Deadly Embrace: Sovereign and Financial Balance Sheets Doom Loops

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Abstract

The recent unravelling of the Eurozone's financial integration raised concerns about feedback loops between sovereign and banking insolvency, and provided an impetus for the European banking union. This paper provides a "double-decker bailout" theory of the feedback loop that allows for both domestic bailouts of the banking system by the domestic government and sovereign debt forgiveness by international creditors. Our theory has important implications for the re-nationalization of sovereign debt, macroprudential regulation, and the rationale for banking unions.

Keywords: feedback loop, sovereign and corporate spreads, bailouts, sovereign default, strategic complementarities, debt maturity.

JEL numbers: F34, F36, F65, G28, H63.

1 Introduction

Rarely does an economic idea gather so wide a consensus as the evilness of the "deadly embrace", also called "vicious circle" or "doom loop". The feedback loop between weak bank balance sheets and sovereign fragility now faces almost universal opprobrium, from

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the IMF[1] and central bankers to the entire political establishment and the European Commission, providing a major impetus for shared supervision through the creation of the European banking union.

This paper seeks to analyze these developments by proposing a "double-decker" bailout theory of the doom loop that allows for both domestic bailouts of the banking system by the government and sovereign debt forgiveness by international creditors. The theory has important implications for the re-nationalization of sovereign debt, macroprudential regulation, and the rationale for a banking union.

Section 2 sets up the framework and defines equilibrium. Our framework has three dates, 0, 1 and 2. At date 0, the Sovereign issues domestic bonds that (in this basic version) mature at date 2; the expectation of the date-2 fiscal capability affects the sovereign spread. Banks, which will need money for their date-1 banking activities, manage their liquidity by holding domestic sovereign bonds and (again, in the basic version of the model) foreign sovereign bonds. Foreign bonds are safe while domestic sovereign bonds are risky, and so the standard diversification argument would call for holding no domestic bonds. The Sovereign has some supervisory capability, but can choose to be more lenient in its monitoring of (direct and indirect) bank exposures itself.

News accrues at date 1, that affects the banks' solvency (a financial shock) and/or the state's date-2 fiscal capability (a fiscal shock). A fiscal shock compounds the financial shock if banks are exposed to their Sovereign. While the government puts less weight on banks than on consumers and so ex ante dislikes transferring resources to the banking sector, it cares sufficiently about economic activity that in bad states of nature, it cannot refrain from bailing out banks when facing the fait accompli of a banking liquidity shortfall.

Equilibrium welfare is equal to the difference between two terms $\mathcal{W}_0 = \mathcal{E}_0 - \mathcal{R}_0$: an efficiency term $\mathcal{E}_0$, equal up to a constant to consumer welfare (which includes economic activity benefits from bank lending) and the social cost $\mathcal{R}_0$ of the rents left to banks (equal to these rents times 1 minus the weight on the bank's payoff). Off the equilibrium path, a corrective term must be added that captures the surprise expected transfer from foreign creditors to banks that arises when the supervision is unexpectedly lenient: $\mathcal{W}_0 = \mathcal{E}_0 - \mathcal{R}_0 + \mathcal{C}_0$.

Section 3 first shows that in bad states of nature the bank bailout further degrades the sovereign's ability to reimburse its debt at date 2, lowers the bond price and reduces bank solvency, etc., an amplification mechanism. The multiplier reflecting the loss in sovereign bond price when a bailout is required increases with the extent of home bias. Section

[1] See e.g. Lagarde (2012).

3 then investigates the banks' and the government's incentives to seek and prevent risk taking, respectively. When banks can count on government bailouts, they optimally diversify as little as supervision allows them to, so as to enjoy the maximal put on taxpayer money. Conversely, the government would like to limit risk and force diversification on the banks. From an ex-ante perspective, a tougher supervision reduces bailouts and therefore the social cost of bankers' rents $\mathcal{R}_0$; it also reduces the occurrence of default and raises $\mathcal{E}_0$. But the social benefit of a tougher supervision is shown to extend to the ex-post stage at which Sovereign debt has already been issued and the government chooses the leniency of its supervision: $\mathcal{C}_0$ decreases with leniency since bankers must now buy debt at date 0 at a price that exceeds expected repayment.

We connect these results with the celebrated Bulow-Rogoff (1988, 1991) argument against buybacks. In an economy with no cost of default and no financial intermediaries, Bulow and Rogoff show that debt buybacks are a giveaway to legacy foreign creditors and reduce the country's welfare. The banks' purchase of domestic bonds is de facto a buyback. We first show that if the default cost is sufficiently large, the Bulow-Rogoff result is overturned. But when the buyback is operated through financial intermediaries that may require a government bailout, the Bulow-Rogoff result is reinstated. This result emphasizes the need for not consolidating the balance sheets of the Sovereign and its banks even when the former fully bails out the latter.

While holdings of domestic bonds are easily measured during a stress test, they may not be so on a continuous basis; furthermore, and mainly, banks may have shrouded exposures to the domestic bond market though derivatives, guarantees or a correlation of the banking book with domestic bonds. We accordingly assume that supervision is imperfect and study the extent to which banks are willing to incur costs so as to evade diversification regulation. In the process, we develop a new argument in favor of macroprudential policies.[2] The consequences of individual banks' undiversified portfolios, and therefore the desirability of intense supervision, depend crucially on the other banks' behaviors. We show that the banks' choices of opaqueness, and thereby their exposures to the sovereign, are strategic complements: Incurring the cost of making one's balance sheet more opaque is more tempting if the put on taxpayer money is more attractive; in turn, this put is attractive when the sovereign bond price is more volatile, which it is when the other banks take a larger gamble. The corollaries to this insight are the existence of collective moral hazard and the necessity of macroprudential regulation: The social cost of

[2] Standard arguments for going beyond the analysis of stand-alone bank solvency include the possibility of fire sales, interconnectedness and the policy response to, say, widespread maturity mismatches. Neglected risk (Gennaioli et al. 2012) can also vindicate macroprudential policies.

poor monitoring of a bank's domestic exposure is higher when other financial institutions are themselves exposed. This is particularly true for institutions that the government is eager to rescue.

Section 4 explores the incentives of legacy creditors to engage in debt forgiveness, how these incentives affect the regulatory stance of the government, and develops a rationale for a banking union. When fiscal prospects are bleak, the legacy debt ends up on the wrong side of the Laffer curve once likely bailouts and debt increases are factored in; investors thus have an incentive to forgive some debt, the more so, the worse the state of nature. This "double-decker bailout" in turn induces the government to turn a blind eye to undiversified bank portfolios. This however occurs only when the situation looks grim, a prediction that fits well with the recent re-nationalization of government debt in the Eurozone. We obtain a simple result: The amount of date-1 debt after debt forgiveness and issuance associated with the peak of the legacy Laffer curve is at the peak of the issuance Laffer curve.

We then show that, under some conditions, the only full equilibrium with endogenous supervisory leniency features maximal supervisory leniency. The analysis thus provides a new argument in favor of shared supervision. Indeed, if the ex-post leniency of domestic supervisors is anticipated ex ante at the time of sovereign debt issuance, then it is priced in the form of higher spreads. The government is better off committing ex ante to a tough ex-post regulatory stance, but is tempted to relax it ex post. A government that lacks commitment benefits from relinquishing its supervisory powers to a supranational supervisor.

Finally, we study five interesting extensions of the basic model, two in Section 5, and three in the Appendix. First, we relax the assumption that sovereign debt maturity matches that of fiscal capability. We compare our economy with long-term sovereign bonds which are claims to coupons accruing at date 2 with an economy where sovereign bonds are short-term one-period bonds which are rolled over at date 1, assuming that the same amount is raised at date 0. We show that a short maturity has both benefits and costs. The cost is that a short maturity is bad for fiscal hedging. The benefit is that a short maturity reduces the risk-shifting possibilities of banks and therefore their rents. A short maturity is therefore an inefficient substitute to regulation. As a result, a long maturity is preferable when supervision of bailout-prone financial entities is efficient enough.

Second, we relax the assumption that foreign assets are all safe. This relaxation is motivated by the multiplicity of troubled countries during the European crisis. Spanish banks, say, could purchase Portuguese bonds and not only German ones. Therefore risk shifting by Spanish banks could have occurred through the purchase of Portuguese bonds rather than through a re-nationalization of the Spanish financial market. We therefore extend the model to allow for multiple risky countries. We show that, provided that balance sheet shocks and fiscal shocks within a country are at least slightly positively correlated and that fiscal shocks across countries are imperfectly correlated (a reasonable assumption), risk shifting solely through domestic bond holding is a strict equilibrium, implying that the re-nationalization result is robust to multiple risky countries. We also show that, with multiple risky countries, our "double-decker bailout" theory predicts that when the fiscal outlook is bad, governments in risky countries have an incentive to relax supervision and let their banks load up on risky domestic debt (and not risky foreign debt). All in all, this extension shows that the multiple forces that we have identified for risk shifting in the baseline model occur through the purchase of risky domestic debt rather than risky foreign debt, implying that the re-nationalization results of the baseline model are robust to multiple risky countries.

The three robustness checks performed in the Appendix concern leverage, limited bailout capabilities and holdings of sovereign debt by banks located in safe countries. When banks can obtain refinancing in markets at date 1, the feedback loop is then stronger, the higher the leverage. This is especially so when sovereign defaults come together with defaults on banks' private debt contracts: As sovereign risk rises, banks have to reduce leverage because the probability of a default on the private debt that they issue also rises. This requires a larger bailout, which puts further pressure on the government budget etc. ad infinitum.

There may be really adverse shocks for which the government can only undertake a partial bailout, as a full one would compromise public finances too much. We then show that banks enter a "rat race". While they wish to remain undiversified so as to enjoy the largest possible put on taxpayer money, they also try to jump ahead of the bailout queue by being a bit more solvent, and therefore cheaper to rescue in the race for bailouts in bad states of nature. Their holdings of foreign bonds are akin to "bids" in a first-price auction, but the analysis is richer than the standard first-price auction in that the focus of competition- the pot of subsidies to be distributed- depends on the distribution of "bids", namely the distribution of holdings of foreign bonds.

