When Fiscal Consolidation Meets Private Deleveraging

Javier Andrés   Óscar Arce   Carlos Thomas[*]

University of Valencia   Banco de España   Banco de España[†]

This draft September 2016

Abstract

We analyze the interaction between fiscal consolidations and private sector deleveraging in an economy inside a monetary union. Preexisting long-term collateralized private debt –a core ingredient of the deleveraging process- plays a critical role in shaping fiscal multipliers. By buffering the short-run fall in debtors' spending capacity, long-run private debt reduces the short-run multipliers of aggressive (large and/or fast) consolidations. However, absent credibility concerns, aggressive consolidations raise the intensity and length of private deleveraging, causing higher output losses over the medium-run. In terms of discounted output losses and welfare, this latter effect dominates, so that larger and faster consolidations are relatively costlier than smaller and more gradual ones. Also, in this environment, alternative budgetary instruments generate sizable differences in terms of their incidence on private deleveraging dynamics and, hence, on their overall output costs of fiscal consolidations.

1

1   Introduction

Since the beginning of the recent crisis, public debt as a percentage of GDP has increased by more than 20 points on average in the OECD countries, up to levels hardly ever seen before in peacetime. For instance, in the euro area, public debt at the end of 2015 was almost 1.5 times higher than in 2007, with a relatively large number of countries where this multiple was well above 2 and, even, some where debt has more than tripled during this period. Absent inflation, and with the prospects of fragile growth in the coming years, considerable budgetary adjustments are called for to reduce public debt-to-GDP ratios towards more sustainable levels. This raises a number of questions about the macroeconomic impact of fiscal consolidations and the best strategy to conduct these processes.

Issues like the appropriate pace of fiscal consolidations, or whether these should be based on spending cuts or tax hikes, have been thoroughly discussed in the literature. In the aftermath of the financial crisis, these questions acquire a new dimension since the effectiveness and costs of fiscal retrenchments are likely to be affected by the lack of room of manoeuvre of conventional monetary policy at the zero lower bound (ZLB) and by the legacy of high private debt. The relevance of the ZLB for fiscal policy has been extensively studied but the literature has largely ignored the bidirectional links between public and private debt-consolidation processes. The mutual interaction between public and private deleveraging has occupied a central role in policy discussions, but there is scant academic research that considers how consolidation efforts by the government impinge upon private debt reduction and spending decisions and, conversely, how private deleveraging shapes the impact of fiscal consolidations. Placing private and public debt consolidation under the same umbrella helps understand better the effect of alternative fiscal consolidation strategies and the determinants of the length, depth and costs of private deleveraging. In this paper we develop a framework to analyze this interaction.

Inspired by the current macrofinancial landscape faced by a number of countries in the euro area that are embarked in ambitious fiscal consolidations, we build a general equilibrium model of an economy that belongs to a monetary union. The lack of an independent monetary policy produces effects similar to those of a binding ZLB but of a more structural nature. In the model, private debt is long-term and borrowers face collateral constraints. The combination of long-term debt and collateral restrictions give rise to an asymmetric double debt-regime. When the value of collateral is sufficiently high, borrowers receive new credit flows, in an amount determined by such value; when the latter is low, credit flows freeze and outstanding debt is reduced at the pre-set contractual amortization rate, thus pushing the economy into a slow deleveraging path. Only sufficiently large negative shocks (like e.g. an intense credit-crunch or a large fiscal contraction) drive the economy into this last regime. As time passes and legacy debts are progressively paid back, debtors' net worth recovers up to a point at which the value of collateral is sufficiently high so as to sustain new credit flows, thus bringing the deleveraging process to its end. In this way, the length and intensity of private deleveraging are endogenously determined in the model.

In this context, we analyze how the size, speed and composition of fiscal consolidations affect the economy, with particular attention on how consolidations interact with private deleveraging. Our main results can be summarized as follows. First, the size of fiscal consolidation has different effects on fiscal sacrifice ratios at different horizons. On the one hand, larger fiscal consolidations imply *lower* fiscal sacrifice ratios (i.e. output losses per unit of reduction in the long-run government debt-to-GDP ratio) in the *short* run. When the fiscal shock is small, constrained agents have to reduce their spending along with the falling 'excess collateral' (collateralizable value of their real estate holdings net of outstanding debt) as long as the latter is positive. If the fiscal contraction is large enough, borrowers' excess collateral vanishes as it does the flow of new loans. As mortgage contracts imply that borrowers do not have to reduce their outstanding debt down to the lower collateral value, in this regime, they pay the amortization rate without receiving new credit. In this way, long-term mortgages prevent debtors' net debt flows and hence their spending capacity from falling proportionally to the size of the shock. Thus, by breaking the link between

debt dynamics and collateral values while in a deleveraging phase, long-term debt contracts cushion the impact of negative fiscal shocks on borrowers' spending capacity. We label this mechanism as the '*buffering effect*'. On the other hand, larger consolidations imply *higher* relative output losses over the *medium* run, by endogenously increasing the duration and depth of private deleveraging; we call this the '*duration effect*'.

Second, as regards the speed of a fiscal consolidation (of a given size), we find that *front-loaded* fiscal consolidations aimed at lowering the government debt-to-GDP ratio faster entail *higher* welfare costs as compared with more gradual adjustments. This is because the short-run utility costs of more aggressive consolidations dominate the medium-run gains in present-discounted terms. Key to this result is the fact that faster consolidations produce larger falls of collateral values which, in turn, prolong the duration of borrowers' deleveraging phase and postpone the recovery following a large negative shock. Third, and finally, fiscal adjustments based on either expenditure cuts or capital tax hikes have a disproportionate effects on the duration and the macroeconomic costs of private deleveraging *vis-à-vis* those based on consumption or labor income tax hikes.

Our results shed some light on the ongoing debate about the appropriate design of fiscal consolidations in terms of their size, speed and composition. Some pundits have advocated large and quick fiscal adjustments as a means of bringing public debt back into a sustainable path. Such a strategy is typically defended upon the grounds that a quick adjustment reduces the period of fiscal pain and gets public finances rapidly back in good shape so that they can play a countercyclical role. According to this view, even if they accentuate the depth of the recession, sharp fiscal adjustments might make it shorter and eventually less painful. Other authors support milder and/or more gradual consolidations when possible, arguing that fiscal multipliers are larger in recessions so that postponing the bulk of the fiscal retrenchment until the economy starts recovering would be the appropriate recipe. The presence of long term debt in our model reinforces the latter policy prescription, although it turns the argument around. Smaller consolidations actually increase the (relative) output costs in the short-run, precisely when the recession is more severe, due to

4

the above-mentioned buffering effect of long-term debt on private spending. Rather, what makes small consolidations more benign in relative terms over the medium run is the fact that, by favoring a faster recovery in the value of collateral, they shorten the duration of the deleveraging phase, thus bringing forward the time at which borrowers regain access to fresh credit.

The impact of fiscal consolidation on the length and intensity of private deleveraging are key implications of our model. In that sense, our results must be interpreted as uncovering an often neglected channel that might be relevant in the design of consolidation packages, namely the endogenous response of private deleveraging. Other channels, such as deflationary effects or the lack of monetary accommodation, are of lesser importance in our context. Likewise, fiscal rules in our model always render public debt sustainable and we do not consider an endogenous response of sovereign spreads. Thus, risks to sustainability or imperfect credibility of fiscal policy announcements, which might constitute powerful arguments in favor of fast consolidation programs, are absent here.

The rest of the paper is organized as follows. We briefly describe the related literature in Section 2. The model and the baseline calibration are presented in Section 3. In Section 4 we analyze the impact of alternative consolidation strategies and in Section 5 we perform a similar exercise against the backdrop of a *credit crunch* scenario. Section 6 concludes.

2   Related literature

There is a vast literature assessing the effects of consolidations. A fair reading of the evidence suggests that fiscal consolidations are successful in delivering conditions more favorable for growth in the medium term, although they might exert non-negligible short term output losses.[1] Also, adjustments in public spending, rather

---

[1]There is also a strand of the empirical literature that finds that, under some conditions, fiscal consolidations might even be beneficial in the short run. See, among others, Alesina and Ardagna (2009), Cogan, Taylor, Wieland and Wolters (2013), Giavazzi and Pagano (1990), Hemming, Kell and Mahfouz (2002) and Perotti (1999).

than tax hikes, make these consolidations more effective and lasting, and impose a lesser drag on the economy in the short run.[2] For instance, Kumar, Leigh and Plekhanov (2007) study fiscal consolidations in 24 OECD countries and find that they can have positive long-run effects, particularly when the gain in fiscal space is used to cut capital income taxes. However, these long-run gains may not occur if the consolidation involves cuts in public infrastructure spending. Forni, Monteforte and Sessa, (2009) find that fiscal consolidations in the euro area increase GDP and all its components over the long run by around $5\% - 7\%$.

Wieland and Wolters (2013) have simulated the effect of a fiscal consolidation strategy in the United States that brings the budget to balance and find also a positive and significant effect on GDP. Almeida, Castro, Mourinho and Maria (2013) simulate the impact on economic activity and welfare of a permanent decline in the ratio of public debt to GDP in a small euro area economy with no independent monetary policy. They find that spending cuts (in government consumption and transfers to households) combined with a reduction in the labor income tax have positive long-run impacts on economic activity and significantly improve the net foreign asset position. This policy mix is also found to penalize real GDP by less than tax hikes. As for the timing, they conclude that smooth fiscal adjustments reduce the negative impact on output but prolong the period before the economy reaps their full benefits.

Some of these results have been called into question in the aftermath of the financial crisis. For one thing, in many jurisdictions, the capacity of monetary policy to mitigate the short run costs of fiscal retrenchments is limited by ZLB, which is thought to raise the magnitude of fiscal multipliers (Christiano, Eichenbaum and Rebelo, 2011 and Woodford, 2011). Moreover, Eggertsson (2010) argues that, unlike outside the ZLB, the output effect of spending cuts is then higher than that of tax rate hikes. Erceg and Lindé (2013) find similar conclusions in a model of a monetary union, in which small open economies lose control of monetary policies.[3] In the same

---

[2] von Hagen, Hallet and Strauch (2001) find that government spending cuts are key in successful fiscal consolidations and point out to the importance of institutional arrangements in pursuing these adjustments.

[3] See also Farhi and Werning (2012) and IMF, Fiscal Monitor, October 2014, c.2

vein, the IMF (2014) has reported that government spending cuts have more negative effects on employment than tax-based consolidations after a protracted recession. But the ZLB and fiscal policy are intertwined in more complex ways. For instance, Erceg and Lindé (2014) show that the size of the fiscal multiplier also depends on the incidence of fiscal shocks on the duration of the ZLB regime. Also, Bi, Leeper and Leith (2013) study the conditions under which consolidations subject to uncertainty about their composition and intensity may be expansionary in the short run, and conclude that such conditions are likely to be more demanding when they are based on spending cuts and the interest rate approaches the ZLB.

The recent financial crisis has left behind a landscape of heavily indebted households and firms. This situation is likely to affect spending decisions and, hence, it seems natural to incorporate considerations about private indebtedness in the analysis of fiscal policies. In this vein, Eggertsson and Krugman (2012) show how fiscal multipliers increase in the presence of high private debt.[4] Despite this, there have been few attempts to analyze jointly the dynamics of private and pubic debt. Batini, Melina and Villa (2016) is one exception. They find that the drag that high private debt imposes on economic growth is more severe than the one caused by public indebtedness. As for fiscal policy, they argue that following a contractionary shock it should keep the appropriate pace to sustain public finances without inflicting an additional burden on the recovery process. Whereas some authors argue that, despite their stronger short-term impact on GDP, front-loading adjustments might entail lower costs in present value terms (ECB, Monthly Bulletin, 2014), others defend gradual consolidations, upon the hypothesis that short term fiscal multipliers in the aftermath of a financial crisis are likely to be higher than long-term ones.[5] In this spirit, Blanchard and Leigh (2013) advocate modulating fiscal adjustments in order to avoid the large fiscal multipliers that are typically associated to recessions.[6]

---

[4]Other authors have confirmed this conection in a variety of theoretical and empirical settings. Andrés, Boscá and Ferri (2015) discuss this effect in the context of collateralized debt and labor market frictions. Kaplan, Violante and . Weidner (2014) and Cloyne and Surico (2014) analyze the relationship between household financial position and spending.

[5]See Fletcher and Sandri (2015) and Corsetti, Kuester Meier and Müller (2010).

[6]Corsetti, Kuester, Meier, and Müller, (2013) emphasize that this may be particularly true when sovereign spreads are not too high or too sensitive to Debt/GDP ratios.

Moreover, large multipliers in the short run can cast a long shadow on output in the presence of hysteresis channels.[7]

To the best of our knowledge, this literature does not allow for the endogenous determination of the duration and intensity of deleveraging of the private sector. The presence of long term collateralized debt in our model has a profound effect on the way private spending adjusts to exogenous shocks, fiscal or otherwise. Long-term debt contracts induce non-linearities in the multipliers associated to fiscal retrenchments and, what is more important, establish a theoretical link between the intensity and timing of fiscal adjustments and the duration and costs of private deleveraging processes and, hence, of recessions.

3 Model

We present a general equilibrium model of a small open economy that belongs to a monetary union. The real side of the economy is standard and there are several types of consumers who differ in the intensity with which they discount the future. Entrepreneurs produce an intermediate good using labor and consumption goods and sell it to retailers, who transform it into consumption good varieties. Construction firms produce real estate, both for residential and commercial use, whereas equipment capital is produced by capital goods producers. Retailers in the consumption-goods sector are characterized by monopolistic competition and nominal rigidities, all other goods markets operate under perfect competition. The fiscal authority collects taxes on households and entrepreneurs, consumes, and issues non-contingent nominal debt, according to a fiscal rule. Public debt is held by patient domestic agents and foreigners.

In what follows, we describe the basic elements of the model. In the notation used herein, all variables are expressed in real terms unless otherwise specified, with the consumption goods basket acting as the numeraire. Appendix A contains the

---

[7]See See Auerbach and Gorodnichenko (2012) and Hernández de Cos and Moral-Benito (2013) on the size of multipliers in difficult times and DeLong and Summers (2012) and Fatás and Summers (2015), reggarding the size of hysteresis effetcs.

whole set of equilibrium conditions.

3.1 Households

Households obtain utility from consumption goods and from occupying housing units. There are three types of consumers: patient households, impatient households, and (impatient) entrepreneurs. In the class of equilibria analyzed in the following sections, the latter two groups borrow from the former and from the rest of the monetary union by issuing long-term nominal debt. In periods in which borrowers are able to receive new credit flows, they do so subject to collateral constraints. Real estate is the only collateralizable asset. We will henceforth refer to impatient and patient households as 'constrained' and 'unconstrained' households, respectively. There is a representative constrained household and a representative unconstrained household, denoted respectively by superscripts c and u.

