, 301-306.

Nautz, Dieter and Joerg Oechsler [1999], "The Repo Auctions of the European Central Bank and the Vanishing Quota Puzzle," SFB 373 working paper, Humboldt-University Berlin.

Nyborg, Kjel G. and Strebulaev, Ilya A. [2001], "Collateral and Short Squeezing of Liquidity in Fixed rate tenders", Memo.

Perez-Quiros, G. and Rodriguez, H. [2000], "The daily market for funds in Europe: Has something changed with the EMU", *ECB Working Paper Series No. 67.*

Valimaki, T. [2001], "Fixed rate tenders and the overnight money market equilibrium", *Bank of Finland discussion paper 8/2001.*

Van Huyck, John B., R.C. Batalio, and R. O. Beil [1990], "Tacit Coordination games, Strategic uncertainty, and co-ordination failures", *American Economic Review*.

Wuertz, F. [2001], "The spread between the EONIA rate and the mid of the corridor set by standing facilities", memo.

Annex: The likelihood of underbidding under pure variable rate tenders: some illustrative evidence.

The crucial assumption that was behind the comparative properties of pure variable rate tenders in an environment of rate cut expectations was that underbidding would normally not occur in variable rate tenders. While the idea is rather intuitive, it appears useful to collect some supportive evidence.

The ECB has conducted so far pure variable rate tenders only in its longer term refinancing operations (LTROs). The longer term refinancing operations have a three-months maturity and aim at providing additional longer term refinancing to the financial sector. The Eurosystem does not, as a rule, intend to send signals to the market and therefore normally acts as a rate taker by pre-announcing allotment volumes and specifying them as pure variable rate tenders. Of course, the bidding in these operations may be driven by different considerations than the bidding in the MROs. Bidding could appear less attractive than in MROs, ceteris paribus, since the allotment volumes are smaller (currently EUR 20 billion). Indeed, less banks participate (on average 271, against 635 in the MROs in the first three years of the euro). Hence, one would expect underbidding to be more likely in full variable rate tender LTROs than in corresponding MROs. However, in none of the 37 LTROs conducted so far, underbidding has been observed, also not when rate cut expectations were intense, as on several occasions in 2001. On average, the bid volume exceeded the allotment volume by EUR 34.6 billion, with a standard deviation of EUR 18.0 billion and a minimum value of EUR 0.9 billion.

The MROs conducted by the ECB as variable rate tenders with minimum bid rate can be considered as genuine variable rate tenders in circumstances of rate hike expectations, when the relevance of the minimum bid rate as restriction to the bidding vanishes. Rate hike expectations prevailed during the variable rate tender period basically in the second half of 2000 (indeed, rates were increased twice in the course of this semester). No case of underbidding could be observed in this semester, the average amount of excess bids being EUR 70.0 billion, the minimum being EUR 1.9 billion and the standard deviation EUR 34.4 billion. In contrast, in 2001, when rate cut expectations prevailed (indeed, rates were cut 4 times), and the minimum bid rate restriction turned the MROs in something much closer to fixed rate tenders, four occurrences of underbidding were observed.

Finally, one may seek evidence that underbidding is extremely unlikely under variable rate tenders from other central banks which apply (or applied) pure variable rate tenders. A natural candidate for this is the Bundesbank, which conducted, since 1988, normally weekly reverse open market operations with 2-week maturity.¹⁶ As the Eurosystem's MROs, these open market operations were the main instrument of liquidity management¹⁷. Until 1996, the tenders were conducted most of the time as variable rate tenders (with one longer period of use of the fixed rate tenders, namely from July 1994 to April 1995; from February 1996 on to the start of the euro, the Bundesbank conducted fixed rate tenders). In the more than 300 pure variable rate tender regular open market operations of the Bundesbank since 1988, no case of underbidding ever occurred.

In sum, one may conclude that the pieces of illustrative evidence presented here unambiguously confirm the intuition that underbidding should be an extremely unlikely event in the case of a pure variable rate tender. Hence, this tender procedure allows the central bank to precisely steer liquidity conditions according to its wishes.

¹⁶ I wish to thank Henner Asche from the Deutsche Bundesbank for information on the Bundesbank's experience.

¹⁷ One difference may be mentioned which argues in favor of a more likely underbidding in the Bundesbanks' tenders: bids had to be fully covered by collateral, while in the Eurosystem's framework, only actual allotments have to be covered by collateral. This difference tends to make bidding itself less costly for banks under the Eurosystem framework.

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