Finally, we introduce foreign banks in the foreign (safe) country. Because of the bailout guarantees, foreign banks also have an incentive to load up on risky domestic debt. The foreign government has an incentive to supervise foreign banks so that they do not take on too much domestic sovereign risk. The analysis then uncovers an additional rationale for a banking union. Domestic supervision has positive external effects for the foreign (safe) country. These effects are not internalized by the domestic government, and as a result, supervision is too lax in the domestic economy. By transferring supervisory decisions from the national to the international level, a banking union allows these effects to be internalized, leading to a toughening of supervision in the domestic country and an improvement of welfare.

Relationship to the literature. Several papers have analyzed doom-loops and have identified a feedback loop similar to the one described in our paper. In Acharya et al (2013), the banks hold government bonds; the government's bailout of its financial sector so as to preserve the latter's lending to the non-financial sector reduces the financial sector credit risk in the short run, but increases the impact of future negative shocks to fiscal capacity on the credit risk of the financial sector. As the paper's title indicates, the stabilization of the financial sector is a Pyrrhic victory as it has deleterious long-term effects. The theoretical model is a closed-economy model, in which default costs are internalized by the government; it does not investigate topics such as re-nationalization, joint default, and domestic vs. international regulation of banks that feature prominently in our analysis.

Cooper-Nikolov (2013) builds a model in which sovereign defaults are the outcome of, as in Calvo (1988), self-fulfilling prophecies; similarly, banks fail because of Diamond-Dybvig (1983) runs. This model allows them to demonstrate the potential existence of a doom loop: Worries about sovereign default generate concerns about the viability of banks holding sovereign bonds; conversely, bank failures require bailouts, increasing the volume of sovereign debt. The paper shows that equity cushions eliminate bad equilibria. Our paper differs from Cooper-Nikolov in several respects; on the technical side, crises are in our paper associated with fundamentals (although we of course find much interest in Cooper-Nikolov self-fulfilling crises as well); this allows us to identify a multiplier and to make unique predictions. Like Acharya et al, Cooper and Nikolov focus on a closed economy.

Chari et al (2014) focus on a different aspect of bank holdings of government debt in a closed economy. They show that forcing banks to hold a minimum fraction of their assets in the form of government debt at or below market interest rates is dominated by other fiscal instruments when the government can commit to a debt repayment policy, but can lead to welfare improvements when the government lacks such commitment because it increases credibility. There are no doom loops and instead a disciplining effect of bank holdings of domestic debt on sovereign debt repayment. Moreover, the increased holdings of government debt by banks are driven by coercion of banks by the government.

Several recent contributions look at the contagion from sovereigns to banks in an open economy, offering different hypotheses for sovereign debt re-nationalization and therefore sets of predictions and policy implications that differ from the unique ones summarized in Section 6; in this sense, our contribution is complementary with existing works. The overall picture is the richness of the economics of interactions between sovereign and bank solvency.

Broner et al (2013) consider environments in which the domestic government can default selectively on foreign investors. Selective default then makes domestic debt comparatively attractive to domestic residents in risky times, implying a re-nationalization. In turn, increased domestic holdings of sovereign debt crowd out domestic banks' investment in the real economy. The contagion channel (discrimination) differs from ours (bailouts) and is one-way[3] (from sovereign debt fragility to banks) rather than two-way; so does the rationale for a banking union, as Broner et al view a union as a reduction in discrimination between domestic and foreign investors while we focus on prudential supervision.

Uhlig (2014)'s model of financial repression, like ours, features banking supervision and no discrimination among investors. It assumes a monetary union, whose central bank is jointly backed by the member states and bails out commercial banks. Like in our paper, a country may allow its banks to load up on domestic sovereign debt as the adverse consequences will be shared abroad. Tolerating/ encouraging risk-shifting by banks that have access to the union's central bank's repurchase facility enables the risky country to borrow more cheaply, a mechanism which bears some resemblance to our rationale for strategic debt re-nationalization whereby governments allow domestic banks to buy up their bonds in order to extract concessions from legacy creditors.

In Gennaioli et al (2014), domestic banks find domestic bonds attractive for a different reason than in our paper: The sovereign's internal cost of default (the drying-up of domestic banks' liquidity as there is neither discrimination nor bank bailouts) is high when banking productivity is high; so sovereign repayment discipline is endogenously positively correlated with the banks' marginal utility of liquidity. This implies a re-nationalization of sovereign debt in bad times. There is no feedback loop but instead a disciplining effect of bank holdings of domestic debt on sovereign debt repayment. Unlike our paper, the emphasis is not on prudential supervision and feedback loops.

Bocola (2014) emphasizes two different channels for the impact on banks of news regarding the possibility of a sovereign default. News that the government may default in the future has adverse effects on the funding ability of exposed banks (liquidity channel),

[3]A two-way feedback loop arises in an extension of their model in which the cost of default is proportional to the amount of defaulted debt, with the proportion decreasing with the capital stock.

and it raises the risk associated with lending to the productive sector (risk channel). There is no feedback loop. In our model as in most aforementioned models that investigate the links between banks and sovereigns, there is no risk channel.

An important ingredient of our analysis, as in Farhi-Tirole (2012) is that direct and indirect exposures may be hard to assess, leading to supervisory failures, and that banks will exploit the supervisory loopholes to secure cheaper financing and thereby increase their return on equity. This ingredient is also shared by Mengus (2013a, b), who shows that if furthermore banks in equilibrium (endogenously) choose heterogeneous portfolios, defaults involve an internal cost, and so a country may a) prefer not to default even in the absence of sanctions, and b) may want to rescue another country despite the subsidy to third-party lenders to the defaulting country. The focus in Mengus is thus on the impact of sovereign default on banks rather than on the doom-loop. Bolton-Jeanne (2011) also study the international contagion of sovereign debt crises through the financial sector and their international fiscal implications. The focus in Bolton-Jeanne is on the impact of sovereign default on banks and the role of banks in contagion rather than on the doom-loop. Philippon-Skreta (2012) and Tirole (2012) study the design of government interventions to rescue the financial sector in the presence of asymmetric information on banks' balance sheets. The focus is on ex-post interventions rather than on the doom loop.

In our model, the combination of limited commitment on the part of the government, and ex-post bailouts gives rise to strategic complementarities in financial risk-taking, and provides a rationale for macroprudential regulation. This occurs through a general equilibrium effect on the pricing of sovereign debt and the occurrence of default. This is an important difference with other papers emphasizing strategic complementarities arising from bailout guarantees, such as Schneider-Tornell (2004), Acharya-Yorulmazer (2008), Ranciere et al. (2008), Diamond-Rajan (2012), Farhi-Tirole (2012), and Chari-Kehoe (2013), which instead rely on mechanisms by which bailouts are extended only when sufficiently many banks are in trouble.

In our model, ruling out bailouts, if possible, would be desirable. Bianchi (2013), Stavrakeva (2013), and Keister (2014) argue that bailouts can have desirable properties despite the associated moral hazard. In Bianchi (2013) and Stavrakeva (2013), this occurs because bailouts help relax borrowing constraints in crises. In Keister (2014), this happens because bailouts mitigate the incentives of depositors to run on banks in an environment a la Diamond-Dybvig (1983). These papers stress that the optimal policy mix might involve bank bailouts combined with macroprudential policy. This possibility could arise in our model, but we mostly focus on the case where it does not by assuming that banks have enough net worth to take advantage of future investment opportunities provided that they manage their liquidity prudently.

2 Model

2.1 Setup

We consider the following economy. There are three dates $t \in \{0,1,2\}$ and a single good at every date.

The economy is populated by international investors, a continuum of mass one of domestic bankers and a continuum of mass one of domestic consumers. In addition, there is a domestic government.

Uncertainty is gradually resolved over time. At date 1, a state of the world is realized $s \in S$, with (full support) probability distribution $d\pi(s)$, where S is an interval of $\mathbb{R}^+$. The bankers' balance sheets and the fiscal capacity of the government depend on the realization of the state of the world s.

Private agents: international investors, bankers and consumers. International investors have a large endowment in every period. Their utility $V_t^* = \mathbb{E}_t[\sum_{s=t}^2 c_s^*]$ at date t is linear over consumption.

Consumers have a random endowment $E \in [0,\infty)$ at date 2, with probability distribution function $f(E|s)$ and cumulative distribution function $F(E|s)$. The government's only fiscal resources are at date 2: The government can tax the (random) endowment E of domestic consumers. The endowment E can hence be interpreted as the fiscal capacity of the government. We assume that $\frac{\partial(f(E|s)/(1-F(E|s)))}{\partial s} \leq 0$ and that $\frac{\partial(f(E|s)/(1-F(E|s)))}{\partial E} > 0$. The first inequality will imply that decreases in s are bad news for the fiscal capacity of the government; the second is a monotone hazard rate condition that will imply a quasi-concave Laffer curve. The two conditions are equivalent if s shifts the distribution uniformly so that $F(E|s) = F(E-s)$. Consumers' utility $V_t^C = \mathbb{E}_t[c_2^C]$ at date t is linear over consumption at date 2. As usual, one can think of E as the consumers' disposable income beyond some incompressible level of consumption.

bankers have an endowment A at date 0. At date 1, in state s, they have a fixed-size investment opportunity which pays off at date 2. They can invest $I(s)$ with a payoff $\rho_1(s)I(s)$ where $\rho_1(s) > 1$. The dependence of I and ρ_1 on s more generally stands for liquidity (or financial) shocks faced by banks. We assume that $\frac{dI(s)}{ds} \leq 0$ so that low s states are states in which banks badly need cash. The utility of bankers $V_t^B = \mathbb{E}_t[c_2^B]$ at date t is linear over consumption at date 2.

We assume for the moment that the return from the investment project of bankers cannot be pledged to outside investors, and as result, bankers cannot raise outside funding at date 1 (see Section A.2 for a relaxation of this assumption). Instead, they must self-finance the investment project $I(s)$. Therefore, at date 0, bankers trade their endowment A for financial assets (stores of value), part or all of which they sell at date 1 to finance their investment project.[4] We assume that $A \geq \bar{I}$ where $\bar{I} = \max_{s \in S} I(s)$ so that if bankers manage or decide to preserve their wealth between dates 0 and 1, they can always finance their investment project.

Assets. In the basic model, these financial assets are assumed to come in two forms, domestic sovereign bonds in amount B_0, and foreign bonds in unlimited supply. Both domestic and foreign bonds are claims to a unit of good at date 2.