3.1.1 Cost minimization

Households consume a basket of home and foreign goods, denoted respectively by subscripts H and F,

$$c_t^x = \left(\omega_H^{1/\varepsilon_H} \left(c_{H,t}^x \right)^{(\varepsilon_H-1)/\varepsilon_H} + (1 - \omega_H)^{1/\varepsilon_H} \left(c_{F,t}^x \right)^{(\varepsilon_H-1)/\varepsilon_H} \right)^{\varepsilon_H/(\varepsilon_H-1)}, \qquad (1)$$

for $x = c, u$; $c_{H,t}^x$ is a basket of domestic good varieties,

$$c_{H,t}^x = \left(\int_0^1 c_{H,t}^x (z)^{(\varepsilon_p-1)/\varepsilon_p} \, dz \right)^{\varepsilon_p/(\varepsilon_p-1)}, \qquad (2)$$

where $\varepsilon_p > 1$ is the elasticity of substitution across consumption varieties $z \in [0,1]$. Let $P_{H,t}(z)$ denote the price of home good variety z, and $P_{F,t}$ the price of the foreign goods basket. Household $x = c, u$ minimizes nominal consumption expenditure, $\int_0^1 P_{H,t}(z) c_{H,t}^x (z) \, dz + P_{F,t} c_{F,t}^x$, subject to (1) and (2). The first order conditions of

the static cost minimization problem can be expressed as

$$c_{H,t}^x = \omega_H \left(\frac{P_{H,t}}{P_t} \right)^{-\varepsilon_H} c_t^x, \quad c_{F,t}^x = (1 - \omega_H) \left(\frac{P_{F,t}}{P_t} \right)^{-\varepsilon_H} c_t^x, \quad c_{H,t}^x (z) = \left(\frac{P_{H,t}(z)}{P_{H,t}} \right)^{-\varepsilon_p} c_{H,t}^x,$$

(3)

where

$$P_t = \left(\omega_H P_{H,t}^{1-\varepsilon_H} + (1 - \omega_H) P_{F,t}^{1-\varepsilon_H} \right)^{1/(1-\varepsilon_H)}, \quad P_{H,t} = \left(\int_0^1 P_{H,t}(z)^{1-\varepsilon_p} \, dz \right)^{1/(1-\varepsilon_p)}$$

are the *consumer price index* (CPI) and the *producer price index* (PPI), respectively. Nominal spending in domestic goods equals $\int_0^1 P_{H,t}(z) c_{H,t}^x(z) \, dz = P_{H,t} c_{H,t}^x$, whereas total nominal consumption spending equals $P_{H,t} c_{H,t}^x + P_{F,t} c_{F,t}^x = P_t c_t^x$.

As noted before, consumption goods are also used as inputs by construction firms and equipment capital producers. The latter are assumed to combine home and foreign goods analogously to households, and similarly for domestic good varieties. This gives rise to investment demand functions analogous to (3).

3.1.2 Unconstrained households

The unconstrained household maximizes

$$E_0 \sum_{t=0}^\infty (\beta^u)^t \left\{ \log (c_t^u) + \vartheta \log (h_t^u) - \chi \int_0^1 \frac{n_t^u (i)^{1+\varphi}}{1 + \varphi} di \right\},$$

(4)

where $n_t^u(i)$ are labor services of type $i \in [0, 1]$ and h_t^u are housing units, subject to the following budget constraint (expressed in units of the consumption goods basket),

$$(1 + \tau_t^c) c_t^u + d_t + b_t^{gu} + p_t^h \left[h_t^u - (1 - \delta_h) h_{t-1}^u \right] = \frac{R_{t-1}}{\pi_t} \left(d_{t-1} + b_{t-1}^{gu} \right)$$
$$+ (1 - \tau_t^w) \int_0^1 \frac{W_t(i)}{P_t} n_t^u(i) \, di - T_t,$$

where d_t is the real value of net private international debt holdings, b_t^{gu} is the real value of nominal domestic government debt holdings, R_t is the gross riskless nominal

10

interest rate, δ_h is the depreciation rate of real estate, p_t^h is the real price of real estate, $\pi_t \equiv P_t/P_{t-1}$ is gross CPI inflation, $W_t(i)$ is the nominal wage for labor services of type i, τ_t^c and τ_t^w are tax rates on consumption and labor income, respectively, and T_t are lump-sum taxes. The first order conditions are standard (see Appendix A).

3.1.3 Constrained households

The constrained household's preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log(c_t^c) + \vartheta \log(h_t) - \chi \int_0^1 \frac{n_t^c(i)^{1+\varphi}}{1+\varphi} di \right\}, \tag{5}$$

where $\beta < \beta^u$, i.e. the constrained household is relatively impatient; h_t and $n_t^c(i)$ are housings units owned and type-i labor services supplied by the constrained household, respectively. The household faces the following budget constraint,

$$(1+\tau_t^c) c_t^c + \frac{R_{t-1}}{\pi_t} b_{t-1} + p_t^h [h_t - (1-\delta_h) h_{t-1}] = b_t + (1-\tau_t^w) \int_0^1 \frac{W_t(i)}{P_t} n_t^c(i) di - T_t,$$

where b_t is the real value of household debt outstanding at the end of period t.

Unlike in most of the literature, which typically assumes short-term (one-period) debt, we assume that private debt contracts are *long-term*. In the interest of tractability, we assume that at the beginning of time t the household repays a fraction $1 - \gamma$ of all nominal debt outstanding at the end of period $t - 1$, regardless of when that debt was issued.[8] This type of perpetual debt is similar to the one proposed by Woodford (2001) as a tractable way of modelling long-term debt. In *real* terms, the outstanding principal of household debt then evolves as follows,

$$b_t = \frac{b_{t-1}}{\pi_t} + b_t^{new} - (1-\gamma) \frac{b_{t-1}}{\pi_t} = b_t^{new} + \gamma \frac{b_{t-1}}{\pi_t}, \tag{6}$$

where b_t^{new} is gross new credit net of *voluntary* amortizations, i.e. amortizations

---

[8]Total debt repayments in each period are then $(1 - \gamma) + (R_{t-1} - 1)$ times nominal debt outstanding, i.e. the sum of amortization and interest payments.

beyond the contractual debt repayment $(1 - \gamma) b_{t-1}/\pi_t$.

Following Iacoviello (2005) and Kiyotaki and Moore (1997), outstanding debt b_t cannot exceed a fraction m_t (the 'loan-to-value ratio', which we assume to be exogenously time-varying) of the expected discounted value of the household's residential stock: $b_t \leq m_t R_t^{-1} E_t \pi_{t+1} p_{t+1}^h h_t$. For brevity, we will refer to such pledgeable value of collateral as *collateral value*. This debt limit, however, is only effective as long as it exceeds $\gamma b_{t-1}/\pi_t$, which we will henceforth refer to as the *contractual amortization path*. We may refer to the gap between collateral value and the contractual amortization path as 'excess collateral'. If the collateral value falls below such path (i.e. if excess collateral becomes negative), lowering b_t to the value of collateral would require lenders not only to reduce gross new credit to zero (its lower bound), but also to *impose* additional amortizations beyond those agreed in the contract (i.e. $b_t^{new} < 0$). Since lenders cannot force borrowers to pay back faster than the contractual amortization rate, the contractual amortization path becomes the effective debt limit. Therefore, long run debt implies the following *asymmetric borrowing constraint*,

$$
b_t \leq R_t^{-1} m_t E_t \pi_{t+1} p_{t+1}^h h_t, \text{ if } \frac{m_t}{R_t} E_t \pi_{t+1} p_{t+1}^h h_t \geq \gamma \frac{b_{t-1}}{\pi_t}, \tag{7}
$$

$$
b_t \leq \gamma \frac{b_{t-1}}{\pi_t}, \qquad\qquad \text{if } \frac{m_t}{R_t} E_t \pi_{t+1} p_{t+1}^h h_t < \gamma \frac{b_{t-1}}{\pi_t}. \tag{8}
$$

This asymmetry gives rise to a *double debt regime*. In 'normal times', in which collateral values exceed the contractual amortization path, debt is restricted by the former. In this baseline regime, households can receive new credit against their housing collateral, with the constraint that such new credit does not exceed the gap between collateral values and the amortization path.[9] However, in the face of shocks that reduce collateral values sufficiently, the economy switches to an alternative regime, in which new credit dries up and debt is restricted by the contractual amortization path.[10] An important element of our framework is that changes from one regime to

---

[9]Indeed, from (6) and (7) we obtain $b_t^{new} \leq m_t R_t^{-1} E_t \pi_{t+1} p_{t+1}^h h_t - \gamma b_{t-1}/\pi_t$.

[10]From (6) and (8): $\left(m_t R_t^{-1} E_t \pi_{t+1} p_{t+1}^h h_t - \gamma b_{t-1}/\pi_t \right) < 0 \Rightarrow b_t^{new} = 0$.

the other take place *endogenously.*

3.2 Production

3.2.1 Entrepreneurs

A representative entrepreneur produces an intermediate product and sells it to retailers at a perfectly competitive real (CPI-deflated) price mc_t. The entrepreneur maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \log c_t^e, \tag{9}$$

with the consumption basket c_t^e defined analogously to (1), subject to a budget constraint. Entrepreneurs, who obtain operating profits from their activities, are also assumed to own the firms in the other productive sectors of the economy. All operating profits (net of capital depreciation) are taxed at rate τ_t^k; since profits accrue to entrepreneurs and these are the sole owners of productive capital in the model, we will henceforth refer to τ_t^k as the *capital income tax*. The entrepreneur's budget constraint is given by

$$
\begin{aligned}
(1 + \tau_t^c) c_t^e \;=\;& \left(1 - \tau_t^k\right) \left(mc_t y_t^e - \frac{W_t}{P_t} n_t^e \right) + b_t^e - \frac{R_{t-1}}{\pi_t} b_{t-1}^e - p_t^h \left[h_t^e - (1 - \delta_h) h_{t-1}^e \right] \\
&- q_t \left[k_t - (1 - \delta_k) k_{t-1} \right] + \tau_t^k \left(\delta_h p_t^h h_{t-1}^e + \delta_k q_t k_{t-1} \right) + \left(1 - \tau_t^k\right) \sum_{s=r,h,k} \Pi_t^s,
\end{aligned}
$$

$$y_t^e = k_{t-1}^{\alpha_k} \left(h_{t-1}^e \right)^{\alpha_h} \left(n_t^e \right)^{1 - \alpha_k - \alpha_h},$$

where y_t^e is output of the intermediate good, k_{t-1} is capital equipment with unit price q_t and a depreciation rate δ_k, h_{t-1}^e is commercial real estate, n_t^e is a basket of labor services, W_t is a nominal wage index, b_t^e is the real value of entrepreneurial debt outstanding at the end of period t, and $\{\Pi_t^s\}_{s=r,h,k}$ are real profits from the retail, construction and equipment goods-producing sectors.

Entrepreneurs' maximization is also subject to an asymmetric borrowing con-

straint analogous to the one on constrained households,

$$b_t^e \leq R_t^{-1} m_t^e E_t \pi_{t+1} p_{t+1}^h h_t^e, \text{ if } \frac{m_t^e}{R_t} E_t \pi_{t+1} p_{t+1}^h h_t^e \geq \gamma^e \frac{b_{t-1}^e}{\pi_t}, \tag{10}$$

$$b_t^e \leq \gamma^e \frac{b_{t-1}^e}{\pi_t}, \qquad\qquad \text{if } \frac{m_t^e}{R_t} E_t \pi_{t+1} p_{t+1}^h h_t^e < \gamma^e \frac{b_{t-1}^e}{\pi_t}, \tag{11}$$

where we allow for a different loan-to-value ratio (m_t^e) and contractual amortization rate $(1 - \gamma^e)$ for entrepreneurs.

3.2.2 Retailers

A continuum of monopolistically competitive retailers indexed by $z \in [0,1]$ purchase the intermediate input from entrepreneurs at the real price mc_t, and transform it one for one into final good varieties. Retailers' real marginal cost is thus mc_t. Each retailer z faces a demand curve

$$y_t(z) = \left(\frac{P_{H,t}(z)}{P_{H,t}} \right)^{-\varepsilon_p} y_t \equiv y_t^d(P_{H,t}(z)), \tag{12}$$

where y_t is aggregate demand of the consumption basket (to be derived below). Let $\lambda_t^e \equiv 1/[c_t^e(1 + \tau_t^c)]$ denote the entrepreneur's marginal utility of real income. Assuming Calvo (1983) price-setting, a retailer that has the chance of setting its nominal price at time t solves

$$\max_{P_{H,t}(z)} E_t \sum_{s=0}^{\infty} (\beta\theta_p)^s \frac{\lambda_{t+s}^e}{\lambda_t^e} \left(1 - \tau_{t+s}^k\right) \left[\frac{P_{H,t}(z)}{P_{t+s}} - mc_{t+s} \right] y_{t+s}^d(P_{H,t}(z)),$$

where θ_p is the probability of not adjusting the price and τ_p is a tax rate on retailers' revenue. The first-order condition is standard, with all time-t price setters choosing a common optimal price $\tilde{P}_{H,t}$.

3.2.3 Construction and capital producing firms

A representative construction firm maximizes its expected discounted stream of profits, $E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t^e}{\lambda_0^e} \left(1 - \tau_t^k\right) \Pi_t^h$, where $\Pi_t^h = p_t^h I_t^h - \frac{W_t}{P_t} n_t^h - i_t^h$, subject to the production technology

$$I_t^h = \left(n_t^h\right)^\omega \left\{ i_t^h \left[1 - \frac{\Phi_h}{2} \left(\frac{i_t^h}{i_{t-1}^h} - 1\right)^2\right] \right\}^{1-\omega},$$

where n_t^h are labor services, i_t^h are consumption goods, and I_t^h are new real estate units.[11]

Also, a representative equipment capital producer maximizes its expected discounted stream of profits, $E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t^e}{\lambda_0^e} \left(1 - \tau_t^k\right) \Pi_t^k$, where $\Pi_t^k = q_t I_t - i_t$, subject to the technology

$$I_t = i_t \left[1 - \frac{\Phi_k}{2} \left(\frac{i_t}{i_{t-1}} - 1\right)^2\right],$$

where i_t are consumption goods, and I_t are new equipment capital goods.