Except in Section A.3, in which we will introduce competition among banks for access to bailout funds, we look for a symmetric equilibrium, in which banks all choose the same portfolio. We denote by b_0 and b_0^* the representative bank's holdings of domestic sovereign bonds and foreign bonds. We assume that there are no short sales so that $b_0^* \geq 0$ and $b_0 \geq 0$.

Foreign bonds—which could be either private bonds or foreign government bonds—are safe, and hence their price is always 1. By contrast, we assume that domestic bonds are risky because the domestic government might default. We denote their price in period 0 by p_0 and their price in period 1 by $p_1(s)$. We assume that $p_0 B_0 > A$ so that the marginal holder of domestic bonds is an international investor.

Welfare. At each point in time, the government evaluates welfare by subtracting default costs (to be introduced below) from $\mathcal{W}_t = \mathbb{E}_t[c_2^C + \beta^B c_2^B + \beta^I(s)\mu(s)I(s)]$, where $\mathcal{W}_t$ is a weighted average of consumer welfare, banker welfare and investment $\mu(s)I(s)$, where $\mu(s)$ is the mass of bankers who undertake their investment project. We assume that $\beta^B < 1$, and so pure consumption transfers to bankers are costly.

The term $\beta^I(s)\mu(s)I(s)$ in the social welfare function captures an externality on other agents in the economy, namely the welfare benefit for other banking stakeholders (borrowers, workers), from the banks' ability to invest.[5] This modeling of social preferences

[4]This structure resembles Woodford (1990).

[5]Imagine that, say, three categories of banking stakeholders' benefit from the banks' ability to invest. First, and most obviously the bankers themselves: They receive $\rho_1(s)\mu(s)I(s)$, where $\rho_1(s)$ is the banks' stake in continuation. Second, the higher $\mu(s)I(s)$, the better off their borrowers. Third, the workers working in banks and industrial companies; to the extent that they are better off employed (e.g., they receive an efficiency wage) and that preserved employment is related to $\mu(s)I(s)$, then workers' welfare grows with $\mu(s)I(s)$. Thus if $\rho_1^F(s)$ and $\rho_1^W(s)$ denote the stakes of the industrial firms and the workers, and if $\tilde{\beta}^B$, $\tilde{\beta}^F$

thus allows for a wide range of preferences among economic agents.

Government debt, bailouts, defaults. The domestic government makes decisions sequentially, without commitment. At date 1, the government decides whether or not to undertake a bailout of its domestic banks. At date 2, the government decides whether to repay its debt or to default.

The government has some outstanding bonds B_0 at date 0. We assume for the moment that these bonds mature at date 2. In Section 5.1, we will investigate whether conclusions are altered by a shorter maturity and whether the government optimally issues long-term bonds.

At date 1, the government chooses whether or not to undertake a bailout of the financial sector. We assume that at date 1, the government inspects the balance sheets of banks that apply for a bailout and so can, if it so desires, tailor individual bailout levels to specific liquidity shortages of applying banks (which in equilibrium will end up being identical because of equilibrium symmetric portfolios).[6] We denote by $X(s)$ the total transfer to the banks. In order to finance this transfer, the government must issue new bonds $B_1(s) - B_0$. Throughout the paper, we maintain the assumption that the government can always raise enough resources to bail out all the banks $p_1(s)(B_1(s) - B_0) = X(s)$. We relax this assumption in Section A.3.[7]

We assume that the weight $\beta^I(s)$ on investment is high enough so that the government always chooses to bail out the financial sector if such a bailout is needed, implying that $X(s) = \max\{I(s) - (b_0^* + p_1(s)b_0), 0\}$. Finally, we assume that the government can always raise enough funds at date 1 to finance the desired bailout (see Section A.3 for a relaxation of this assumption).

At date 2, the government decides whether or not to default on its debt. The government cannot discriminate between foreign and domestic bond holders, and hence cannot selectively default on foreigners. The government incurs a fixed cost Φ if it defaults on its debt, which we assume is high enough $\Phi > B_0$. This implies in particular that the government only defaults if it cannot pay its debt, that is if and only if $B_1(s) > E$.[8]

and $\tilde{\beta}^W$ denote the three categories of stakeholders' welfare weights or political influence, then $\beta^B = \tilde{\beta}^B$ and $\beta^I = (\tilde{\beta}^F \rho_1^F(s) + \tilde{\beta}^W \rho_1^W(s))$. This "credit crunch" interpretation can be formalized further along the lines of Holmström-Tirole (1997) (see Appendix A.1).

[6] Alternatively, we could have followed Farhi and Tirole (2012) or Mengus (2013a, b) in assuming that individual portfolios are imperfectly observed at the bailout date and that these portfolios are endogenously heterogeneous. This would make bailouts more costly and the analysis more complex, without altering the basic insights in our context.

[7] The assumption that the government sets the amount it promises to reimburse, $B_1(s)$, rather than the amount it borrows eliminates any multiplicity associated with erratic expectations as in Calvo (1988).

[8] The government would never issue $B_1(s) > \Phi$. Such an issuance would yield zero revenues since it

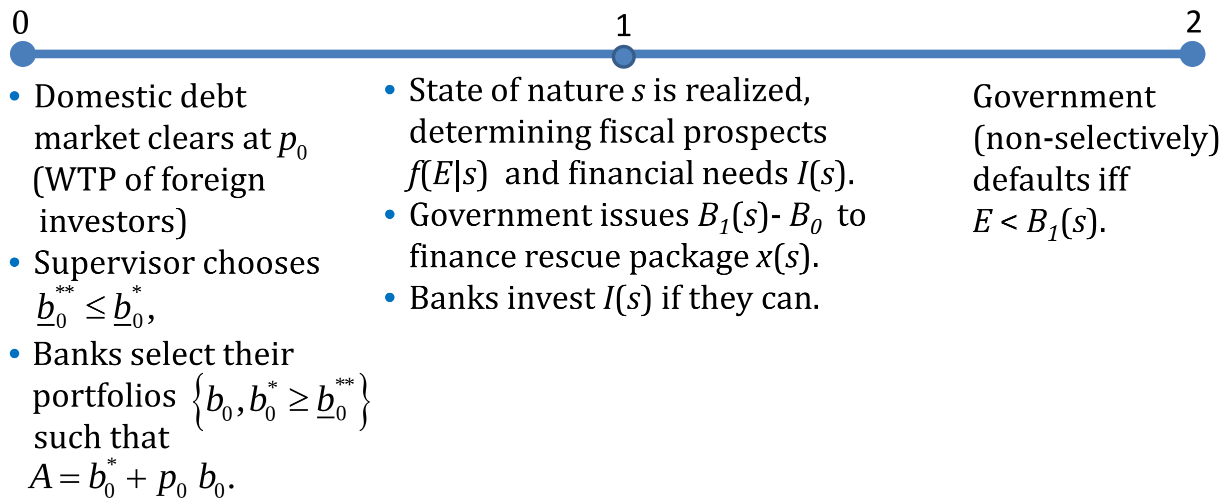

Figure 1: Timeline.

Supervisory game. As we will note, banking bailouts provide banks with an incentive to take risk. Conversely, and in the absence of sovereign debt forgiveness, the supervisor would like to limit risk taking. In general, an individual bank's exposure to domestic bonds depends on a costly supervisory effort to detect hidden exposures and on the bank's costly effort to make these exposures opaque.

Rather than formalize the supervisory game in its entire generality we will assume that there is an exogenous supervisory capability in the form of a minimum diversification requirement, i.e. a lower bound on foreign holdings $\underline{b}_0^*$. For a given supervisory capability $\underline{b}_0^*$, the supervisor can then decide to be lenient by setting an effective minimum diversification requirement $\underline{b}_0^{**} \leq \underline{b}_0^*$.

Except in Section 3.5, we assume that the supervisor can perfectly enforce the effective minimum diversification requirement $\underline{b}_0^{**}$, so that a banker must set $b_0^* \geq \underline{b}_0^{**}$. In Section 3.5, we will allow supervisory evasion by the banks by more generally assuming that a banker can invest b_0^* in safe foreign bonds and $A - b_0^*$ in domestic bonds at cost $\Psi(\underline{b}_0^{**} - b_0^*)$, where Ψ is a strictly increasing and convex function on $\mathbb{R}$ with $\Psi(x) = 0$ for $x \leq 0$.

Informational assumptions. We make the following informational assumptions. The supervisory capability $\underline{b}_0^*$ and the amount of legacy debt B_0 are publicly observable at date 0. The decision regarding supervisory leniency $\underline{b}_0^{**}$ and the portfolios of banks are not observable to international investors at date 0 but are publicly observable at date 1, and so are the bailout $X(s)$ and the amount of debt $B_1(s)$.

would lead to default with probability 1 and hence be associated with a zero price $p_1(s) = 0$.

Supervisory information is in general kept confidential. Why this is so is clear for proprietary information embodied in the bank's portfolio. But aggregate information is no released either as regulators are also concerned by runs on an institution which is assessed as risky by the supervisor. For example, in the US, CAMEL and RFI ratings are not released to the public but only to the top management. The non-observability assumption is therefore reasonable; but we have checked that the qualitative results also hold in the slightly more complex case in which supervisory intensity is observed by the market at date 0.

2.2 Equilibrium

In this section, we characterize the equilibrium of the model. Figure 1 summarizes the timeline.

Bond prices and Laffer Curve. Because the marginal investor of domestic bonds is a risk-neutral international investor, the prices of domestic bonds at dates 0 and 1 simply reflect the relevant conditional default probability:

$$p_1(s) = 1 - F(B_1(s)|s), \tag{1}$$

$$p_0 = \int p_1(s) d\pi(s). \tag{2}$$

At date 1 in state s, the government can thus collect $(B_1 - B_0)[1 - F(B_1|s)]$ by issuing $B_1 - B_0$. This revenue is strictly quasi-concave in B_1 and increasing in s from our assumptions on the distribution of the date-2 endowment E. It is always optimal for the government to pick $B_1 = B_1(s)$ so as to be in the upward sloping part of the Laffer curve in state s.

Bankers' portfolios and supervision. Bankers invest their net worth into foreign bonds $b_0^* \geq 0$ and domestic bonds $b_0 \geq 0$ so that

$$A = b_0^* + p_0 b_0.$$

At date 1, their pre-bailout net worth is $b_0^* + p_1(s)b_0$. If their pre-bailout net worth falls short of the investment size $I(s)$, they receive a government bailout. In a symmetric equilibrium, they receive $X(s) = I(s) - (b_0^* + p_1(s)b_0)$. If their pre-bailout net worth exceeds the investment size $I(s)$, they simply save the difference by acquiring either domestic or international bonds (at this stage, they are indifferent between both since they are risk neutral over date-2 consumption).