3.3 Wage setting

Both entrepreneurs and construction firms use a basket of labor services offered by constrained and unconstrained households,

$$n_t^s = \left(n_t^{s,c}\right)^{\mu_s} \left(n_t^{s,u}\right)^{1-\mu_s},$$

where $n_t^{s,x}$ are labor services provided by type-x household, $x = c, u$, to each sector $s = e, h$. We assume that both worker types (constrained and unconstrained) earn the same wage. Cost minimization then implies $(1 - \mu_s) n_t^{s,c} = \mu_s n_t^{s,u}$, for $s = e, h$. From each household type, each sector demands in turn a basket of labor service

---

[11]We include labor services in the production function of construction firms so as to allow for long-run changes in real estate prices. Without labor in construction ($\omega = 0$), real estate prices are always unity in the long run. More generally, it can be shown that $p_{ss}^h = (w_{ss})^\omega \omega^{-\omega} (1 - \omega)^{-(1-\omega)}$.

varieties,

$$n_t^{s,x} = \left(\int_0^1 n_t^{s,x}(i)^{(\varepsilon_w - 1)/\varepsilon_w} \, di \right)^{\varepsilon_w/(\varepsilon_w - 1)},$$

for $x = c, u$ and $s = e, h$, where $\varepsilon_w > 1$ is the elasticity of substitution across labor varieties $i \in [0, 1]$. Cost minimization implies $n_t^{s,x}(i) = (W_t(i)/W_t)^{-\varepsilon_w} n_t^{s,x}$, for $x = c, u$ and $s = e, h$, where $W_t \equiv (\int_0^1 W_t(i)^{1-\varepsilon_w} \, di)^{1/(1-\varepsilon_w)}$ is the nominal wage index. Total demand for each variety of labor services is thus

$$n_t^x(i) \equiv n_t^{e,x}(i) + n_t^{h,x}(i) = \left(\frac{W_t(i)}{W_t} \right)^{-\varepsilon_w} \left(n_t^{e,x} + n_t^{h,x} \right) \equiv n_t^{d,x}(W_t(i)),$$

for $x = c, u$. Total nominal wage income earned by each type-x household equals $\int_0^1 W_t(i) \, n_t^x(i) \, di = W_t n_t^x$, where $n_t^x \equiv n_t^{e,x} + n_t^{h,x}$.

As in Erceg, Henderson and Levin (2000; EHL), nominal wages are set à la Calvo (1983). In particular, a union representing all type-i workers maximizes the utility of the households to which such workers belong. Let $\lambda_t^x \equiv 1/[c_t^x(1 + \tau_t^c)]$ denote the marginal utility of real income for each household type $x = c, u$. Then, a union that has the chance to reset the nominal wage at time t chooses $W_t(i)$ to maximize

$$\sum_{x=c,u} E_t \sum_{s=0}^{\infty} (\beta^x \theta_w)^s \left[\lambda_{t+s}^x \left(1 - \tau_{t+s}^w \right) \frac{W_t(i)}{P_{t+s}} n_{t+s}^{d,x}(W_t(i)) - \chi \frac{\left(n_{t+s}^{d,x}(W_t(i)) \right)^{1+\varphi}}{1 + \varphi} \right],$$

where θ_w is the probability of not adjusting the wage and $\beta^c = \beta$. All time-t wage-setters choose a common optimal wage $\tilde{W}_t$; see the first-order condition in the Appendix.

3.4 International linkages

A representative exporter produces a basket of domestic consumption goods: $x_t = (\int_0^1 x_t(z)^{(\varepsilon_p - 1)/\varepsilon_p} \, dz)^{\varepsilon_p/(\varepsilon_p - 1)}$, where $x_t(z)$ is demand for each domestic good variety. Cost minimization implies that the exporter's demand for each variety is $x_t(z) = (P_{H,t}(z)/P_{H,t})^{-\varepsilon_p} x_t$, and total spending is $\int_0^1 P_{H,t}(z) \, x_t(z) \, dz = P_{H,t} x_t$.

The exporter sells the basket x_t in export markets under perfect competition. The zero profit condition implies that the market price of the export basket is exactly $P_{H,t}$. Assuming that foreign consumers' preferences are analogous to those of domestic consumers, foreign demand for the basket of domestic goods is given by

$$x_t = \zeta \left(\frac{P_{H,t}}{P_{F,t}} \right)^{-\varepsilon_F} y_{F,t},$$

where $P_{F,t}$ and $y_{F,t}$ are the foreign price level and aggregate demand (both exogenous) and ε_F is the price elasticity of exports. Defining the *terms of trade* $p_t^* \equiv P_{H,t}/P_{F,t}$, the latter evolve according to $p_t^* = p_{t-1}^* \pi_{H,t}/\pi_{F,t}$, where $\pi_{F,t} \equiv P_{F,t}/P_{F,t-1}$ is foreign inflation.

As mentioned before, domestic agents can lend to and borrow from foreigners and other domestic agents at the riskless nominal rate R_t. Following standard practice in the literature, in order to guarantee stationarity of the country's net foreign asset position, we assume that R_t is given by

$$R_t = R^* \exp \left(-\psi \frac{P_t nfa_t}{P_{H,t} gdp_t} \right),$$

for $\psi > 0$, where R^* is the area-wide nominal interest rate (which is assumed to be constant here), and nfa_t and gdp_t are the country's real (CPI-deflated) net foreign asset position and real (PPI-deflated) GDP, both to be defined later.

3.5 Fiscal authority

Real (CPI-deflated) government debt b_t^g evolves as follows,

$$
\begin{aligned}
b_t^g &= \frac{R_{t-1}}{\pi_t} b_{t-1}^g + \frac{P_{H,t}}{P_t} g_t - 2T_t - \tau_t^w \frac{W_t}{P_t} \left(n_t^C + n_t^U \right) - \tau_t^c \left(c_t^C + c_t^U + c_t^e \right) \quad (13) \\
&\quad - \tau_t^k \left[mc_t y_t^e - \frac{W_t}{P_t} n_t^e - \left(\delta_h p_t^h h_{t-1}^e + \delta_k q_t k_{t-1} \right) + \sum_{s=r,h,k} \Pi_t^s \right].
\end{aligned}
$$

We have assumed full home bias in government consumption, such that its nominal value equals $P_{H,t} g_t$. A fiscal rule ensures stability of government debt. In particular, we will consider rules of the form

$$fi_t = fi_{t-1} + \phi_b \left(b_{t-1}^{gy} - \bar{b}^{gy} \right) + \phi_{\Delta b} \left(b_t^{gy} - b_{t-1}^{gy} \right), \tag{14}$$

where $b_t^{gy} \equiv \frac{P_t b_t^g}{P_{H,t} gdp_t}$ is the government debt-to-GDP ratio, $\bar{b}^{gy}$ is its long-run target, and $fi \in \{g, \tau^w, \tau^c, \tau^k\}$ is the fiscal instrument that is endogenized through the fiscal rule, to be specified in each case below. The other fiscal instruments are held constant at the steady state level $\overline{fi} \in \{\bar{g}, \bar{\tau}^w, \bar{\tau}^c, \bar{\tau}^k\}$.

3.6 Aggregation and market clearing

Each retailer z demands $y_t^d (P_{H,t}(z))$ units of the intermediate input, as given by (12). Total demand for the latter equals $\int_0^1 y_t^d (P_{H,t}(z)) \, dz = y_t \Delta_t$, where $\Delta_t \equiv \int_0^1 (P_{H,t}(z) / P_{H,t})^{-\varepsilon_p} \, dz$ denotes relative price dispersion. Market clearing in the intermediate good market thus requires

$$k_{t-1}^{\alpha_k} \left(h_{t-1}^e \right)^{\alpha_h} \left(n_t^e \right)^{1-\alpha_h-\alpha_k} = y_t \Delta_t.$$

As noted before, investment-goods producers and exporters demand the same combination of domestic consumption goods as consumers. Therefore, aggregate demand for the basket of domestic consumption goods is given by,

$$y_t = c_{H,t}^c + c_{H,t}^u + c_{H,t}^e + i_{H,t} + i_{H,t}^h + g_t + x_t. \tag{15}$$

Total demand for real estate must equal total supply,

$$h_t + h_t^u + h_t^e = I_t^h + (1 - \delta_h) \left(h_{t-1} + h_{t-1}^u + h_{t-1}^e \right).$$

Total demand for equipment capital must equal total supply

$$k_t = I_t + (1 - \delta_k) k_{t-1}.$$

Labor market clearing requires

$$n_t^c + n_t^u = n_t^e + n_t^h.$$

Total supply of government debt equals total demand by nationals, i.e. unconstrained households (b_t^{gu}), and by foreigners (b_t^{g*}): $b_t^g = b_t^{gu} + b_t^{g*}$. We may combine all market clearing conditions and budget constraints to obtain the current account identity,

$$nfa_t = \frac{R_{t-1}}{\pi_t} nfa_{t-1} + \frac{P_{H,t}}{P_t} x_t - \frac{P_{F,t}}{P_t} \left(c_{F,t}^c + c_{F,t}^u + c_{F,t}^e + i_{F,t} + i_{F,t}^h \right),$$

where

$$nfa_t \equiv d_t - b_t - b_t^e - b_t^{g*} = (d_t + b_t^{gu}) - b_t - b_t^e - b_t^g$$

is the real (CPI-deflated) net foreign asset position.[12] We finally define real (PPI-deflated) GDP as

$$
\begin{aligned}
gdp_t &\equiv y_t + \frac{P_t}{P_{H,t}} (q_t I_t - i_t) + \frac{P_t}{P_{H,t}} \left(p_t^h I_t^h - i_t^h \right) \\
&= \frac{P_t}{P_{H,t}} c_t^{tot} + \frac{P_t}{P_{H,t}} \left(q_t I_t + p_t^h I_t^h \right) + g_t + \left[x_t - \frac{P_{F,t}}{P_{H,t}} \left(c_{F,t}^{tot} + i_{F,t} + i_{F,t}^h \right) \right],
\end{aligned}
$$

where in the second equality we have used (15) and $z_{H,t} = \frac{P_t}{P_{H,t}} z_t - \frac{P_{F,t}}{P_{H,t}} z_{F,t}$ for $z = c^c, c^u, c^e, i, i^h$, and where $c_t^{tot} \equiv c_t^c + c_t^u + c_t^e$ is total consumption (total consumption imports $c_{F,t}^{tot}$ are defined analogously).

3.7   Calibration and solution method

We calibrate the model to the Spanish economy. As explained in the introduction, we are motivated by the recent experience of developed countries, which are still

---

[12] Notice that the distribution of unconstrained households' financial wealth between international and government bonds (d_t, b_t^{gu}) is undeterminate, as only their sum $d_t + b_t^{gu}$ is pinned down in equilibrium. This implies that the distribution of the government debt stock (b_t^g) between domestic and foreign holders (b_t^{gu}, b_t^{g*}) is also undeterminate, although it does not affect equilibrium dynamics either.

embarked in a lengthy process of both fiscal consolidation and private sector deleveraging, Spain being a good example thereof. The time period is a quarter. We match the model's steady state to a number of empirical targets in 2007, the year prior to the start of the financial crisis. Hence, our model's steady state should be interpreted as the economy's initial condition for the purpose of our simulation exercises.

The discount factor of impatient agents is set to $\beta = 0.98$, following Iacoviello (2005). For patient households, we choose $\beta^u = 1.025^{-1/4}$, which is consistent with a steady state nominal interest rate of $R_{ss} = 1.025^{1/4} \pi_{ss} = R^* e^{-\psi(nfa_{ss}^y)}$. The union-wide inflation is $\pi_{F,ss} = 1$, which implies $\pi_{H,ss} = \pi_{ss} = 1$ in a stationary equilibrium. The union's nominal interest rate is $R^* = 1.02^{1/4}$ and we set ψ to replicate net foreign assets over GDP in 2007, $nfa_{ss}^y = -79.3\%$. The inverse labor supply elasticity is set to $\varphi = 4$, consistently with a large body of micro evidence. The weight parameter in the consumption basket, ω_H, is set to match gross exports over GDP in 2007 (26.9%). The price elasticity of exports and imports is set to $\varepsilon_F = \varepsilon_H = 1$ (García *et al.*, 2009) and the scale parameter in export demand, ζ, is chosen such that steady-state terms of trade p_{ss}^* are normalized to 1.

The elasticities of substitution across varieties of consumption goods and labor services, ε_p and ε_w, determine the desired markups in product and labor markets, respectively. We set $\varepsilon_p = 7$, consistent with an initial price markup of $\varepsilon_p/(\varepsilon_p - 1) = 1.17$, which is consistent with the findings of Montero and Urtasun (2013) for Spanish firm-level data. To calibrate wage markups we follow Galí (2011) who interprets EHL model of wage-setting in a way that delivers equilibrium unemployment (see Appendix B for details). Targeting an unemployment rate of 8.6% in 2007, we obtain an initial wage markup of $\varepsilon_w/(\varepsilon_w - 1) = 1.43$, which we achieve by setting $\varepsilon_w = 3.31$.

The elasticity of entrepreneurial output with respect to capital and real estate are set to $\alpha_k = 0.11$ and $\alpha_h = 0.21$, which help to replicate the labor share of GDP in 2007 (61.6%) and the share of equipment capital in the total stock of productive capital.[13] Following Iacoviello and Neri (2010) we set $\delta_h = 0.01$, whereas δ_k is set to a standard value of 0.025. The elasticity of construction output with respect to labor ω is chosen

---

[13]The value of equipment capital was estimated at 21.4% of the total value of productive capital in 2007 (using data from BBVA Research).