Their expected utility is therefore $V_0^B = \int [\rho_1(s)I(s) + \max\{b_0^* + p_1(s)b_0 - I(s), 0\}] \, d\pi(s)$. Because $p_0 = \int p_1(s)d\pi(s)$, $I(s)$ is decreasing in s, $A \geq \bar{I}$, bankers always choose $b_0^* = \underline{b}_0^{**}$.[9] This is intuitive: Bankers have an incentive to take as much risk as possible to extract the biggest possible expected bailout from the government.

Bailouts and date-1 bond issuance. To finance the bailout at date 1 in state s

$$X(s) = \max\{I(s) - \underline{b}_0^{**} - (A - \underline{b}_0^{**})\frac{p_1(s)}{p_0}, 0\}.$$

requires issuing $B_1(s) - B_0$ new bonds at date 1 with

$$p_1(s)[B_1(s) - B_0] = X(s).$$

Date-1 debt $B_1(s) \geq B_0$ is the smallest solution of the following fixed-point equation

$$[B_1(s) - B_0][1 - F(B_1(s)|s)] = \max\{I(s) - \underline{b}_0^{**} - (A - \underline{b}_0^{**})\frac{1 - F(B_1(s)|s)}{p_0}, 0\}. \quad (3)$$

The solution $B_1(s) \geq B_0$ is necessarily on the upward sloping part of the Laffer curve, and we assume that equation (3) has a unique solution $B_1(s) \geq B_0$ on this upward sloping part of the Laffer curve. If $B_1(s) > B_0$, this solution is then necessarily locally stable, by which we mean that the slope of the left-hand side of (3) is greater than that of the right-hand side.

There exists a cutoff $\tilde{s}$ such that $B_1(s) > B_0$ if $s < \tilde{s}$ and $B_1(s) = B_0$ for $s \geq \tilde{s}$. Furthermore, we can show that $\frac{dB_1(s)}{ds} < 0$ for $s < \tilde{s}$ and similarly that $\frac{dp_1(s)}{ds} > 0$ for $s < \tilde{s}$, and for all s if $\frac{\partial(f(E|s)/(1-F(E|s)))}{\partial s} < 0$ (strict inequality) .

Supervisory leniency. When setting the effective minimum diversification requirement $\underline{b}_0^{**} \leq \underline{b}_0^*$, the supervisor seeks to maximize welfare

[9]Because $p_1(s)$ is increasing in s and $I(s)$ is decreasing in s, there exists $\tilde{s}$ such that $b_0^*(1 - \frac{p_1(s)}{p_0}) + \frac{p_1(s)}{p_0}A - I(s) \geq 0$ if and only if $s \geq \tilde{s}$. Note that if $p_1(s) \geq p_0$, then $s \geq \tilde{s}$. Now consider $b_0^{*\prime} > b_0^*$. We necessarily have $\tilde{s}' \leq \tilde{s}$. This implies that $V_0^{B\prime} - V_0^B \leq \int_{s \geq \tilde{s}'}(b_0^{*\prime} - b_0)(1 - \frac{p_1(s)}{p_0})d\pi(s) \leq 0$.

$$\mathcal{W}_0 = \int \left[\int_{B_1(s)}^{\infty} [E - B_1(s)] f(E|s) dE + \int_0^{B_1(s)} [E - \Phi] f(E|s) dE + \beta^I(s) I(s) \right] d\pi(s)$$
$$+ \int \beta^B \left[\rho_1(s) I(s) + \max\{\underline{b}_0^{**} + (A - \underline{b}_0^{**}) \frac{p_1(s)}{p_0} - I(s), 0\} \right] d\pi(s),$$

taking p_0 as given but taking into account the impact of $\underline{b}_0^{**}$ on $p_1(s)$ and $B_1(s)$ through

$$p_1(s) = 1 - F(B_1(s)|s),$$

$$p_1(s)[B_1(s) - B_0] = \max\{I(s) - \underline{b}_0^{**} - (A - \underline{b}_0^{**}) \frac{p_1(s)}{p_0}, 0\}.$$

Welfare decomposition. Using the equilibrium equations (1), (2), and (3), we can derive an enlightening decomposition of equilibrium welfare

$$\mathcal{W}_0 = \mathcal{E}_0 - \mathcal{R}_0, \tag{4}$$

with

$$\mathcal{E}_0 = \int \left[\int_{B_1(s)}^{\infty} [E - B_0] f(E|s) dE + \int_0^{B_1(s)} [E - \Phi] f(E|s) dE \right] d\pi(s)$$
$$+ \int \left[\left[\beta^I(s) + \beta^B(\rho_1(s) - 1) \right] A + \beta^B A \right] d\pi(s) \tag{5}$$

and

$$\mathcal{R}_0 = -(1 - \beta^B) \int \min\{\underline{b}_0^{**} + (A - \underline{b}_0^{**}) \frac{p_1(s)}{p_0} - I(s), 0\} d\pi(s), \tag{6}$$

which using the equilibrium martingale property of prices (2), also has the alternative expression

$$\mathcal{R}_0 = (1 - \beta^B) \int \left[\max\{\underline{b}_0^{**} + (A - \underline{b}_0^{**}) \frac{p_1(s)}{p_0} - I(s), 0\} - [A - I(s)] \right] d\pi(s). \tag{7}$$

The term $\mathcal{E}_0$ is a pure efficiency term. It accounts for the cost $p_0 B_0$ of legacy debt repayment and the cost of defaults $\Phi \int F(B_1(s)|s) d\pi(s)$.[10] The term $\mathcal{R}_0 \geq 0$ is a pure distributive term, which is positive because the banks engage in risk-taking and thereby obtain

[10] The cost of legacy debt repayment $p_0 B_0$ can be seen as the cost of repayment of foreign legacy creditors. It occurs both ex ante at date 0 for the foreign legacy creditors who have sold their debt to bankers and ex post at date 2 for those who have held on to it.

a put on taxpayer money. It accounts for the cost of the rents extracted by bankers at the expense of consumers because of bailouts. These rents reduce welfare because bankers carry a lower welfare weight than consumers $\beta^B < 1$.

It is important to keep in mind that the decomposition $\mathcal{W}_0 = \mathcal{E}_0 - \mathcal{R}_0$ in (4) as well as the alternative expression (7) for $\mathcal{R}_0$ make use of the martingale property of equilibrium prices (2). As a result, they are only valid in equilibrium. They cannot be used off equilibrium to analyze the ex-post incentives of supervisors to set the effective diversification requirement $\underline{b}_0^{**}$, for a given p_0, because (2) might not hold. Instead we then use a different decomposition

$$\mathcal{W}_0 = \mathcal{E}_0 - \mathcal{R}_0 + \mathcal{C}_0 \tag{8}$$

with $\mathcal{E}_0$ given by (5), $\mathcal{R}_0$ given by (6), and $\mathcal{C}_0$ given by

$$\mathcal{C}_0 = \beta^B \int \left[\underline{b}_0^{**} + (A - \underline{b}_0^{**}) \frac{p_1(s)}{p_0} - A \right] d\pi(s). \tag{9}$$

The term $\mathcal{C}_0$ is a corrective term that accounts for the rents that bankers extract at the expense of foreign legacy creditors when the martingale property of prices (2) does not hold. This is because the price p_0 at which banker purchase debt from foreign legacy creditors deviates from the expected future probability of repayment $\int p_1(s) d\pi(s)$. We have $\mathcal{C}_0 = 0$ in equilibrium when (2) holds so that $p_0 = \int p_1(s) d\pi(s)$. But off equilibrium, we have $\mathcal{C}_0 > 0$ if $p_0 < \int p_1(s) d\pi(s)$ and $\mathcal{C}_0 < 0$ if $p_0 > \int p_1(s) d\pi(s)$.

3 Sovereign and Financial Balance Sheets Doom Loops

In this section, we illustrate the amplification mechanism arising from a feedback loop between banks and sovereign balance sheets. We show that for a given supervisory capability, it is never optimal for the supervisor to engage in supervisory leniency. We then characterize optimal first-best frictionless supervision (when the government can force full diversification at no cost). We finally show that when supervision is imperfect, banks' domestic sovereign risk loadings are strategic complements, leading to the possibility of multiple equilibria with varying degrees of banks' domestic sovereign risk exposures, and imparting a macroprudential dimension to supervision.

3.1 Amplification Mechanism

This feedback loop can be seen through the following fixed-point equation for the date-1 price of government bonds

$$p_1(s) = 1 - F(B_1(s)|s), \tag{10}$$

where

$$B_1(s) = B_0 + \max\{\frac{I(s) - \underline{b}_0^{**}}{p_1(s)} - \frac{A - \underline{b}_0^{**}}{p_0}, 0\}. \tag{11}$$

Using the implicit function theorem, we can then derive the following comparative static result, assuming that a bailout occurs in state s, i.e. that $s < \tilde{s}$.

Proposition 1 (Feedback Loop)**.** *The sensitivity of date-*1 *bond prices* $p_1(s)$ *to the state* $s < \tilde{s}$ *when a bailout is required is given by*

$$\frac{dp_1(s)}{ds} = \frac{-\frac{\partial F(B_1(s)|s)}{\partial s} - \frac{1}{p_1(s)} f(B_1(s)|s) \frac{dI(s)}{ds}}{1 - \frac{I(s) - \underline{b}_0^{**}}{p_1^2(s)} f(B_1(s)|s)}. \tag{12}$$

The numerator encapsulates the direct effect of the change in s on the debt price $p_1(s)$ if there were no change in the price at which the government issues bonds to finance the bailout and at which bankers liquidate their government bond holdings. The first term in the numerator captures the direct change in the probability of no-default at constant investment size $I(s)$. The second term in the numerator captures the direct impact of the change in the investment size $I(s)$.

The denominator is positive because of the local stability of the selected fixed-point solution to equations (10) and (11). It takes the form of a multiplier, which represents the indirect effect of a change in s on the debt price $p_1(s)$ through the change in the price at which the government issues bonds and at which bankers liquidate their government holdings. The multiplier is higher, the larger the amount of foreign-held debt $B_1(s) - (B_0 - b_0) = \frac{I(s) - \underline{b}_0^{**}}{p_1(s)}$ that must be issued to finance the bailout (and hence the higher the amount of domestic debt held by domestic banks, i.e. the lower is $\underline{b}_0^{**}$), and the larger the semi-elasticity $\frac{1}{p_1(s)} f(B_1(s)|s)$ of the debt price $p_1(s)$ to additional debt issuances. This multiplier captures the feedback loop between banks and sovereigns as an amplification mechanism: An increase in the default probability reduces the price $p_1(s)$ which increases the required bailout $X(s)$ and hence the quantity of bonds $B_1(s) - B_0$ that must be issued at date 1, which further reduces the price $p_1(s)$ etc. ad infinitum.

Consider for example the case where $\frac{dI(s)}{ds} = 0$ so that there are no variation in investment needs as we vary s, and assume that $\frac{\partial (f(E|s)/(1-F(E|s)))}{\partial s} < 0$ (strict inequality).

Decreases in s are then just bad news for the fiscal capacity of the government. The effect of a bad fiscal shock $ds < 0$ on bond prices $p_1(s)$ is then amplified because some of these bonds are held by the banking system, which increases the size of the required bailout, worsening the fiscal problems etc. ad infinitum.

Similarly, consider the case where $\frac{\partial f(E|s)}{\partial s} = 0$ so that there are no variations in fiscal capacity as we vary s, and assume that $\frac{dI(s)}{ds} < 0$ (strict inequality). Decreases in s are then just increases in the liquidity needs of entrepreneurs.[11] Again, the effect of a bad financial shock $ds < 0$ on bond prices $p_1(s)$ is amplified because some of these bonds are held by the banking system, which must then be bailed out, worsening the fiscal problems etc. ad infinitum.[12]

3.2 No Supervisory Leniency

In the model, it is never optimal for the supervisor to engage in supervisory leniency. In other words, it is always optimal to set the effective minimum diversification requirement $\underline{b}_0^{**}$ as high as allowed by supervisory capability $\underline{b}_0^{*}$.

Ex post taking p_0 as given, the effect of engaging in supervisory leniency on welfare can be analyzed using the decomposition $\mathcal{W}_0 = \mathcal{E}_0 - \mathcal{R}_0 + \mathcal{C}_0$ given in (8). First, it has an efficiency benefit by reducing expected legacy debt repayments because it leads to more defaults. Second, it has an efficiency cost by increasing expected default costs. Under our maintained assumption of large default costs, the net effect is a reduction in $\mathcal{E}_0$. Third, it has a distributive cost by increasing the rents extracted by bankers at the expense of consumers because of bank bailouts. This increases $\mathcal{R}_0$. Fourth, it has a distributive cost

[11] Although this is not essential, in order for decreases in s to represent bad news, we also assume that $\frac{d(\rho_1(s)I(s))}{ds} > 0$, and $\frac{d(\beta^I(s)I(s))}{ds} > 0$.

[12] It is instructive to view the doom loop through the lens of balance sheet consolidation. At date 1, domestic agents (the government, consumers, and banks) need to finance $I(s)$. To that end, they can use foreign liquid resources b_0^* and fresh borrowing, as captured by the consolidated budget constraint

$$b_0^* + p_1(s)[B_1(s) - (B_0 - b_0)] = I(s),$$

which is just a re-arrangement of the bailout equation (11). Therefore, domestic balance sheets can always be consolidated ex post at date 1. But the ex-post consolidated balance sheet at date 1 depends on the ex-ante disaggregated balance sheets at date 0. In particular, the ex-ante decisions of banks at date 0 determine the amount of available liquid foreign resources b_0^* ex post at date 1 and hence government debt $B_1(s)$ at date 1 and the price $p_1(s)$ of government debt at date 1. This variable is masked in the ex-ante consolidated balance sheet at date 0, which indicates a domestic net foreign asset position of

$$b_0^* - p_0(B_0 - b_0) = A - p_0 B_0.$$

The doom loop is therefore consistent with ex-ante and ex-post balance sheet consolidation. But it also shows the limits of balance sheet consolidation in the sense that the doom loop originates in ex-ante developments that have implications ex post and that are not revealed in consolidated balance sheets.

because bankers must now purchase debt from foreign legacy creditors at a price which exceeds expected repayment. This reduces $\mathcal{C}_0$. Overall, the total effect of supervisory leniency is therefore a reduction in welfare.

Proposition 2 (No Supervisory Leniency). *As long as there are bailouts, it is never optimal for the supervisor to engage in supervisory leniency and so $\underline{b}_0^{**} = \underline{b}_0^{*}$.*

Proof. The choice of $\underline{b}_0^{**}$ for a given p_0 involves off-equilibrium calculations. We therefore cannot make use of the decomposition (4) provided earlier, which only holds in equilibrium when the martingale property of prices (2) holds. Instead, we use the decomposition (8), which holds both in equilibrium when the martingale property of prices (2) holds and off equilibrium when it does not. Hence we write

$$\mathcal{W}_0 = \mathcal{E}_0 - \mathcal{R}_0 + \mathcal{C}_0,$$

where

$$\mathcal{E}_0 = \int \left[\int_{B_1(s)}^{\infty} [E - B_0] f(E|s) dE + \int_0^{B_1(s)} [E - \Phi] f(E|s) dE \right] d\pi(s) \\ + \int \left[\left[\beta^I(s) + \beta^B(\rho_1(s) - 1) \right] I(s) + \beta^B A \right] d\pi(s),$$

$$\mathcal{R}_0 = -(1 - \beta^B) \int \min\{\underline{b}_0^{**} + (A - \underline{b}_0^{**}) \frac{p_1(s)}{p_0} - I(s), 0\} d\pi(s),$$

$$\mathcal{C}_0 = \beta^B \int \left[\underline{b}_0^{**} + (A - \underline{b}_0^{**}) \frac{p_1(s)}{p_0} - A \right] d\pi(s).$$

The supervisor sets $\underline{b}_0^{**} \leq \underline{b}_0^{*}$ in order to maximize W_0 taking p_0 as given but subject to the two constraints

$$p_1(s) = 1 - F(B_1(s)|s),$$

$$p_1(s)[B_1(s) - B_0] = \max\{I(s) - \underline{b}_0^{**} - (A - \underline{b}_0^{**}) \frac{p_1(s)}{p_0}, 0\}.$$

We denote the resulting dependence of the solution on $\underline{b}_0^{**}$ given p_0 as $p_1(s, \underline{b}_0^{**}; p_0)$, $B_1(s, \underline{b}_0^{**}; p_0)$, $\mathcal{W}_0(\underline{b}_0^{**}; p_0)$, $\mathcal{E}_0(\underline{b}_0^{**}; p_0)$, $\mathcal{R}_0(\underline{b}_0^{**}; p_0)$ and $\mathcal{C}_0(\underline{b}_0^{**}; p_0)$.

Towards a contradiction, consider a candidate equilibrium with $\underline{b}_0^{**} < \underline{b}_0^{*}$, and suppose that there are bailouts with strictly positive probability. The date-0 prices p_0 satisfies the fixed point equation $p_0 = \int p_1(s, \underline{b}_0^{**}; p_0) d\pi(s)$. Given this price p_0, consider setting $\underline{b}_0^{**\prime} \in (\underline{b}_0^{**}, \underline{b}_0^{*})$. We now proceed to show that $\mathcal{W}_0(\underline{b}_0^{**\prime}; p_0) > \mathcal{W}_0(\underline{b}_0^{**}; p_0)$, a contradiction. We do so by showing that $\mathcal{E}_0(\underline{b}_0^{**\prime}; p_0) > \mathcal{E}_0(\underline{b}_0^{**}; p_0)$, $\mathcal{R}_0(\underline{b}_0^{**\prime}; p_0) < \mathcal{R}_0(\underline{b}_0^{**}; p_0)$, and

$\mathcal{C}_0(\underline{b}_0^{**\prime}; p_0) > \mathcal{C}_0(\underline{b}_0^{**}; p_0)$.

It is easy to see that $p_1(s, \underline{b}_0^{**\prime}; p_0) \geq p_1(s, \underline{b}_0^{**}; p_0)$ and $B_1(s, \underline{b}_0^{**\prime}; p_0) \leq B_1(s, \underline{b}_0^{**}; p_0)$ for all s, where the inequalities are strict with positive probability. Using $B_1(s, \underline{b}_0^{**\prime}; p_0) \leq B_1(s, \underline{b}_0^{**}; p_0)$ with a strict inequality with positive probability, we get

$$\int_{B_1(s,\underline{b}_0^{**\prime};p_0)}^{\infty} [E - B_0] f(E|s) dE + \int_0^{B_1(s,\underline{b}_0^{**\prime};p_0)} [E - \Phi] f(E|s) dE \geq \int_{B_1(s,\underline{b}_0^{**};p_0)}^{\infty} [E - B_0] f(E|s) dE + \int_0^{B_1(s,\underline{b}_0^{**};p_0)} [E - \Phi] f(E|s) dE,$$

where the inequality is strict with positive probability. This implies that $\mathcal{E}_0(\underline{b}_0^{**\prime}; p_0) > \mathcal{E}_0(\underline{b}_0^{**}; p_0)$.

Using the fact that $\underline{b}_0^{**\prime} > \underline{b}_0^{**}$ and $p_1(s, \underline{b}_0^{**\prime}; p_0) \geq p_1(s, \underline{b}_0^{**}; p_0)$ with a strict inequality with positive probability, we get

$$\min\{\underline{b}_0^{**\prime} + (A - \underline{b}_0^{**\prime}) \frac{p_1(s, \underline{b}_0^{**\prime}; p_0)}{p_0} - I(s), 0\} \geq \min\{\underline{b}_0^{**} + (A - \underline{b}_0^{**}) \frac{p_1(s, \underline{b}_0^{**}; p_0)}{p_0} - I(s), 0\}$$

where the inequality is strict with positive probability. This follows because given that $A \geq \bar{I}$, we necessarily have $\frac{p_1(s,\underline{b}_0^{**};p_0)}{p_0} < 1$ whenever $\underline{b}_0^{**} + (A - \underline{b}_0^{**}) \frac{p_1(s,\underline{b}_0^{**};p_0)}{p_0} - I(s) < 0$. This implies that $\mathcal{R}_0(\underline{b}_0^{**\prime}; p_0) < \mathcal{R}_0(\underline{b}_0^{**}; p_0)$.