Table 1: Baseline calibration

Parameter	Value	Description
Preferences		
β^u	0.994	unconstrained household discount factor
β	0.98	constrained household discount factor
φ	4	(inverse) labor supply elasticity
ϑ	0.35	weight on housing utility
ε_p	7	elasticity of subst. across consumption varieties
ε_w	3.31	elasticity of substitution across labor varieties
ω_H	0.65	weight home goods in consumption basket
ε_H	1	elasticity of imports wrt terms of trade
ε_F	1	elasticity of exports wrt terms of trade
ζ	0.83	scale parameter export demand
Technology		
α_h	0.21	elasticity output wrt real estate
α_k	0.11	elasticity output wrt equipment
ω	0.50	elasticity construction wrt labor
δ_h	0.01	depreciation real estate
δ_k	0.025	depreciation equipment
μ_e, μ_h	0.5	share of constr. households in labor baskets
Φ_h	12.0	investment adjustment costs construction
Φ_k	9.9	investment adjustment costs equipment
Price/wage setting		
θ_p	0.67	fraction of non-adjusting prices
θ_w	0.75	fraction of non-adjusting wages
Debt constraints		
$\bar{m}$	0.85	household LTV ratio
$\bar{m}^e$	0.69	entrepreneur LTV ratio
γ	0.98	amortization rate household debt
γ^e	0.97	amortization rate entrepreneurial debt
Fiscal policy		
$\bar{\tau}^c$	0.08	initial tax rate on consumption
$\bar{\tau}^w$	0.16	initial tax rate on labor income
$\bar{\tau}^k$	0.18	initial tax rate on capital income
$\bar{g}$	0.57	initial government spending
$\bar{b}^{gy}/4$	0.80	long-run target for gov't debt-to-annual GDP
$\phi_{b^{gy}}$	0.035	response coefficient in fiscal rule
$\phi_{\Delta b^{gy}}$	0.387	response coefficient in fiscal rule

to match the construction share of total employment in 2007 (13.4%). We set the weight of utility from housing services, ϑ, to replicate gross household debt over annual GDP (80.2%). The shares of constrained and unconstrained workers in the labor baskets are set to $\mu_h = \mu_e = 1/2$. The scale parameters of convex investment adjustment costs, Φ_h and Φ_k, are chosen such that the dynamics of construction and equipment capital investment in our baseline deleveraging scenario resembles their behavior during the crisis.[14]

The Calvo parameters are set to $\theta_p = 2/3$ and $\theta_w = 3/4$, such that prices and wages are adjusted every 3 and 4 quarters on average, respectively. This is consistent with survey evidence for the Spanish economy (see e.g. Druant et al., 2009).

Regarding the debt contract we set $\bar{m} = 0.85$ for the household's initial loan-to-value ratio, consistently with Spanish evidence on pre-crisis LTV ratios for new mortgages,[15] while the entrepreneurial initial loan-to-value ratio, $\bar{m}^e = 0.69$, is chosen to match the ratio of gross non-financial corporate debt to annual GDP (125.4% in 2007). We calibrate the contractual amortization rates, $1 - \gamma$ and $1 - \gamma^e$, to replicate the average age of the stock of outstanding mortgage debt prior to the crisis: $1 - \gamma = 0.02$ and $1 - \gamma^e = 0.03$ per quarter.[16]

As explained above, all fiscal instruments other than the one in the fiscal rule (equation 14) are held constant at some initial values $\{\bar{\tau}^s\}_{s=c,w,k}$ and $\bar{g}$. We calibrate the initial tax rates $\{\bar{\tau}^s\}_{s=c,w,k}$ as in Stähler and Thomas (2012), who calculate pre-crisis average implicit tax rates for different tax figures. We set $\bar{g}$ such that $\bar{g}/gdp_{ss}$ equals the government spending share of GDP in 2007 (18.3%). In the fiscal rule, we set the long-run target for the government debt-to-(quarterly) GDP

---

[14]The accumulated fall in construction and equipment capital investment 8 quarters after the financial shock replicate their accumulated fall 8 quarters after their peak in 2007:Q4 (24.5% and 28% respectively).

[15]See e.g. Masier and Villanueva (2011, Table A1), and Akin et al. (2014, Table A.1).

[16]Under our debt contracts (with a constant fraction of outstanding debt amortized each period), the average age of the debt stock converges in the steady state to $\gamma / (1 - \gamma)$ and $\gamma^e / (1 - \gamma^e)$ for households and entrepreneurs, respectively. According to calculations by Banco de España, based on data from the Land Registry office and large financial institutions, the average age of outstanding mortgage debt prior to the crisis was close to 12.5 years for households and 8 years for nonfinancial corporations and entrepreneurs. This yields $\gamma = 12.5 \times 4 / (12.5 \times 4 + 1) = 0.98$ and $\gamma^e = 8 \times 4 / (8 \times 4 + 1) = 0.97$.

ratio to $\bar{b}^{gy} = 0.80 \times 4$, consistently with levels reached in Spain only a few years after the start of the crisis and with our focus on scenarios that involve a reduction in government indebtedness towards the EU Treaty target (60% of GDP). The response coefficients in the rule, ϕ_b and $\phi_{\Delta b}$, are calibrated to make the dynamic change in current deficit roughly comparable across consolidation scenarios based on the different fiscal instruments. Table 1 summarizes the calibration.

Solution algorithm. We assume perfect foresight in all our simulations. We solve for the fully nonlinear equilibrium path, using a variant of the Newton-Raphson algorithm developed by Laffargue (1990), Boucekkine (1995) and Juillard (1996) (LBJ).[17] Our assumption of long-run debt contracts gives rise to two debt regimes for households and entrepreneurs. If collateral values are above the contractual debt amortization paths, then debt levels are restricted by the former, according to equations (7) and (10). If the opposite holds, then new credit flows collapse to zero and debt is restricted by the contractual amortization path (equations 8 and 11). We have therefore modified the LBJ algorithm to allow for endogenous change of debt regime. In particular, the dates at which the regime changes take place (which we will denote by T^* for entrepreneurs and T^{**} and households) are solved as equilibrium objects.

4   Effects of a fiscal consolidation

We first look at the effects of fiscal consolidations abstracting from any other disturbances or policy changes. This allows us to isolate some channels of transmission that will play a critical role in the full analysis of fiscal consolidations in a private debt overhang presented later. To further clear the desk, for now we focus on one specific fiscal instrument in (14), government spending ($fi = g$).[18]

In particular, we first consider the effects of reducing the long-run target for the

---

[17]See also Juillard et al. (1998) for an application of the LBJ variant of the Newton-Raphson algorithm.

[18]The results using alternative fiscal instruments are qualitatively similar and are available on request.

government debt-to-GDP ratio, $\bar{b}^{gy}$, from its initial value (80% of annual GDP). We will refer to the numerical reduction in such target as the *size* of the fiscal consolidation. Debt reduction is achieved through the fiscal rule in equation (14). In the rest of the paper, a key outcome variable we will focus on is the *fiscal sacrifice ratio*, which following Erceg and Lindé (2013) is defined as the change in output (consumption) relative to the size of the fiscal consolidation: $\Delta y_t / |\Delta \bar{b}^{gy}|$ $(\Delta c_t / |\Delta \bar{b}^{gy}|)$.

Figures 1 and 2 depict the dynamic fiscal sacrifice ratio associated to two different consolidation sizes. The blue line represents this ratio when the fiscal adjustment is designed to reduce the long-run government debt-to-GDP ratio $\bar{b}^{gy}$ by 1 percentage point (pp), whereas the red line corresponds to a much larger consolidation effort aimed at reducing that ratio by 20 pp. Before the (credible) reduction of the debt target is announced ($t = 0$), the economy rests in the steady state of the baseline regime, where debt levels equal pledgeable collateral values.[19] The exercise in Figure 1 is simulated under the standard assumption of *one-period* private debt ($\gamma = \gamma^e = 0$), whereas in Figure 2 we simulate our benchmark model featuring *long-term* household and entrepreneurial debt in which the gross flow of new credit is non-negative. The comparison of both models is useful to clarify the type of non-linearity that arises here in the presence of long-term debt, due to the asymmetry in the borrowing constraints and the resulting potential for debt-regime changes. In our model, unlike in models with one-period debt, the response to shocks need not be symmetric (as it may depend on the sign of the shocks) nor proportional to the shock size. This has implications for fiscal sacrifice ratios across different consolidation scenarios (size, gradualism, etc.).

In both cases, the contraction in public spending lowers GDP in a significant and persistent manner, due to the combination of the rise in the real interest rate and the fall in asset prices and, hence, in borrowers' net worth. This effect is common to models with a binding zero lower bound on nominal interest rates (e.g. Erceg and Lindé, 2013 and Eggertsson, 2010). In the absence of monetary accommodation,

---

[19]Indeed, the fact that constrained households and entrepreneurs are both more impatient than unconstrained households, $\beta < \beta^u$, guarantees that the collateral constraint binds for both agents in the steady state.

fiscal multipliers are large. Moreover, they get amplified in our setting by the persistent deterioration of borrowers' net worth. These effects, combined with the negative impact on labor income (due both to lower real wages and employment) induce a prolonged recession.

Beyond this similarity, the dynamic path of output varies substantially across these two different environments. In the case of short term debt (Figure 1), the consolidation effort has a homogenous effect on the sacrifice ratio with respect to the size of the consolidation. On the contrary, when private debts are long-term (Figure 2), the output response depends substantially on the size of the consolidation program. More precisely, a small scale fiscal adjustment has a stronger short-run relative effect on output than larger consolidations, but this pattern is reversed in the medium term, as a comparatively larger fiscal adjustment gives rise to higher relative output losses.

Figure 1

One period debt. g-based consolidation, no financial shock

Figure 2

Long term debt. g-based consolidation, no financial shock
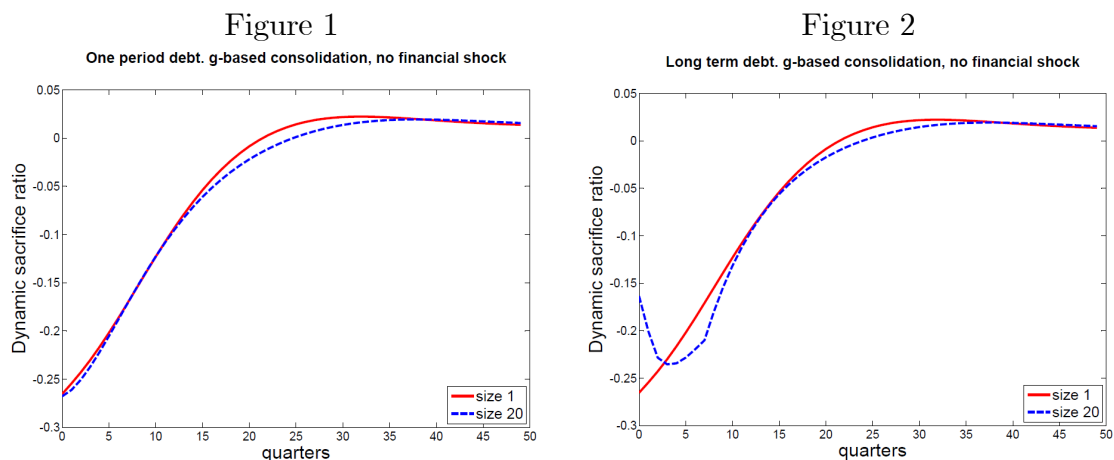
To account for the differences observed in Figure 2, we must bear in mind that long-run debt contracts imply that debtors' debt repayments are bounded by the contractual amortization rate, which dampens the impact on their spending and consumption capacity of shocks that reduce the net worth and the available collateral relative to outstanding debt. Since the response of consumption plays a central role in shaping the overall fiscal sacrifice ratio, it is interesting to focus on how long-term

debt shapes the response of this component of aggregate spending. Focusing on the case of indebted households (the argument applies analogously to entrepreneurs), when the fiscal shock is small so is the fall in collateral values and *excess collateral* remains positive, i.e. $\frac{m_t}{R_t}\pi_{t+1}p^h_{t+1}h_t - \gamma\frac{b_{t-1}}{\pi_t} > 0$. This allows borrowers to tap new credit up to the limit dictated by collateral constraint (7), in which case the time-t flow of new debt obeys the following expression,

$$b_t - \frac{R_{t-1}}{\pi_t}b_{t-1} = \frac{m_t}{R_t}\pi_{t+1}p^h_{t+1}h_t - \frac{R_{t-1}}{\pi_t}b_{t-1}.$$

Along this path, the fall in collateral values, $\frac{m_t}{R_t}\pi_{t+1}p^h_{t+1}h_t$, triggered by the fiscal contraction reduces households' net debt flow one for one. This adds to the standard debt-deflation channel operating through the fall in inflation that raises the real value of the burden of outstanding debt, $\frac{R_{t-1}}{\pi_t}b_{t-1}$. Hence, the resulting contraction in net debt flows reduces debtors' spending capacity.

By contrast, if the size of the fiscal consolidation is large enough to drive excess collateral below zero, then the contractual amortization path becomes the effective debt limit: $b_t \leq \gamma\frac{b_{t-1}}{\pi_t}$. Since this constraint holds in equilibrium as an equality, households' net debt now evolves according to

$$b_t - \frac{R_{t-1}}{\pi_t}b_{t-1} = -\left(\frac{R_{t-1} - \gamma}{\pi_t}\right)b_{t-1}.$$

This last expression shows how net debt flows, and hence consumption, are not directly affected by changes in collateral values while in this debt-regime. In fact, in this debt regime the only effect of the fiscal consolidation on net debt flows is through the standard debt deflation effect, which in this case is second order since $R_{t-1} - \gamma$ is small.

The previous mechanism, by which long-term debt contracts isolate borrowers spending capacity from the effect of large falls in collateral values, may be interpreted as a *'buffering effect'* of long-run debt. This effect induces a some disconnection between debtors' net worth and collateral holdings, on the one hand, and consumption, on the other. This disconnection becomes apparent in Figure 3 that shows that for

26

small fiscal adjustments (e.g. sizes 1 and 5) the downward adjustment of these variables is stronger, relative to the (absolute value of) the fall of net worth, whereas for larger consolidations, these components of spending adjust much less. Thus, in the short run large consolidations have a *milder* relative effect on output than smaller ones do.[20]

<div align="center">Figure 3</div>
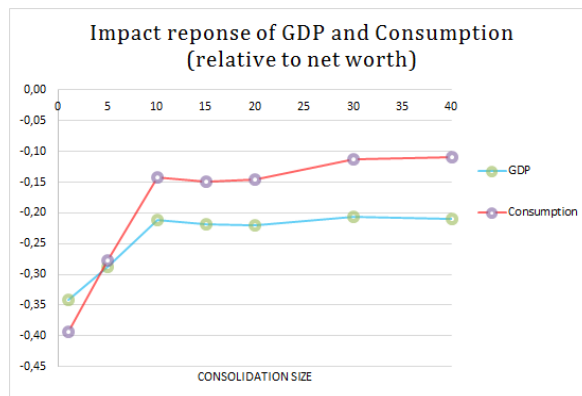
However, the impact reaction of GDP only provides a small piece of information about the total output loss caused by a fiscal consolidation. While the short-term sacrifice ratio of larger consolidations gets moderated through the previous buffering effect of long-term debt, larger consolidations cause a larger contraction of asset prices and collateral values. For a sufficiently large consolidation, indebted consumers are pushed into the negative excess collateral regime, in which fresh credit is zero requiring a prolonged period of spending moderation until the relative levels of debt and collateral are restored and new credit becomes accessible again.