Finally note that using the fact that $\underline{b}_0^{**\prime} > \underline{b}_0^{**}$, $p_1(s, \underline{b}_0^{**\prime}; p_0) \geq p_1(s, \underline{b}_0^{**}; p_0)$ with a strict inequality with positive probability and $p_0 = \int p_1(s, \underline{b}_0^{**}; p_0) d\pi(s)$, we get

$$\int \left[\underline{b}_0^{**\prime} + (A - \underline{b}_0^{**\prime}) \frac{p_1(s, \underline{b}_0^{**\prime}; p_0)}{p_0} - A\right] d\pi(s) = \underline{b}_0^{**} + (A - \underline{b}_0^{**}) \frac{\int p_1(s, \underline{b}_0^{**\prime}; p_0) d\pi(s)}{p_0} - A > 0 = \int \left[\underline{b}_0^{**} + (A - \underline{b}_0^{**}) \frac{p_1(s, \underline{b}_0^{**}; p_0)}{p_0} - A\right] d\pi(s).$$

This implies that $\mathcal{C}_0(\underline{b}_0^{**\prime}; p_0) > \mathcal{C}_0(\underline{b}_0^{**}; p_0)$. □

In Section 4, we show that this conclusion can be overturned in the presence of debt forgiveness. It can then be optimal to engage in supervisory leniency in order to extract concessions from legacy creditors. We use the result established by Proposition 2 that $\underline{b}_0^{**} = \underline{b}_0^{*}$ throughout the paper except in Sections 4, as well as in the extensions in Sections A.4 and 5.2 which also consider the possibility of debt forgiveness.

3.3 First-Best Frictionless Supervision

In this section, we investigate a related but different question. We perform a comparative statics exercise with respect to supervisory capability $\underline{b}_0^*$. This is different because the government internalizes its impact on the date-0 price of debt p_0 when choosing supervisory capacity $\underline{b}_0^*$, but not when choosing actual supervision $\underline{b}_0^{**} \leq \underline{b}_0^*$ given supervisory capacity $\underline{b}_0^*$. We perform this comparative static under two alternative assumptions: (a) that the the face value of debt B_0 is kept constant; or (b) that the market value of debt $p_0 B_0$ is kept constant. Which of (a) or (b) is most reasonable depends on the situation one is trying to capture. One could argue that (b) is more relevant for long-run comparisons across countries or regulatory regimes, while (a) is more relevant to analyze short-run responses to unanticipated shocks or situations as in Bulow-Rogoff (1988,1991) (see below).

As we argued earlier, the government may have limited ability to force banks to diversify. Nonetheless, it is instructive to investigate optimal supervision in the ideal theoretical situation where such limits to supervision are absent. We refer to this situation as first-best, costless or frictionless supervision. The occurrence of default is minimized when $\underline{b}_0^* = \bar{I}$ so that bankers can always finance their investment $I(s)$ without requiring a bailout.

The welfare of bankers as well as total welfare are then independent of the amount $b_0^* \geq \underline{b}_0^*$ invested in foreign bonds above the floor $\underline{b}_0^*$, where we have used the fact that $\underline{b}_0^{**} = \underline{b}_0^*$. Reducing $\underline{b}_0^*$ below $\bar{I}$ on the other hand would reduce welfare.

Indeed, the effect of reducing $\underline{b}_0^*$ below $\bar{I}$ on welfare can be analyzed using the decomposition $\mathcal{W}_0 = \mathcal{E}_0 - \mathcal{R}_0$ given in (4). First, if the face value of debt is kept constant, it has an efficiency benefit by reducing expected repayments to foreigners because it leads to more defaults (this efficiency benefit is inexistent if instead the market value of debt is kept constant). Second, it has an efficiency cost by increasing expected default costs. Under our maintained assumption of large default costs, the net effect is a reduction in $\mathcal{E}_0$. Third, it has a distributive cost by increasing the rents extracted by bankers at the expense of consumers because of bank bailouts. This increases $\mathcal{R}_0$. Overall, the total effect of lower supervisory capability is therefore a reduction in welfare.

Proposition 3 (First-Best Frictionless Supervision)**.** *Setting $\underline{b}_0^* = \bar{I}$, if feasible, maximizes ex-ante welfare $\mathcal{W}_0$.*

Proof. We use the decomposition given (4) and write

$$\mathcal{W}_0 = \mathcal{E}_0 - \mathcal{R}_0.$$

Suppose first that B_0 is kept constant as we consider lowering $\underline{b}_0^*$ below $\bar{I}$. We have the following bounds

$$\mathcal{E}_0 \leq \int \left[\int_{B_0}^{\infty} [E - B_0] f(E|s) dE + \int_0^{B_0} [E - \Phi] f(E|s) dE \right] d\pi(s)$$
$$+ \int \left[\left[\beta^I(s) + \beta^B(\rho_1(s) - 1) \right] I(s) + \beta^B A \right] d\pi(s),$$
$$\mathcal{R}_0 \geq 0,$$

with strict inequalities if there are bailouts with positive probability and equalities otherwise. Both bounds are attained for $\underline{b}_0^* = \bar{I}$ where there are no bailouts. The result in the proposition follows. In the case where $p_0 B_0$ is kept constant instead, the result is even stronger since B_0 increases (and p_0 decreases) as we lower $\underline{b}_0^*$. □

The optimal frictionless first-best supervision actually prevents the feedback loop from occurring in the first place by prohibiting domestic banks from holding domestic sovereign debt to an extent that could make them illiquid. We have already discussed in Section 2.1 some reasons why we might observe suboptimal supervision $\underline{b}_0^* < \bar{I}$, creating the possibility of the feedback loops that are the focus of this paper. These considerations lead us to adopt a pragmatic position and treat $\underline{b}_0^*$ as a parameter.

3.4 Connection to Bulow-Rogoff

Engaging in supervisory leniency, or lowering supervisory capacity triggers a form of debt buyback of domestic sovereign debt by domestic banks. Propositions 2 and 3 and establish that such debt buybacks are undesirable, ex post and ex ante respectively.

It is interesting to relate these results to the well known result by Bulow-Rogoff (1988, 1991) that debt buybacks are undesirable. Their result is derived in a model that has similarities and differences with ours. Like our model, their model has mechanical defaults (defaults occur if the government cannot pay). Unlike our model, their model has zero default costs, and it has only consumers but no banks and no bailouts. We now proceed to unpack the respective roles of these different assumptions. As we shall see, our result, and the logic behind it, are very different from those of Bulow-Rogoff. But in order to maximize the comparability with their result, we focus on Proposition 3 where there is an effect of the associated debt buyback on the date-0 price of debt which is internalized by the government. For the same reason, we focus on the case where the face value of debt, rather than the market value of debt, is kept constant. We rely on the decomposition

$\mathcal{W}_0 = \mathcal{E}_0 - \mathcal{R}_0$ given in (4).

The essence of the Bulow-Rogoff argument is as follows. Consider the same environment as in our model, but with no default costs ($\Phi = 0$) and no banks ($A = \bar{I} = 0$). A debt buyback which reduces debt from B_0 to $B_0 + \Delta B_0$ with $\Delta B_0 < 0$ results in new no-default states $\Delta ND = \{(s, E) | E \in [B_0 + \Delta B_0, B_0)\}$. This leads to a positive change in foreign welfare

$$\Delta \mathcal{W}_0^* = \mathbb{E}_0[B_0 1_{\{(s,E) \in \Delta ND\}}] > 0$$

and a negative change in domestic welfare

$$\Delta \mathcal{W}_0 = \Delta \mathcal{E}_0 = -\Delta \mathcal{W}_0^* < 0.$$

Basically, the game is zero sum between domestics and foreigners. Foreigners gain from the buyback because it reduces the amount of outstanding debt, increases the probability of repayment, and increases the date-0 price of debt.[13] The gains of foreigners are at the expense of domestics, who end up repaying more often. A debt buyback is therefore a bad deal.

Now introduce default costs ($\Phi > 0$), but continue to assume that there are only consumers but no banks ($A = \bar{I} = 0$). Repeating the same exercise, we still have

$$\Delta \mathcal{W}_0^* = \mathbb{E}_0[B_0 1_{\{(s,E) \in \Delta ND\}}] > 0$$

but we now have

$$\Delta \mathcal{W}_0 = \Delta \mathcal{E}_0 = \mathbb{E}_0[(\Phi - B_0) 1_{\{(s,E) \in \Delta ND\}}] = \mathbb{E}_0[\Phi 1_{\{(s,E) \in \Delta ND\}}] - \Delta \mathcal{W}_0^* > -\Delta \mathcal{W}_0^*.$$

Because of default costs, the game between domestics and foreigners is not zero sum anymore: Debt-buybacks have efficiency gains because they economize on default costs. With large enough default costs, debt buybacks are a good deal: Both domestics and foreigners gain from a debt buyback.

Now introduce not only default costs ($\Phi > 0$) but also banks ($A > 0$ and $\bar{I} > 0$) to get our model as analyzed in Proposition 3. Start from $\underline{b}_0^* = \bar{I}$ and lower supervisory capacity to $\underline{b}_0^* + \Delta \underline{b}_0^*$ with $\Delta \underline{b}_0^* < 0$. This leads to a debt buyback of domestic sovereign debt by domestic banks. Despite the presence of default costs, this reduces domestic welfare. The reason is as follows. The debt buyback leads to greater bank bailouts $\Delta X(s) \geq 0$

[13] Foreigners are indifferent between selling and holding on to domestic sovereign debt. Those who sell benefit from the increased date-0 price. Those who do not sell benefit from the reduced probability of default.

and additional post-bailout debt $\Delta B_1(s) \geq 0$. As a result, there are new default states $\Delta D = \{(s, E) | E \in [B_1(s), B_1(s) + \Delta B_1(s))\}$ instead of new no-default states. This reduces foreign welfare

$$\Delta \mathcal{W}_0^* = -\mathbb{E}_0[B_0 1_{\{(s,E)\in\Delta D(s)\}}] \leq 0$$

and domestic welfare

$$\Delta \mathcal{W}_0 = \Delta \mathcal{E}_0 - \Delta \mathcal{R}_0 = -\mathbb{E}_0[(\Phi - B_0) 1_{\{(s,E)\in\Delta D(s)\}}] - (1 - \beta^B)\mathbb{E}_0[\Delta X(s)] \leq 0.$$

The reduction in domestic welfare arises from efficiency costs in the form of larger expected default costs net of debt repayments $\Delta \mathcal{E}_0 = -\mathbb{E}_0[(\Phi - B_0) 1_{\{(s,E)\in\Delta D(s)\}}] \leq 0$, and from distributive costs in the form of extra rents extracted by bankers at the expense of consumers $\Delta \mathcal{R}_0 = (1 - \beta^B)\mathbb{E}_0[\Delta X(s)] \geq 0$.