Figure 4 illustrates this, comparing the evolution of household debt (as percentage of GDP) in a consolidation program that aims at reducing the public debt to GDP ratio by 20 and 40 pp respectively.[21] Following the implementation of a sufficiently large fiscal consolidation (20 pp), the excess collateral of the representative

---

[20]In our calibration, the impact response of aggregate consumption, relative to the size of the consolidation program is three times as large in the case of small consolidations ($\nabla \bar{b}^{gy} = 1pp$) than in the case of larger ones ($\nabla \bar{b}^{gy} = 20pp$).

[21]The effects of the smallest consolidation, equivalent to the reduction of the target public debt to GDP ratio by 1 pp are not shown in Figure 4. Small fiscal adjustments keep the economy in the

constrained household becomes negative. As this situation prevails, the household cannot access new loans and repays the existing debts at the contractual rate. After 8 quarters, the reduction in debt is sufficient to guarantee the accumulation of a positive excess collateral and the household then regains access to new loans, giving rise to a "releveraging" process, along which the evolution of the stock of debt becomes linked to that of collateral holdings. This turning point is represented in our model by the statistic T^{**}. Thus, T^{**} (T^*) represents the endogenous *duration* of the deleveraging phase for households (entrepreneurs). For a larger consolidation (40 pp), the initial fall in borrowers' net worth and available collateral is significantly larger than in the previous case. The combination of an initial sharper fall in borrowers' net worth and collateral and a stronger debt-deflation effect implies that the fall of (negative) excess collateral gets amplified. Since the initial debt level, and the contractual path of nominal debt amortization are the same as in the case of a milder fiscal adjustment, it takes now much longer (14 quarters) for the same household to recover the borrowing capacity.

Figure 4

Household debt, consolidations size 20pp vs 40pp, g-based
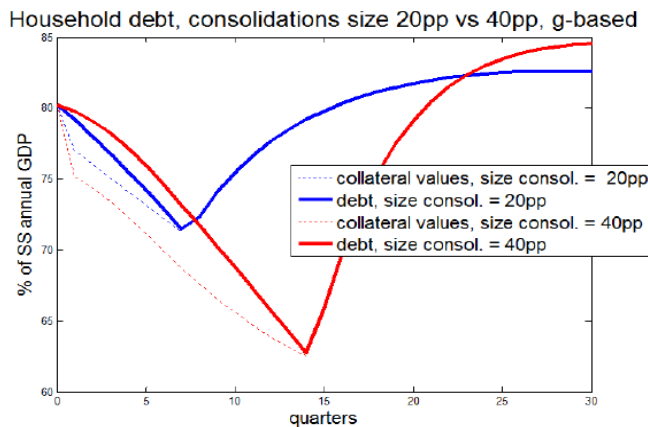
Fiscal adjustments that accompany public debt consolidations produce a negative

---

positibe excess collateral regime in which fresh credit (b_t^{new}) is positive and there is not deleveraging as such.

impact on borrowers' assets and income flows, but the evolution of long-term preexisting debts in the short run is largely unaffected by the size of the fiscal shock if the latter is big enough. Thus, larger consolidations produce a disproportionately large negative impact on net worth, implying that indebted consumers find themselves with negative 'excess collateral' for a potentially long period. Given an initial level of debt, sharp fiscal retrenchments lengthen the deleveraging phase for the private sector until additional spending can be financed with fresh credit. This is corroborated by the response of T^* and T^{**} for consolidations of different sizes depicted in Figure 5. Larger fiscal consolidations produce more persistent deleveraging, thus postponing the recovery of credit and GDP. This 'duration effect' explains why severe consolidations produce larger medium-term relative output losses, as shown in Figure 2.

Figure 5
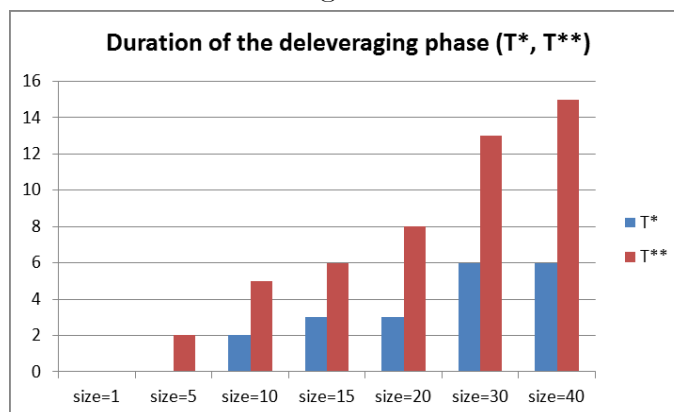
Summing up, milder short run relative output costs of larger consolidations come with a higher sacrifice ratio in the long run. In light of this dynamic trade-off, it is useful to summarize the overall effects of alternative consolidations into two statistics. A first natural candidate is social welfare, which we define as

$$E_0 \sum_{t=0}^{\infty} \left[(\beta^u)^t U \left(c_t^u, h_t^u, \{n_t^u\} \right) + \beta^t U \left(c_t^c, h_t, \{n_t^c\} \right) + \beta^t \log c_t^e \right],$$

29

where U is the period utility function of households and entrepreneurs as introduced in equations (4), (5) and (9). In particular, we compute the welfare loss of a fiscal consolidation as the percentage change in permanent consumption that is required to compensate for the effects of the consolidation.

Table 2. Relative welfare and output losses				
Fiscal shock				
Size	1%	5%	20%	40%
welfare loss	0,016	0,017	0,020	0,023
discounted wel. loss	-1,94	-1,96	-2,06	-2,27
$[T^*, \ T^{**}]$	[0, 0]	[0, 4]	[3, 8]	[6, 13]

Table 2 shows how the welfare loss, defined in this way and scaled by the consolidation size, increases monotonically as we move from smaller to larger consolidations. These numbers reveal that the duration effect (larger consolidations give rise to longer and more deeper private deleveraging) dominates the buffering effect (larger consolidations trigger a relatively lower GDP loss in the short run). As a reflection of this, the welfare loss is positively correlated with the duration of the period of private debt adjustment. The same pattern is observed when the output loss is computed in present value terms, despite the fact that the extra cost of larger consolidations concentrates in the medium term.

5   Fiscal consolidations in a credit-crunch environment

In this section we exploit the framework introduced earlier to address three issues regarding the connection between fiscal consolidation and private debt against the backdrop of a process of private deleveraging process that is already unfolding as a consequence of a negative financial shock. In mind we have the recent experience of a

number of European countries where the main motivation for consolidating public finances came from the worsening of their macrofinancial landscape that followed from the crisis initiated in 2008. In this sense, although we do not model the decision for consolidating, we analyze its macroeconomic effects in a more realistic environment in which the trigger of private deleveraging is not primarily the result of the fiscal retrenchment itself.

In this richer context, that combines fiscal policy shocks and financial shocks, we first analyze, parallel to the exercise in the previous section, the relative cost of large versus small consolidation programs; second, we study the relative cost of gradual versus front-loaded consolidation programs; and, finally, we compare the effects of alternative instruments used to adjust the level of public debt to its target: public spending, taxes on capital (τ^k) and consumption taxes (τ^c).[22]

Before that, to get a glimpse of the macroeconomic effects of the financial shock, Figure 6 depicts the response of the main variables of the model in absence of the fiscal consolidation. The baseline scenario is now one in which the model economy is subject to an unexpected, gradual, permanent drop in the LTV ratios of both households and entrepreneurs, as way of reflecting the financial origin of last global crisis. In particular, we assume an autoregressive process: $x_t = (1 - \rho^x) \bar{x} + \rho^x x_{t-1}$, $x = m, m^e$, where we set $\rho^m = \rho^{m^e} = 0.75$. We then simulate an unanticipated fall in the long-run LTV ratios $(\bar{m}, \bar{m}^e)$ of 5 percentage points from their baseline values in Table 1, a conservative choice in the light of recent experience in Spain.[23]

This financial shock is large enough to push the economy into the deleveraging regime, even in the absence of any fiscal adjustment. Total consumption declines as a result of the deleveraging process, and then it experiences successive recoveries when, first, entrepreneurs and, then, households regain access to new loans. The shock has also a negative impact on total investment, driven by lower expenditure in both real estate and equipment capital. Notice that investment recovers from $t = 8$ onwards i.e. before the process of entrepreneurial debt reduction is actually over

---

[22]In the first two exercises we focus just on consolidations based in adjustments in public spending. The results with fiscal rules defined in τ^k and τ^c are qualitatitevely similar.

[23]Data from the Spanish Land Registry office shows that average LTV ratios for new mortgages declined by 7.7 percentage points in the 6 years between 2007:Q3 and 2013:Q3.

$(t = 9)$. This initial creditless recovery in investment is financed with an increase in borrowers' internal saving.[24]

Figure 6

**Macroeconomic adjustment to a financial crisis**
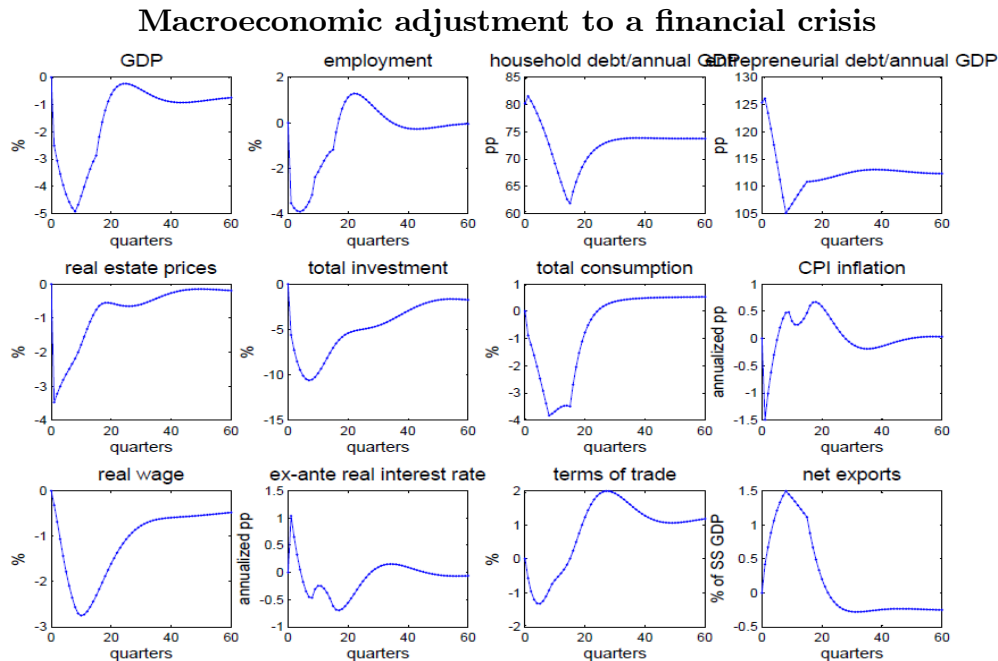
The deflationary process caused by the financial shock leads to a temporary depreciation of the terms of trade, which fosters gross exports. On the other hand, imports fall due to the combined effect of the terms-of-trade depreciation and the severe contraction in domestic demand. Both effects give rise to a substantial improvement in net exports during the deleveraging period. The positive contribution of the external sector, however, is not sufficient to avoid a protracted recession that lasts two years. As shown in Andrés, Arce and Thomas (2015), the presence of long-run debt allows the model to replicate reasonably well the dynamics of private debt observed during historical deleveraging episodes.

---

[24]From $t = 0$ until T^* entrepreneurs reduce their consumption, which in our framework may be interpreted as dividend payments, thus increasing their retained earnings.

5.1 The size of the fiscal adjustment

We discuss now the output effects of the size of the consolidation program. Figure 7 depicts the *fiscal sacrifice ratio* associated to spending-based consolidations of two different sizes: 1 pp (blue line) and 20 pp (red line). Since now there are two shocks operating simultaneously, in what follows we calculate the GDP effect of a fiscal consolidation (Δy_t^{fc}) as the difference between the GDP path in the scenario with both financial and fiscal shocks and that in the baseline scenario, that only incorporates the financial shock. The *sacrifice ratio* is then given by $\Delta y_t^{fc} / \left| \Delta \bar{b}^{gy} \right|$.

Figure 7

**Long term debt. g-based consolidation, financial shock**
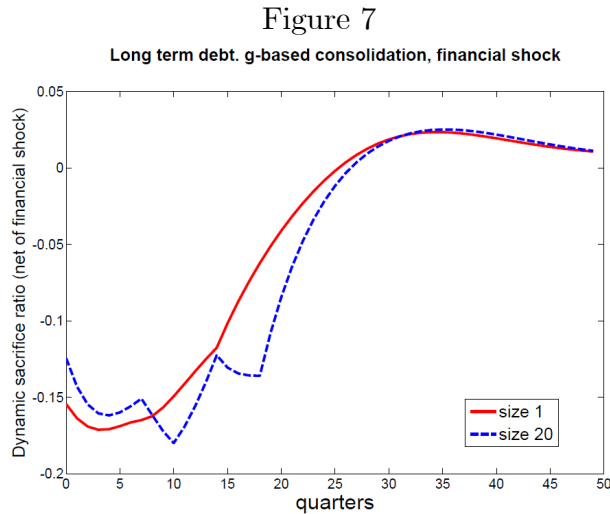
Notice first that the main results from the analysis in section 4 are preserved in the presence of a separate financial shock: (i) the fiscal adjustment makes the recession deeper and more persistent (*vis-à-vis* the financial-shock-only scenario); (ii) smaller adjustments have greater relative effects in the short run; and (iii) larger consolidations are more costly in relative terms over the medium run. Moreover, a quick comparison with Figure 2 reveals that the additional medium-run costs caused by the fiscal consolidation is somewhat higher when the economy is contemporaneously undergoing a private deleveraging process.

Both, the buffering effect and the duration effect show up in this context too. Compared with the sacrifice ratio in the fiscal solo case in Figure 2, the former ef-

fect seems to be weaker now, whereas the duration effect remains strong.[25] The latter effect can be observed in more detail in Figure 8 that portrays the response to the deleveraging shock of collateral values (dashed lines) and the actual equilibrium path of outstanding debt (thick solid lines) of entrepreneurs and households. The deleveraging shock is accompanied by a government spending-based fiscal consolidation of 1 pp (red lines) and 20 pp (blue lines) respectively. Before these shocks take place ($t = 0$) the economy rests in the steady state of the baseline regime, but the credit crunch shock drives collateral values *below* the contractual amortization paths already on impact ($t = 1$), an effect that is amplified by the fiscal contraction. Therefore, the economy enters the alternative regime in which entrepreneurial and household debt stocks decay at the contractual amortization rates. In this phase, the economy undergoes a gradual and prolonged deleveraging process. Eventually, collateral values rise again above the contractual amortization path, at which point borrowers are able to regain access to fresh funds (periods T^* and T^{**}). Deleveraging lasts longer for households than for entrepreneurs, which mainly reflects the slower amortization rate assumed for the former ($1 - \gamma < 1 - \gamma^e$) and, more importantly, the bigger fiscal shock associated to the consolidation of 20 pp amplifies the strength and the duration of private sector deleveraging.

---

[25]Differences in the short run multiplier among the two consolidation exercises, are now dampened by the fact that the financial shock gets the economy into the deleveraging regime even for the small consolidation program.

Figure 8

Collateral and debt in a financial crisis
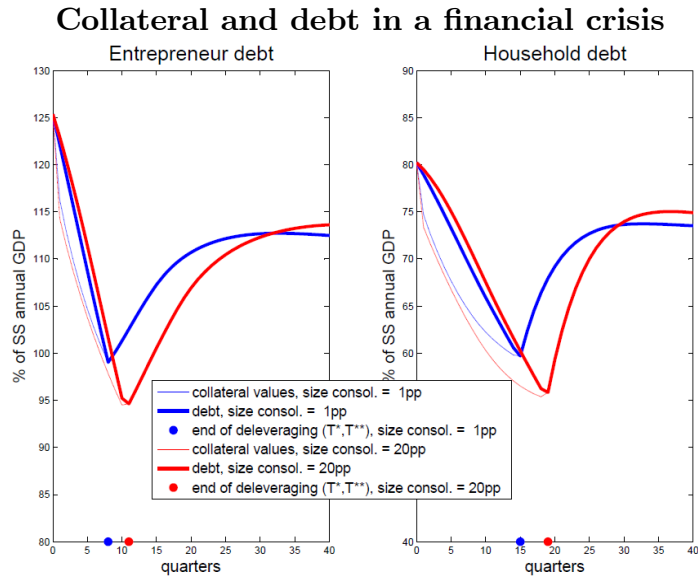
Since the sacrifice ratios plotted in Figure 7 are in differences with respect to the baseline in which there is no fiscal shock, the downward spikes in the red line (size 20 pp) represent the additional GDP loss caused by the larger fiscal consolidation through the lengthening of the deleveraging process (i.e. higher T^* and T^{**}). These spikes do not show up in the consolidation of size 1 pp since in this case the fiscal shock is small enough to leave the duration of the deleveraging phase unaffected. On the contrary, the spending cut associated to the 20 pp consolidation postpones the exit from deleveraging by four quarters for both entrepreneurs and households with respect to baseline scenario of no fiscal policy change. This in turn drags down the recovery of GDP, producing significant and persistent losses over the medium run.

As in the no financial shock case analyzed earlier, both the welfare loss and the present discounted value of the GDP loss (per unit of $\bar{b}^{gy}$ reduction) rise with the size of the consolidation (Table 3), since the duration effect that delays the exit from private deleveraging outweighs the short-run buffering effect.

35

| Table 3. Relative welfare and output losses		
Cum financial shock		
Size	1%	20%
Rel. welf. loss	0,024	0,026
Rel. PV(∇y)	-2,1	-2,5
$[T^*,\ T^{**}]$	[8, 15]	[11, 19]

5.2 Gradualism vs front-loading

There is another dimension of fiscal consolidations that has been much discussed in policy circles, namely the appropriate pace of debt (or deficit) adjustment. So far we have kept constant the speed of fiscal consolidation as captured by the (instrument-adjusted) coefficients in the fiscal rule, and focused on differences across consolidation sizes. In this section we fix the consolidation size at 20 percentage points and consider different values of the response coefficient (ϕ_b) to debt deviations from target ($b_{t-1}^{gy} - \bar{b}^{gy}$). In this way we analyze two types of fiscal adjustments: front-loaded consolidations (high ϕ_b), and gradual consolidations (low ϕ_b).[26]

The key question is to what extent the speed of adjustment affects the response of GDP and interacts with the duration and intensity of private-sector deleveraging (T^*, T^{**}). As can be seen in Figure 9, a higher degree of gradualism reduces the short/medium-run costs of the fiscal consolidation, but raises the longer-run costs.

---

[26]Given our assumption that government debt is riskless, we assume away any risk premium on its price and, hence, we abstract from any hypothetical effect of different consolidation paths on the spread. In mind we have a scenario in which the government credibly commits to reducing its debt ratio in the first place, irrespectively of the specific rule chosen to accomplish the fiscal consolidation and its pace. Alternatively, the previous assumption can be understood as reflecting a situation in which the stock of government debt is never so high so as to generate any appreciable doubt about its sustainability (see Bi, 2012).

Figure 9
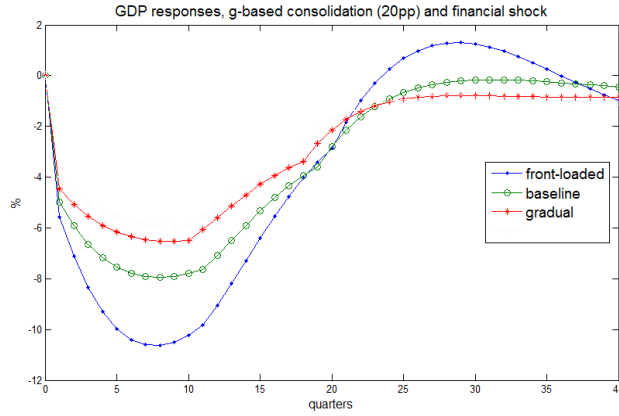
Some authors defend a more gradual pace of consolidation, arguing that gradualism helps to avoid large fiscal shocks in the short run, i.e. at the time when the fiscal multiplier is presumably larger, specially in a context with high levels of private debt and lack of room for maneuver on the side of monetary policy. The main reason why gradualism is less costly in our model is different. Front-loaded consolidations are more costly in the short run since they entail larger fiscal shocks in the early years of the fiscal program that, in turn, prolong the duration and the intensity of the private deleveraging period. The deleveraging process and the recession last longer as the speed of the fiscal consolidation increases, which has a significant impact on current spending of forward looking households and firms. As shown in Table 4 below, front-loaded consolidations delay the exit from the deleveraging phase substantially: relative to a gradual consolidation, it takes 4 and 2 additional quarters for households and entrepreneurs to regain access to new credit respectively.[27] Welfare losses of fiscal consolidation, relative to the no-consolidation scenario, increase as a larger part of the debt-reduction effort is exerted in the early years of the program and are more intense for constrained households and entrepreneurs than for unconstrained

---

[27]Similar results are obtained if gradualism concerns the smoothing parameter $\rho_{\bar{b}^{gy}}$ in a generalized specification that allows for a time-varying debt ratio target $(\bar{b}_t^{gy})$,

$$\bar{b}_t^{gy} = \rho_{\bar{b}^{gy}} \bar{b}_{t-1}^{gy} + (1 - \rho_{\bar{b}^{gy}}) \bar{b}^{gy}.$$

These results are also available upon request.

households.

	Welfare losses				T^*, T^{**}
Scenario	Aggr.	U. HHs	C. HHs	C. Entr.	
Front-loaded	0.67	0.15	1.47	1.58	[12, 22]
Baseline	0.55	0.9	1.18	1.37	[11, 19]
Gradual	0.42	0.6	0.91	1.08	[10, 18]

Table 4. Welfare costs of fiscal consolidation in a financial crisis

Note: welfare costs in % of permanent consumption

5.3 Alternative instruments

Finally we look at the comparative effects of a consolidation of a given size (20 pp) based on alternative fiscal instruments: government spending (g_t), consumption tax (τ_t^c), and capital income tax (τ_t^k).[28] As can be seen in Figure 10, different instruments give rise to distinct patterns in the impact and medium-run output response, out of which several features stand out.[29] First, consolidations implemented trough adjustments in taxes induce more moderate output losses in the short run than those based on government spending cuts; second, other than the more pronounced fall in GDP following adjustments in g_t, the dynamic pattern of output is very similar to the one following changes in τ_t^c; and, third, while the response of output in the short run is very similar for consolidations based on τ_t^k and τ_t^c, as the negative effect of capital income tax hikes on debtor's net worth and collateral accumulation gathers momentum, adjustments in τ_t^k becomes the costliest alternative.

---

[28]The coefficients of the rules have been chosen so that adjustment in different instruments lead to equal responses of the public deficit at time $t = 1$. The effects of consolidations based on labor income taxes (τ_t^w) are relatively similar to those of a consumption tax-based consolidation, and we thus omit them for brevity.

[29]As discussed in the context of Figure 7, in Figure 10, the spikes in the GDP effects reflect fiscally-induced changes in T^* and T^{**} (and hence in the dates at which the consumption spikes associated to the end of deleveraging take place) relative to the baseline no-consolidation scenario.

Figure 10

Alternative instruments, financial shock
Consolidation 20pp
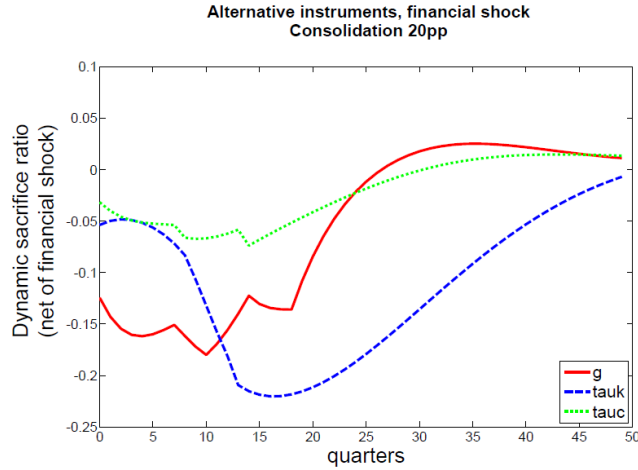
These patterns can be explained by the evolution of the financial conditions of the various agents. Eggertsson (2010) and Erceg and Lindé (2013), among others, have explained why government spending cuts might be more damaging than tax hikes when the monetary policy cannot accommodate the fiscal shock. When the economy is away from the ZLB, tax increases are less deflationary than spending cuts since the negative demand effect is compensated in the case of taxes by increases in the marginal cost. Thus monetary policy accommodation is stronger following spending cuts. By contrast, when the ZLB is binding, the impact of these two fiscal instruments have the opposite effect on the real interest rate: spending cuts raise it more than tax hikes do, hence, the former go hand on hand with higher fiscal multipliers.

Consolidations based on both g-cuts and τ^c-hikes give rise to similar time-patterns for the sacrifice ratio, with the aforementioned differences in their magnitude, since both instruments have a symmetric impact on the length of households and entrepreneurs deleveraging, i.e $\Delta T^* = \Delta T^{**}$ in both cases. Losses caused by spending cuts are larger, in part, because the duration effect on private deleveraging is nonetheless stronger than in the case of consumption tax rises, specifically, $\Delta T^*|_g = \Delta T^{**}|_g = 4$ qrts, versus $\Delta T^*|_{\tau^c} = \Delta T^{**}|_{\tau^c} = 1$ qrt.

Finally, the instrument that exerts the most distinctive impact on the dynamics of GDP is τ^k. This reflects the fact that the adjustment in capital income taxes

unchains a disproportionate effect on the dynamics of deleveraging of entrepreneurs: $\Delta T^*|_{\tau^k} = 4$, $\Delta T^{**}|_{\tau^k} = 1$. Thus the effect of capital income tax hikes on investment and collateral accumulation is stronger than that on consumption, which explains a more gradual but eventually stronger fall of GDP as compared with the impact of τ^c and g. While hikes in capital income taxes have mild short-run effects on consumption and output (similar to those of consumption tax increases), by inhibiting the accumulation of collateralizable capital assets (and keeping their market value low), they lead to longer and deeper private deleveraging and hence to higher medium-and long-run output losses.

6   Concluding remarks

We have analyzed the interaction between fiscal consolidations and private sector deleveraging in an economy inside a monetary union. We have focused on the macro-economic impact of the size, composition and pace (gradualism) of consolidation programs, in particular, thorough the interaction between fiscal consolidations and private debt deleveraging. To this aim, we have developed a model that sheds light on several largely unexplored channels of mutual feedback between private and public debt reduction processes.

In particular, we find that very short-run fiscal multipliers are smaller for large scale consolidation programs, but medium-run multipliers increase with consolidation size. Larger consolidations delay the end of private deleveraging and make it more intense and costly. Long term private secured (mortgaged) debt plays a critical role in these results. As opposed to the canonical assumption of one-period debt, allowing for more realistic maturity profiles for private loans limits the speed of deleveraging, at the cost, however, of making it lengthier. In this way, long-term debt provides some cushioning on private spending after a negative shock. In the case of a fiscal shock induced by a lower target for the long-run public debt ratio this cushioning effect is more important, in relative terms, for large fiscal consolidations, thus giving rise to a negative relation between the size of the consolidation and its associated output cost on impact. On the flip side, larger consolidations depress the value

of collateral and the net worth of borrowers. To the extent that the debt contract prevents a similar downward adjustment in consumption, borrowers' excess collateral plunges into negative territory and it takes longer to recover before they can access to fresh credit. This delays the process of private debt absorption, making the recession longer. Also, through a similar channel, front-loaded consolidation programs delay the recovery, amplifying the short/medium run cost of the fiscal adjustment.

In short, the effect of a fiscal consolidation on the duration and intensity of private debt deleveraging turns out to be a powerful mechanism of transmission of the consolidation costs, and one that has been largely neglected in the previous literature. Indeed, some welfare-based measures reveal that more gradual strategies facilitate a faster recovery of borrowers' net worth and credit. In this way, a more gradual pace for consolidating public finances mitigates the associated costs. Likewise, the previous lengthening effect on the duration of the recession, along with the lack of nominal interest reaction, explains also the large differential GDP effects of fiscal adjustments based on alternative budgetary instruments: public spending, indirect taxes and capital income taxes.

In interpreting these results, it is worth recalling that we have analyzed a model economy where the fiscal authority can credibility commit to alternative consolidation paths, all of which render public debt sustainable. Hence, we do not consider an endogenous response of sovereign spreads. In some real-life contexts, as it could have been the case in a number of euro area countries during the turbulences in the sovereign debt markets in 2010-2012, this last channel can constitute a powerful argument in favor of fast consolidation programs. In this sense, our benchmark economy can be best understood as one where any potential obstacles limiting the credibility of the government to pursue a given fiscal strategy have already been dispelled. Alternatively, one might interpret our exercises as featuring changes in the degree of fiscal gradualism of a magnitude not large enough so as to trigger a reassessment of the credibility of the fiscal strategy by the holders of government debt.