Like in Bulow-Rogoff, in our setting, debt buybacks reduce domestic welfare. But there are important differences. First, in our setting and unlike in theirs, there are default costs. Second, in our setting and unlike in theirs, it matters which institutions perform the debt buyback: Debt buybacks by the government can improve domestic welfare, but debt buybacks by banks reduce domestic welfare. Third, in our setting and unlike in theirs, debt buybacks by banks reduce foreign welfare as well as domestic welfare.

The takeaway from this discussion is that balance sheet consolidation leads to misleading conclusions. In the presence of large default costs, debt buybacks are a good deal, but not if they occur through domestic banks. This result, and the logic behind it, are fundamentally different from those of Bulow-Rogoff. They can only be uncovered in a setting with enough granularity to capture the special position of the financial sector and its relation with the sovereign.

3.5 Collective Moral Hazard

The rationale for liquidity regulation also has a macroprudential dimension. Indeed, the benefits of liquidity regulation depend on the risk taken by the banking system as a whole. For example, if for some reason only a fraction of banks take on domestic sovereign debt, then the benefit from regulating the other banks is reduced (and might even vanish) because the government has more fiscal space, reducing the riskiness of the government bonds and hence the need for bailouts, and also weakening the feedback loop between the remaining banks and the sovereign. We now show that for a given supervisory capacity, the incentives for banks to take on domestic sovereign debt are increased when other banks do so—a manifestation of the strategic complementarities in financial risk-taking at work in the model—so that the effectiveness of supervision depends on the risk taken by the banking system as a whole. This is the manifestation of a collective moral hazard problem as in Farhi-Tirole (2012).[14] The main difference here is that bailouts are perfectly targeted while imperfect targeting was key to the strategic complementarity result of Farhi-Tirole (2012). There are also strategic complementarities in financial risk-taking, which justify macroprudential regulation, but through a different, general equilibrium effect on the pricing of debt and the occurrence of default rather than through untargeted bailouts.

As discussed earlier, a bank's ability to engage in risk taking depends not only on supervisory policy, but also on its own ability to make its balance sheet opaque. Let us capture this idea by taking the supervisory effort as a given and assume that each bank indexed by $i \in [0,1]$ can (locally) select its individual level of foreign holdings $b_0^*(i)$ at non-monetary cost $\Psi(\underline{b}_0^{**} - b_0^*(i))$, a strictly increasing and convex function with $\Psi(x) = 0$ for $x \leq 0$. We look for a symmetric equilibrium in which all banks choose the same $b_0^*(i) = b_0^*$ for all i. For simplicity, we focus on fiscal shocks and assume that $I(s) = \bar{I}$ is independent of s. We also assume that $A = \bar{I}$. It is easy to see that the absence of supervisory leniency $\underline{b}_0^{**} = \underline{b}_0^*$ extends to the setting of this section. We therefore make use of the fact that $\underline{b}_0^{**} = \underline{b}_0^*$ throughout.

The banks' choices of opaqueness, and thereby the exposures to the domestic government, are strategic complements: Incurring the cost of making one's balance sheet more opaque is more tempting if the put on taxpayer money is more attractive; in turn, this put is more attractive when the sovereign bond price is more volatile, which it is when the other banks take a larger gamble.

To show this, note that for an individual bank i, given an aggregate b_0^*, the payoff from investing $b_0^*(i)$ is

$$V_0^B(b_0^*(i); b_0^*) = \int \rho_1(s)\bar{I}d\pi(s) + \int_{\tilde{s}}^{\infty} (A - b_0^*(i))(\frac{p_1(s)}{p_0} - 1)d\pi(s) - \Psi(\underline{b}_0^* - b_0^*(i)),$$

where we have left the dependence of p_0 and $p_1(s)$ on b_0^* implicit.

Proposition 4 (Strategic Complementarities in Banks' Domestic Exposures)**.** *Suppose that $I(s) = \bar{I}$ is independent of s, and that $A = \bar{I}$. There are strategic complementarities across banks in the choice of $b_0^*(i)$, i.e. the marginal benefit $\frac{\partial V_0^B(b_0^*(i); b_0^*)}{\partial b_0^*(i)}$ for a bank of increasing its individual investment $b_0^*(i)$ in foreign bonds is increasing in the aggregate investment b_0^* of banks in foreign*

[14] In Farhi-Tirole (2012), we study a related model where the combination of limited commitment on the part of the government, and ex-post untargeted bailouts gives rise to strategic complementarities in financial risk-taking, and provides a rationale for macroprudential regulation.

bonds.

Proof. Denote by ϵ the random variables $\frac{p_1(s)}{p_0}$. For a given aggregate b_0^*, the random variable ϵ follows some distribution $H(\epsilon)$ such that $\int_0^1(1-\epsilon)dH(\epsilon) = \int_1^\infty(\epsilon-1)dH(\epsilon)$. For an individual bank i, the payoff from investing $b_0^*(i)$ is

$$V_0^B(b_0^*(i); b_0^*) = \int \rho_1(s) A d\pi(s) + \int_1^\infty (A - b_0^*(i))(\epsilon - 1)dH(\epsilon) - \Psi(\underline{b}_0^* - b_0^*(i)).$$

The marginal benefit of reducing $b_0^*(i)$ is given by

$$-\frac{\partial V_0^B(b_0^*(i); b_0^*)}{\partial b_0^*(i)} = \int_1^\infty (\epsilon - 1)dH(\epsilon) - \Psi'(\underline{b}_0^* - b_0^*(i)).$$

Now consider two aggregate level b_0^* and $b_0^{*\prime}$ with $b_0^* > b_0^{*\prime}$ with associated prices p_0, $p_1(s)$, p_0', $p_1'(s)$ and distributions H and H'. Let $\tilde{s}$ be such that $\frac{p_1(\tilde{s})}{p_0} = 1$ (and so bailouts occur if and only if $s < \tilde{s}$). We proceed in two steps.

In the first step, we prove that $p_0' < p_0$, $p_1'(s) = p_1(s)$ for $s \geq \tilde{s}$, and $\frac{p_1'(s)}{p_0'} > \frac{p_1(s)}{p_0}$ for $s \geq \tilde{s}$. Indeed, the price $p_1(s)$ is a locally stable solution of the following fixed-point equation

$$p_1(s) = 1 - F(B_0 + (A - b_0^*)\max\{\frac{1}{p_1(s)} - \frac{1}{p_0}, 0\}|s).$$

Towards a contradiction, suppose that $p_0' \geq p_0$. Then for any $p_1(s)$, the right-hand side of the above equation decreases when b_0^* is replaced by $b_0^{*\prime}$. Hence $p_1'(s) < p_1(s)$ decreases for all s, and strictly decreases for $s < \tilde{s}$. This contradicts the martingale property of prices, and proves that $p_0' < p_0$. For all $s \geq \tilde{s}$, $\frac{p_1(\tilde{s})}{p_0'} > \frac{p_1(\tilde{s})}{p_0} \geq 1$. Hence for all $s \geq \tilde{s}$ the pre-bailout net worth of banks satisfies $b_0^{*\prime} + (A - b_0^{*\prime})\frac{p_1(\tilde{s})}{p_0'} > b_0^* + (A - b_0^*)\frac{p_1(\tilde{s})}{p_0}$. This in turn implies that it is still the case that there are no bailouts for $s > \tilde{s}$ when aggregate debt is $b_0^{*\prime}$. By implication, $p_1'(s) = p_1(s)$ is the same for $s \geq \tilde{s}$.

In the second step, we use the first step to get

$$\int_1^\infty (\epsilon - 1)dH'(\epsilon) \geq \int_1^\infty (\frac{p_0}{\tilde{p}_0}\epsilon - 1)dH(\epsilon) > \int_1^\infty (\epsilon - 1)dH(\epsilon).$$

The incentive to marginally reduce $b_0^*(i)$ is therefore higher when the aggregate foreign debt level is $b_0^{*\prime}$ than when it is b_0^*:

$$-\frac{\partial V_0^B(b_0^*(i); b_0^{*\prime})}{\partial b_0^*(i)} > -\frac{\partial V_0^B(b_0^*(i); b_0^*)}{\partial b_0^*(i)}.$$

□

As is well understood, depending on the exact shape of the cost function Ψ, these strategic complementarities can lead to multiple equilibria: Equilibria with low exposure of domestic banks to domestic sovereign default risk (high b_0^*) and equilibria with high exposure of domestic banks to domestic sovereign default risk (low b_0^*). Because multiplicity is not the focus of this paper, we simply illustrate this possibility with a simple example in Appendix B.2.

The next proposition focuses on a local comparative statics result for a given equilibrium. We consider shifts $F(E|s;\xi)$ and $f(E|s;\xi)$ in the distribution of fiscal capacity E given the state of the world s indexed by the parameter ξ. We perform the local comparative statics around a initial value ξ_0. All equilibrium variables are indexed by ξ, and we consider partial and total derivatives of these variables with respect to ξ at $\xi = \xi_0$.

We say that the shifter ξ is *risk-increasing* at ξ_0 if $\frac{\partial F(B_1(s;\xi)|s;\xi)}{\partial \xi}|_{\xi=\xi_0} > 0$ for $s < \tilde{s}(\xi_0)$ and $\frac{\partial F(B_1(s;\xi)|s;\xi)}{\partial \xi}|_{\xi=\xi_0} = 0$ for $s \geq \tilde{s}(\xi_0)$. Concretely, this means that an infinitesimal increase in ξ leads to an adverse shift in the distribution of fiscal capacity (in the first-order stochastic dominance sense) for states $s < \tilde{s}(\xi_0)$ in which there is a bailout, but not for states $s \geq \tilde{s}(\xi_0)$ for which there is no bailout.