References

[1] Alesina, A. y R. Perotti (1997): "Fiscal Adjustments in OECD Countries: Composition and Macroeconomic Effects". *IMF Staff Papers*, 44(2), 210-248.

[2] Alesina, A. and S. Ardagna (2010): "Large Changes in Fiscal Policy: Taxes versus Spending", in NBER Book Series: *Tax Policy and the Economy*, Volume 24, Jeffrey R. Brown, editor.

[3] Andrés, J., J.E. Boscá and J. Ferri (2016): "Household Debt and Fiscal Multipliers", *Economica*, forthcoming.

[4] Auerbach, Alan J., and Yuriy Gorodnichenko (2012): "Measuring the Output Responses to Fiscal Policy" *American Economic Journal: Economic Policy*, 4(2): 1-27.

[5] Batini, N., G. Melina and S. Villa (2016): "Fiscal Buffers, Private Debt, and Stagnation: The Good, the Bad and the Ugly", International Monetary Fund, WP no. 16/104.

[6] Benigno, P., G. B. Eggertsson and F. Romei (2014). "Dynamic Debt Deleveraging and Optimal Monetary Policy," NBER Working Paper 20556

[7] Benigno, P. and F. Romei, 2014. "Debt deleveraging and the exchange rate," *Journal of International Economics*, 93(1), 1-16.

[8] Bergin, P. (2000): "Fiscal solvency and price level determination in a monetary union," *Journal of Monetary Economics,* Elsevier, vol. 45(1), pages 37-53, February.

[9] Bi, H., (2012): "Sovereign default risk premia, fiscal limits, and fiscal policy," *European Economic Review*, 56, 389–410.

[10] Bi, H., E. Leeper, E.and C. Leith (2013): "Uncertain Fiscal Consolidations," *The Economic Journal* 123(566): F31–F63.

[11] Blanchard, O., C.J. Erceg and J.Lindé (2015): "Jump Starting The Euro Area Recovery: Would a Rise in Core Fiscal Spending Help the Periphery?", Working Paper 21426

[12] Blanchard, O. and F. Giavazzi, 2003. "Macroeconomic Effects Of Regulation And Deregulation In Goods And Labor Markets," *Quarterly Journal of Economics*, 118(3), 879-907.

[13] Blanchard, O. and D. Leigh (2013): "Fiscal consolidation: At what speed?", VoxEu

[14] Boucekkine, R., 1995. An Alternative Methodology for Solving Nonlinear Forward-Looking Models. *Journal of Economic Dynamics and Control*, 19(4), 711-734.

[15] Calvo, G., 1983. "Staggered prices in a utility-maximizing framework," *Journal of Monetary Economics*, 12(3), 383-398.

[16] Christiano, L., Eichenbaum, M. and S. Rebelo (2011) "When is the Government Spending Multiplier Large?", *Journal of Political Economy*, 119, 78-121.

[17] Cloyne, J. and P. Surico (2014): "Household Debt and the Dynamic Effects of Income Tax Changes", Bank of England Working Paper No. 491, Bank of England.

[18] Coenen, G., R. Straub and M. Trabandt (2013): "Gauging the Effects of Fiscal Stimulus Packages in the Euro Area", *Journal of Economic Dynamics and Control*, 37, 367-386.

[19] Cogan, J. F., Taylor, J. B., Wieland, W. and M. H. Wolters (2013): "Fiscal Consolidation Strategy", *Journal of Economic Dynamics and Control*, 37, 404–421.

[20] Corsetti, G.,Kuester, K., Meier, A. and G. J. Müller (2010): "Debt Consolidation and Fiscal Stabilization of Deep Recessions", *American Economic Review*, 100(2), 41-45.

[21] Corsetti, G.,Kuester, K., Meier, A. and G. J. Müller (2013): "Sovereign Risk, Fiscal Policy, and Macroeconomic Stability", *Economic Journal*, 123, p. 99-132.

[22] DeLong, J. Bradford, and Lawrence H. Summers (2012): Fiscal Policy in a Depressed, *Brookings Papers on Economic Activity*, 233–297.

[23] ECB (2014): "Fiscal Multipliers and the Timing of Consolidation", *ECB Monthly Bulletin*, April, European Central Bank, Frankfurt.

[24] Eggertsson, G., (2010): "What fiscal policy is effective at zero interest rates? *NBER Macroeconomics Annual*, 25,59–112.

[25] Eggertsson, G. B. and P. Krugman, 2012. "Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach," *Quarterly Journal of Economics*, 127(3), 1469-1513.

[26] Erceg, C. J., D. W. Henderson and A. Levin, 2000. "Optimal monetary policy with staggered wage and price contracts," *Journal of Monetary Economics*, 46(2), 281-313.

[27] Erceg, C. J. and J. Lindé (2013): "Fiscal Consolidation in a Currency Union: Spending Cuts vs. Tax Hikes", *Journal of Economic Dynamics and Control*, 37, 422-445.

[28] Erceg C. and J. Lindé (2014):. "Is There A Fiscal Free Lunch In A Liquidity Trap?," *Journal of the European Economic Association, European Economic Association*, vol. 12(1), pages 73-107, 02.

[29] Farhi, E. and I. Werning (2012): "Fiscal Multipliers: Liquidity Traps and Currency Unions", NBER Working Papers 18381.

[30] Fatás, A. and L. Summers (2015): The permanent effects of fiscal consolidations", CEPR Working Paper 1092

[31] Fletcher, K. and D. Sandri (2015): "How Delaying Fiscal Consolidation Affects the Present Value of GDP", *IMF, Working Paper,* 151/52.

44

[32] Fornaro, L., 2012, "International Debt Deleveraging", mimeo.

[33] Forni, L., A. Gerali and M. Pisani, 2010. "Macroeconomic Effects Of Greater Competition In The Service Sector: The Case Of Italy," *Macroeconomic Dynamics*, 14(05), 677-708.

[34] Galí, J., 2011. "The Return Of The Wage Phillips Curve," *Journal of the European Economic Association*, 9(3), 436-461.

[35] García, C., E. Gordo, J. Martínez-Martín and P. Tello, 2009. "Una actualización de las funciones de exportación e importación de la economía española", Banco de España, Documento Ocasional 0905.

[36] Guerreri, L. and M. Iacoviello, 2014, "Collateral Constraints and Macroeconomic Asymmetries", Mimeo.

[37] Hernández de Cos, P. and E. Moral-Benito (2013): "Fiscal Multipliers in Turbulent Times: the Case of Spain", Banco de España Working Papers 1309, Banco de España.

[38] Iacoviello, M., 2005. "House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle," *American Economic Review*, 95(3), 739-764.

[39] Iacoviello, M. and S. Neri, 2010. "Housing Market Spillovers: Evidence from an Estimated DSGE Model," *American Economic Journal: Macroeconomics*, 2(2), 125-64.

[40] International Monetary Fund, 2011, "Spain–Staff Report for the 2011 Article IV Consultation," IMF Country Report No. 11/215.

[41] International Monetary Fund, 2013. *World Economic Outlook. Transitions and Tensions*, October.

[42] International Monetary Fund (2014): "Can Fiscal Policies Do More for Jobs?", *Fiscal Monitor*, C.2.

[43] Juillard, M., 1996. DYNARE: A program for the resolution and simulation of dynamic models with forward variables through the use of a relaxation algorithm. CEPREMAP Working Paper No. 9602, Paris, France.

[44] Juillard, M., D. Laxton, P. McAdam, H. Pioro, 1998. An algorithm competition: First-order iterations versus Newton-based techniques, *Journal of Economic Dynamics and Control*, 22, 1291-1318.

[45] Justiniano, A., G. Primiceri and A. Tambalotti, 2014. "Household Leveraging and Deleveraging," forthcoming, *Review of Economic Dynamics*

[46] Kaplan, G., Violante, G. L. and J. Weidner (2014): "The Wealthy Hand-to-Mouth", NBER Working Paper 20073, National Bureau of Economic Research.

[47] Kiyotaki, N. and J. Moore, 1997. "Credit Cycles," *Journal of Political Economy*, 105(2), 211-48.

[48] Kumar M, Leigh D and Plekhanov A. (2007): "Fiscal Adjustments: Determinants and Macroeconomic Consequences". In: IMF (ed) IMF Working Paper. IMF.

[49] Laffargue, J.P., 1990. Résolution d'un modèle macroéconomique avec anticipations rationnelles. *Annales d'Economie et Statistique*, 17, 97-119.

[50] von Hagen, J., A.C. Hallet y R. Strauch (2001): "Budgetary Consolidation in EMU". *Economic Papers*, No. 148. Euroepan Communities.

[51] Woodford, M., 2001. "Fiscal Requirements for Price Stability," *Journal of Money, Credit and Banking*, 33(3), 669-728.

[52] Woodford, M. (2011): "Simple Analytics of the Government Expenditure Multiplier", *American Economic Journal: Macroeconomics*, 3(1), 1-35.

Appendix

A. Equilibrium conditions

Let $\tilde{p}_t \equiv \tilde{P}_{H,t}/P_{H,t}$, $p_{H,t} \equiv P_{H,t}/P_t$, $w_t \equiv W_t/P_t$, $\tilde{w}_t \equiv \tilde{W}_t/W_t$, $\pi_{wt} \equiv W_t/W_{t-1}$. Equilibrium conditions:

- Unconstrained household budget constraint and first-order conditions (d_t, h_t^u),

$$c_t^u + d_t + p_t^h \left[h_t^u - (1 - \delta_h) h_{t-1}^u \right] = \frac{R_{t-1}}{\pi_t} d_{t-1} + (1 - \tau_w) w_t n_t^u - T_t, \qquad (16)$$

$$\frac{1}{c_t^u} = \beta^u E_t \frac{R_t}{\pi_{t+1}} \frac{1}{c_{t+1}^u}, \qquad (17)$$

$$\frac{p_t^h}{c_t^u} = \frac{\vartheta}{h_t^u} + \beta^u E_t \frac{(1 - \delta_h) p_{t+1}^h}{c_{t+1}^u}. \qquad (18)$$

- Constrained household budget constraint, debt constraints, and first-order conditions (b_t, h_t),

$$c_t^c + \frac{R_{t-1}}{\pi_t} b_{t-1} + p_t^h \left[h_t - (1 - \delta_h) h_{t-1} \right] = b_t + (1 - \tau_w) w_t n_t^c - T_t, \qquad (19)$$

$$b_t \leq \begin{cases} R_t^{-1} m_t E_t \pi_{t+1} p_{t+1}^h h_t, & \text{if } m_t R_t^{-1} E_t \pi_{t+1} p_{t+1}^h h_t \geq \gamma b_{t-1}/\pi_t, \\ \gamma b_{t-1}/\pi_t, & \text{if } m_t R_t^{-1} E_t \pi_{t+1} p_{t+1}^h h_t < \gamma b_{t-1}/\pi_t, \end{cases} \qquad (20)$$

$$\frac{1}{c_t^c} = \beta E_t \frac{R_t}{\pi_{t+1}} \frac{1}{c_{t+1}^c} + \xi_t \mathbf{1} (\vartheta_t \geq 0) + \mu_t \mathbf{1} (\vartheta_t < 0) - \beta \gamma E_t \frac{\mu_{t+1}}{\pi_{t+1}} \mathbf{1} (\vartheta_{t+1} < 0), \quad (21)$$

$$\frac{p_t^h}{c_t^c} = \frac{\vartheta}{h_t} + \beta E_t \frac{(1 - \delta_h) p_{t+1}^h}{c_{t+1}^c} + \xi_t \mathbf{1} (\vartheta_t \geq 0) \frac{m_t}{R_t} E_t \pi_{t+1} p_{t+1}^h, \qquad (22)$$

where μ_t is the Lagrange multiplier on constraint (8) in the text, $\mathbf{1}(\cdot)$ is the indicator function and $\vartheta_t \equiv R_t^{-1} m_t E_t \pi_{t+1} p_{t+1}^h h_t - \gamma b_{t-1}/\pi_t$.

- Entrepreneur budget constraint, debt constraints, and first-order conditions

$(b_t^e,\ h_t^e,\ n_t^e,\ k_t)$,

$$c_t^e = mc_t k_{t-1}^{\alpha_k} \left(h_{t-1}^e\right)^{\alpha_h} \left(n_t^e\right)^{1-\alpha_h-\alpha_k} - w_t n_t^e - p_t^h \left[h_t^e - (1-\delta_h) h_{t-1}^e\right]$$
$$+ b_t^e - \frac{R_{t-1}}{\pi_t} b_{t-1}^e - q_t \left[k_t - (1-\delta_k) k_{t-1}\right] + \Pi_t^r + \Pi_t^h + \Pi_t^k, \qquad (23)$$

$$b_t^e \leq \begin{cases} R_t^{-1} m_t^e E_t \pi_{t+1} p_{t+1}^h h_t^e, & \text{if } m_t^e R_t^{-1} E_t \pi_{t+1} p_{t+1}^h h_t^e \geq \gamma^e b_{t-1}^e / \pi_t, \\ \gamma^e b_{t-1}^e / \pi_t, & \text{if } m_t^e R_t^{-1} E_t \pi_{t+1} p_{t+1}^h h_t^e < \gamma^e b_{t-1}^e / \pi_t, \end{cases} \qquad (24)$$

$$\frac{1}{c_t^e} = \beta E_t \frac{R_t}{\pi_{t+1}} \frac{1}{c_{t+1}^e} + \xi_t^e \mathbf{1}\left(\vartheta_t^e \geq 0\right) + \mu_t^e \mathbf{1}\left(\vartheta_t^e < 0\right) - \beta \gamma^e E_t \frac{\mu_{t+1}^e}{\pi_{t+1}} \mathbf{1}\left(\vartheta_{t+1}^e < 0\right), \quad (25)$$

$$\frac{p_t^h}{c_t^e} = \beta E_t \frac{mc_{t+1} \alpha_h k_t^{\alpha_k} \left(h_t^e\right)^{\alpha_h - 1} \left(n_{t+1}^e\right)^{1-\alpha_h-\alpha_k} + (1-\delta_h) p_{t+1}^h}{c_{t+1}^e} + \xi_t^e \frac{m_t^e}{R_t} E_t \pi_{t+1} p_{t+1}^h \mathbf{1}\left(\vartheta_t^e \geq 0\right),$$
$$(26)$$

$$w_t = mc_t \left(1 - \alpha_h - \alpha_k\right) k_{t-1}^{\alpha_k} \left(h_{t-1}^e\right)^{\alpha_h} \left(n_t^e\right)^{-\alpha_h-\alpha_k}, \qquad (27)$$

$$\frac{q_t}{c_t^e} = \beta E_t \frac{mc_{t+1} \alpha_k k_t^{\alpha_k - 1} \left(h_t^e\right)^{\alpha_h} \left(n_{t+1}^e\right)^{1-\alpha_h-\alpha_k} + (1-\delta_k) q_{t+1}}{c_{t+1}^e}, \qquad (28)$$

where μ_t^e is the Lagrange multiplier on constraint (8) in the text, and $\vartheta_t^e \equiv R_t^{-1} m_t^e E_t \pi_{t+1} p_{t+1}^h h_t^e - \gamma^e b_{t-1}^e / \pi_t$.

- Retailers' optimal price decision, and aggregate profits,

$$E_t \sum_{s=0}^{\infty} (\beta \theta_p)^s \frac{c_t^e}{c_{t+s}^e} \left[\frac{(1-\tau_p) \tilde{p}_t}{\prod_{j=1}^s \pi_{H,t+j}} p_{H,t+s} - \frac{\varepsilon_p}{\varepsilon_p - 1} mc_{t+s} \right] \left(\frac{\prod_{j=1}^s \pi_{H,t+j}}{\tilde{p}_t} \right)^{\varepsilon_p} y_{t+s} = 0,$$
$$(29)$$

$$\Pi_t^r = y_t \left((1-\tau_p) p_{H,t} - mc_t \Delta_t\right), \qquad (30)$$

- Dynamics of PPI inflation and price dispersion,

$$1 = (1-\theta) \tilde{p}_t^{1-\varepsilon_p} + \theta \pi_{H,t}^{\varepsilon_p - 1}, \qquad (31)$$

$$\Delta_t \equiv (1-\theta) \tilde{p}_t^{-\varepsilon_p} + \theta \pi_{H,t}^{\varepsilon_p} \Delta_{t-1}. \qquad (32)$$

- Construction firm output, first order conditions (n_t^h, i_t^h), and profits,

$$I_t^h = \left(n_t^h\right)^\omega \left\{ i_t^h \left[1 - \frac{\Phi_h}{2}\left(\frac{i_t^h}{i_{t-1}^h} - 1\right)^2\right]\right\}^{1-\omega}, \tag{33}$$

$$w_t = p_t^h \omega \left(n_t^h\right)^{\omega-1} \left\{ i_t^h \left[1 - \frac{\Phi_h}{2}\left(\frac{i_t^h}{i_{t-1}^h} - 1\right)^2\right]\right\}^{1-\omega}, \tag{34}$$

$$
\begin{aligned}
1 &= p_t^h \left(n_t^h\right)^\omega (1-\omega) \left\{ i_t^h \left[1 - \frac{\Phi_h}{2}\left(di_t^h\right)^2\right]\right\}^{-\omega} \left[1 - \frac{\Phi_h}{2}\left(di_t^h\right)^2 - \Phi_h\left(di_t^h\right)\frac{i_t^h}{i_{t-1}^h}\right] \\
&\quad + \beta\frac{\lambda_{t+1}^e}{\lambda_t^e} p_{t+1}^h \left(n_{t+1}^h\right)^\omega (1-\omega) \left\{ i_{t+1}^h \left[1 - \frac{\Phi_h}{2}\left(di_{t+1}^h\right)^2\right]\right\}^{-\omega} \Phi_h di_{t+1}^h \left(\frac{i_{t+1}^h}{i_t^h}\right)^2 \tag{35}
\end{aligned}
$$

$$\Pi_t^h = p_t^h I_t^h - w_t n_t^h - i_t^h, \tag{36}$$

for $di_t^h \equiv i_t^h / i_{t-1}^h - 1$.

- Equipment capital producers output, first order condition (i_t), and profits,

$$I_t = i_t \left[1 - \frac{\Phi_k}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2\right], \tag{37}$$

$$1 = q_t \left[1 - \frac{\Phi_k}{2}\left(di_t\right)^2 - \Phi_k\left(di_t\right)\frac{i_t}{i_{t-1}}\right] + E_t \frac{\lambda_{t+1}^e}{\lambda_t^e} q_{t+1} \Phi_k di_{t+1} \frac{i_{t+1}^2}{i_t^2}, \tag{38}$$

$$\Pi_t^k = q_t I_t - i_t, \tag{39}$$

for $di_t \equiv i_t / i_{t-1} - 1$.

- Optimal wage decision,

$$\sum_{x=c,u} E_t \sum_{s=0}^{\infty} \left(\beta^x \theta_w\right)^s \left[\frac{(1\text{-}\tau_w)\,\tilde{w}_t}{\prod\limits_{j=1}^{s}\pi_{w,t+j}}\frac{w_{t+s}}{c_{t+s}^x} - \frac{\varepsilon_w\chi\left(n_{t+s}^x\right)^\varphi}{\varepsilon_w\text{-}1}\left(\frac{\tilde{w}_t}{\prod\limits_{j=1}^{s}\pi_{w,t+j}}\right)^{-\varepsilon_w\varphi}\right]\left(\frac{\prod\limits_{j=1}^{s}\pi_{w,t+j}}{\tilde{w}_t}\right)^{\varepsilon_w} n_{t+s}^x = 0, \tag{40}$$

with $\beta^c = \beta$.

- Dynamics of wage inflation and wage dispersion,

$$1 = (1 - \theta_w) \tilde{w}_t^{1-\varepsilon_w} + \theta_w \pi_{wt}^{\varepsilon_w - 1}, \tag{41}$$

$$\Delta_t^{w,n} = (1 - \theta_w) \tilde{w}_t^{-\varepsilon_w} + \theta_w \pi_{wt}^{\varepsilon_w} \Delta_{t-1}^{w,n}. \tag{42}$$

- Fiscal authority's budget constraint,

$$\tau_w w_t \left(n_t^c + n_t^u \right) + \tau_p p_{H,t} y_t + 2T_t = 0.$$

- Aggregate employment,

$$N_t^c = n_t^c \Delta_t^{w,n}, \tag{43}$$

$$N_t^u = n_t^u \Delta_t^{w,n}, \tag{44}$$

$$N_t = N_t^c + N_t^u, \tag{45}$$

- Export demand,

$$x_t = \zeta \left(p_t^* \right)^{-\varepsilon_F} y_{F,t}. \tag{46}$$

- Intermediate good market clearing,

$$y_t \Delta_t = k_{t-1}^{\alpha_k} \left(h_{t-1}^e \right)^{\alpha_h} \left(n_t^e \right)^{1-\alpha_h-\alpha_k}, \tag{47}$$

- Labor market clearing,

$$n_t^c + n_t^u = n_t^e + n_t^h. \tag{48}$$

- Consumption goods basket market clearing,

$$y_t = c_{H,t}^c + c_{H,t}^u + c_{H,t}^e + i_{H,t} + i_{H,t}^h + x_t. \tag{49}$$

- Real estate market clearing,

$$h_t + h_t^u + h_t^e = I_t^h + (1 - \delta_h) \left(h_{t-1} + h_{t-1}^u + h_{t-1}^e \right). \tag{50}$$

- Equipment capital market clearing,

$$k_t = (1 - \delta_k) k_{t-1} + I_t. \tag{51}$$

- Real wages,

$$w_t = w_{t-1} \frac{\pi_{wt}}{\pi_t}, \tag{52}$$

- Terms of trade,

$$p_t^* = p_{t-1}^* \frac{\pi_{H,t}}{\pi_{F,t}}. \tag{53}$$

- Relative demand for domestic goods,

$$c_{H,t}^c = \omega_H p_{H,t}^{-\varepsilon_H} c_t^c, \tag{54}$$

$$c_{H,t}^u = \omega_H p_{H,t}^{-\varepsilon_H} c_t^u, \tag{55}$$

$$c_{H,t}^e = \omega_H p_{H,t}^{-\varepsilon_H} c_t^e, \tag{56}$$

$$i_{H,t} = \omega_H p_{H,t}^{-\varepsilon_H} i_t, \tag{57}$$

$$i_{H,t}^h = \omega_H p_{H,t}^{-\varepsilon_H} i_t^h, \tag{58}$$

- Relative demand for constrained/unconstrained household labor,

$$(1 - \mu) n_t^c = \mu n_t^u, \tag{59}$$

where $\mu \equiv \mu_e = \mu_h$.

- Relative domestic producer prices,

$$p_{H,t} = p_{H,t-1} \frac{\pi_{H,t}}{\pi_t}, \tag{60}$$

51

- CPI inflation,

$$\pi_t^{1-\varepsilon_H} = \frac{\omega_H \left(p_{t-1}^*\right)^{1-\varepsilon_H}}{\omega_H \left(p_{t-1}^*\right)^{1-\varepsilon_H} + 1 - \omega_H} \pi_{H,t}^{1-\varepsilon_H} + \frac{1 - \omega_H}{\omega_H \left(p_{t-1}^*\right)^{1-\varepsilon_H} + 1 - \omega_H}, \quad (61)$$

- Real (PPI-deflated) GDP,

$$gdp_t = y_t + \frac{1}{p_{H,t}} \left(q_t I_t - i_t\right) + \frac{1}{p_{H,t}} \left(p_t^h I_t^h - i_t^h\right), \quad (62)$$

- Gross nominal interest rate,

$$R_t = R^* \exp\left(-\psi \frac{d_t - b_t - b_t^e}{p_{H,t} gdp_t}\right). \quad (63)$$

B. Equilibrium unemployment

Following Galí (2011), we assume that each representative household consists of a unit squared of individuals indexed by $(i,j) \in [0,1] \times [0,1]$, where i represents the variety of labor service provided by the individual and j indexes her disutility from working, given by χj^φ. Let $n_t^x(i)$ denote the number of variety-i workers in household $x = c, u$ employed at time t. Total household disutility from working is given by

$$\chi \int_0^1 \int_0^{n_t^x(i)} j^\varphi dj di = \chi \int_0^1 \frac{n_t^x(i)^{1+\varphi}}{1+\varphi} di,$$

for $x = c, u$. Given the type-specific wage $W_t(i)$, the number of type-i workers that each household *would like to* send to work is given by

$$\arg\max_{n_t^x(i)} \left\{ \lambda_t^x \frac{W_t(i)}{P_t} n_t^x(i) - \chi \frac{n_t^x(i)^{1+\varphi}}{1+\varphi} \right\} = \left(\frac{\lambda_t^x}{\chi} \frac{W_t(i)}{P_t} \right)^{1/\varphi} \equiv l_t^x(i),$$

for $x = c, u$, where $\lambda_t^x \equiv 1/c_t^x$. Unemployment in the market for type-i labor is just the number of workers willing to work at the going wage minus effective labor

52

demand: $u_t(i) \equiv \sum_{x=c,u} l_t^x(i) - \sum_{x=c,u} n_t^x(i)$ .Let

$$l_t^x \equiv \int_0^1 l_t^x(i) \, di = \left(\frac{\lambda_t^x}{\chi} \frac{W_t}{P_t}\right)^{1/\varphi} \int_0^1 \left(\frac{W_t(i)}{W_t}\right)^{1/\varphi} di = \left(\frac{\lambda_t^x}{\chi} \frac{W_t}{P_t}\right)^{1/\varphi} \Delta_t^{w,l},$$

$$N_t^x \equiv \int_0^1 n_t^x(i) \, di = n_t^x \int_0^1 \left(\frac{W_t(i)}{W_t}\right)^{-\varepsilon_w} di = n_t^x \Delta_t^{w,n},$$

denote total household-specific labor supply and labor demand, respectively, for $x = c, u$, where $\Delta_t^{w,l} \equiv \int_0^1 (W_t(i)/W_t)^{1/\varphi} \, di$ and $\Delta_t^{w,n} \equiv \int_0^1 (W_t(i)/W_t)^{-\varepsilon_w} \, di$ are indexes of wage dispersion. Then aggregate unemployment is

$$u_t \equiv \int_0^1 u_t(i) \, di = l_t - N_t.$$

where $l_t \equiv \sum_{x=c,u} l_t^x$ and $N_t \equiv \sum_{x=c,u} N_t^x$ are aggregate labor supply and labor demand, respectively. Finally, the unemployment rate is $u_t^{rate} \equiv u_t/l_t$.

6.1 C. The size of the fiscal adjustment: τ^k and τ^k based consolidations.

Figure C1 and C2 display the GDP effects of fiscal consolidations based on capital income and consumption taxes respectively. In the figure we appreciate a similar pattern in the comparison across different consolidation sizes: a large consolidation produces (slightly) smaller short-run costs, but persistently higher costs in the medium run.
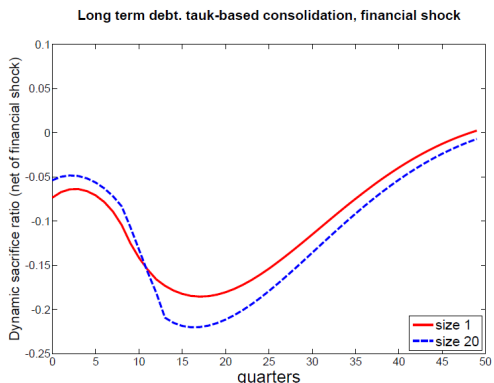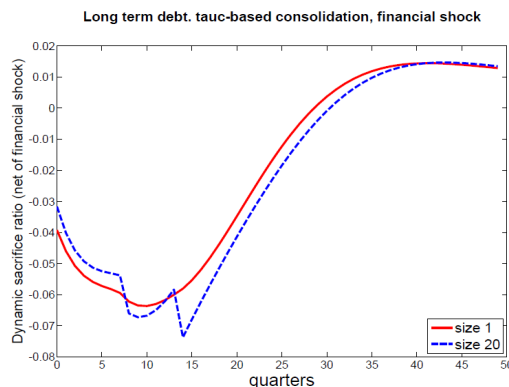
Figure C1

Long term debt. tauk-based consolidation, financial shock

Figure C2

Long term debt. tauc-based consolidation, financial shock

The economics of these responses is similar to that following an adjustment in government spending. As to the differences between both tax-based consolidations, they reflect their distinct effect on T^* and T^{**}. The downward spikes observed in the case of the consumption tax-based consolidation reflect the fact that T^* and T^{**} are both delayed relative to the no-consolidation scenario. This pattern is very similar to the one observed in the case of g adjustment, although the delay in the deleveraging process is less intense in this case (one quarter here versus four quarters in the case of g). When consolidations are implemented via adjustments in capital income taxes, we observe a single downward spike associated to the larger consolidation (20 pp). This is due to the fact that the relevant effect of this tax rate, as far as the deleveraging period is concerned, operates mainly via T^*, which is delayed by 4 quarters. Thus, as discussed above, it is through lower capital accumulation by entrepreneurs that consolidations based on capital income tax hikes produce persistent output losses.