To lighten the notation, whenever this cannot lead to any confusion, we leave the dependence of all variables on ξ implicit. We also leave implicit that the derivatives are taken at $\xi = \xi_0$. Hence for example, we write p_0 instead of $p_0(\xi)$ and $\frac{dp_0}{d\xi}$ instead of $\frac{dp_0(\xi)}{d\xi}|_{\xi=\xi_0}$.

Proposition 5 (Bad Fiscal Shocks and Debt Renationalization)**.** *Suppose that $I(s) = \bar{I}$ is independent of s, that $A = \bar{I}$, and that the shifter ξ is risk-increasing. Then*

$$\frac{dp_0}{d\xi} = \frac{-\int_{s<\tilde{s}} \frac{\frac{\partial F(B_1(s)|s)}{\partial \xi}}{1-\frac{A-b_0^{**}}{p_1(s)^2} f(B_1(s)|s)} d\pi(s)}{1 + \int_{s<\tilde{s}} \frac{f(B_1(s)|s)\frac{A-b_0^*}{p_0^2}}{1-\frac{A-b_0^{**}}{p_1(s)^2} f(B_1(s)|s)} d\pi(s) - \frac{\int_{s\geq\tilde{s}} \frac{p_1(s)}{p_0^2} d\pi(s)}{\Psi''(\underline{b}_0^* - b_0^*)} \int_{s<\tilde{s}} \frac{f(B_1(s)|s)\left[\frac{1}{p_1(s)} - \frac{1}{p_0}\right]}{1-\frac{A-b_0^{**}}{p_1(s)^2} f(B_1(s)|s)} d\pi(s)} < 0, \tag{13}$$

$$\frac{dp_1(s)}{d\xi} = \begin{cases} 0 \quad \text{if} \quad s \geq \tilde{s}, \\ \frac{-\frac{\partial F(B_1(s)|s)}{\partial \xi} + f(B_1(s)|s)\left[\frac{1}{p_1(s)} - \frac{1}{p_0}\right]\frac{db_0^*}{d\xi} - f(B_1(s)|s)\frac{A-b_0^*}{p_0^2}\frac{dp_0}{d\xi}}{1-\frac{A-b_0^{**}}{p_1(s)^2} f(B_1(s)|s)} \quad \text{if} \quad s < \tilde{s}, \end{cases} \tag{14}$$

$$\frac{db_0^*}{d\xi} = \frac{\int_{s\geq\tilde{s}} \frac{p_1(s)}{p_0^2} d\pi(s)}{\Psi''(\underline{b}_0^* - b_0^*)} \frac{dp_0}{d\xi} < 0. \tag{15}$$

Proof. Differentiating the first-order condition for the choice of b_0^* by banks, the equilibrium date-0 and date-1 price conditions (10) and (11) leads to the following linear system of three equations in three unknowns $\frac{dp_0}{d\xi}$, $\frac{dp_1(s)}{d\xi}$, and $\frac{db_0^*}{d\xi}$:

$$\frac{db_0^*}{d\xi}\Psi''(\underline{b}_0^* - b_0^*) = \frac{dp_0}{d\xi}\int_{s\geq\tilde{s}} \frac{p_1(s)}{p_0^2} d\pi(s),$$

$$\frac{dp_1(s)}{d\xi} = \begin{cases} 0 \quad \text{if} \quad s \geq \tilde{s}, \\ \dfrac{-\frac{\partial F(B_1(s)|s)}{\partial \xi} + f(B_1(s)|s)\left[\frac{1}{p_1(s)} - \frac{1}{p_0}\right]\frac{db_0^*}{d\xi} - f(B_1(s)|s)\frac{A-b_0^*}{p_0^2}\frac{dp_0}{d\xi}}{1 - \frac{A-\underline{b}_0^{**}}{p_1(s)^2} f(B_1(s)|s)} \quad \text{if} \quad s < \tilde{s}, \end{cases}$$

$$\frac{dp_0}{d\xi} = \frac{db_0^*}{d\xi}\int_{s<\tilde{s}} \frac{f(B_1(s)|s)\left[\frac{1}{p_1(s)} - \frac{1}{p_0}\right]}{1 - \frac{A-\underline{b}_0^{**}}{p_1(s)^2} f(B_1(s)|s)} d\pi(s) - \frac{dp_0}{d\xi}\int_{s<\tilde{s}} \frac{f(B_1(s)|s)\frac{A-b_0^*}{p_0^2}}{1 - \frac{A-\underline{b}_0^{**}}{p_1(s)^2} f(B_1(s)|s)} d\pi(s)$$
$$- \int_{s<\tilde{s}} \frac{\frac{\partial F(B_1(s)|s)}{\partial \xi}}{1 - \frac{A-\underline{b}_0^{**}}{p_1(s)^2} f(B_1(s)|s)} d\pi(s).$$

Solving this linear system of equations yields the results in the proposition. □

When the shifter ξ is risk-increasing, an infinitesimal increase in ξ does not change the date-1 price of debt $p_1(s)$ in the no-bailout states $s \geq \tilde{s}$, but decreases it on average in the bailout states $s < \tilde{s}$, leading to a decrease in the date-0 price of debt p_0. As a result the returns to holding domestic sovereign debt for a bank $\frac{p_1(s)}{p_0}$ increase in the no-bailout states, and are unchanged in the bailout states because the bank is bailed out. This increases the attractiveness of risky domestic sovereign debt for domestic banks, leading banks to increase their exposure $A - b_0^*$ to risky domestic sovereign debt and decrease their exposure b_0^* to safe foreign sovereign debt. This in turn increases the size of bailouts on average in bailout states, further reduces the date-1 price of debt $p_1(s)$ on average in these states and the date-0 price of debt p_0, further increases the attractiveness of risky domestic sovereign debt for domestic banks, leading banks to further increase their exposure $A - b_0^*$ to risky domestic sovereign debt and to decrease their exposure b_0^* to safe foreign sovereign debt, etc. ad infinitum. This feedback loop amplifies the initial effect on p_0, $p_1(s)$, and b_0^* through a multiplier effect (see the denominator in (13)). There is more amplification, the stronger are the strategic complementarities, as controlled by the inverse of the curvature $\frac{1}{\Psi''(\underline{b}_0^* - b_0^*)}$ of the supervisory evasion cost function.

Proposition 5 defines a precise sense in which bad fiscal news can lead to debt re-nationalization. It offers a possible explanation for the well-known fact that such a re-nationalization of sovereign debt was observed in Europe as the recent crisis intensified.[15] Here this is due to the imperfect ability of the government to limit the exposure of banks to domestic sovereign default risk through regulation. The rationale for re-nationalization is based on the idea that sovereign bonds are more attractive to banks in bad times.[16]

We return to debt re-nationalization in Sections 4.2 and A.3. In Section 4.2, we propose a different mechanism, which relies on the desirability for the government to allow banks to load up on domestic sovereign default risk in order to push legacy creditors to forgive more debt, even if the government can perfectly supervise the banking system.[17]

4 Debt Forgiveness, Lax Supervision, and Banking Unions

This section investigates the possibility of debt forgiveness at date 1. It shows that this can give rise to an incentive for lax supervision whereby the domestic government, anticipating concessions from legacy creditors, turns a blind eye when its banks take on domestic sovereign risk exposures and sets $\underline{b}_0^{**} < \underline{b}_0^{*}$. If the ex-post leniency of domestic supervisors is anticipated ex ante at the time of sovereign debt issuance, then it is priced in the form of higher spreads. The government is better off committing ex ante to a tough ex-post supervisory stance $\underline{b}_0^{**} = \underline{b}_0^{*}$, but is tempted to relax it ex post to $\underline{b}_0^{**} < \underline{b}_0^{*}$. If the government lacks commitment, then it benefits from relinquishing its supervisory powers to a supranational supervisor by joining a banking union.

4.1 Debt Forgiveness

We model date-1 debt forgiveness as follows. We assume that after the state of nature s is observed at date 1, international investors can forgive some of the legacy debt to an arbitrary $\tilde{B}_0 \leq B_0$, before the government undertakes the bailout policy.

We show that it can be in the interest of legacy creditors (international investors who hold the legacy debt $B_0 - \frac{A - \underline{b}_0^{**}}{p_0}$) to forgive some of the debt at date 1, bringing the overall stock of legacy debt to $\tilde{B}_0 \leq B_0$.[18] In other words, there is a legacy Laffer curve, and it is possible for legacy debt B_0 to be on the wrong side of the legacy Laffer curve, i.e.

[15]See Broner et al (2013), Genaioli et al (2014,a,b) and Uhlig (2014) for careful documentations.

[16]In bad times monitoring banks is also more attractive to the supervisor. Proposition 5 nonetheless would still hold as long as the supervisory capability does not adjust rapidly with the state of nature.

[17]In Section A.3, we uncover an opposing mechanism based on limits to the capacity of the government to bail out the banking system.

[18]Of course organizing debt forgiveness requires coordination among legacy creditors to neutralize the free-riding incentives of individual creditors.

$\frac{d(p_1(s;\tilde{B}_0)\tilde{B}_0)}{d\tilde{B}_0}|_{\tilde{B}_0=B_0} < 0$ where we have made the dependence of the date-1 price of debt $p_1(s;\tilde{B}_0)$ on the post-debt forgiveness debt stock $\tilde{B}_0$ explicit.[19,20] Moreover, we show that the feedback loop between sovereign and financial balance sheets that we have characterized in Section 3 makes it more likely that the economy is on the wrong side of the Laffer curve.

We can compute the sensitivity of the value $p_1(s;\tilde{B}_0)\tilde{B}_0$ of legacy debt with respect to (post-debt forgiveness) legacy debt $\tilde{B}_0$, assuming that $s < \tilde{s}$ so that a bailout is required:

$$\frac{d[p_1(s;\tilde{B}_0)\tilde{B}_0]}{d\tilde{B}_0} = p_1(s;\tilde{B}_0) - \tilde{B}_0[1 - \frac{\frac{A-b_0^{**}}{p_0}}{B_0}]\frac{f(B_1(s);\tilde{B}_0|s)}{1 - \frac{I(s)-b_0^{**}}{p_1^2(s;\tilde{B}_0)}f(B_1(s);\tilde{B}_0|s)}. \tag{16}$$

The first term on the left-hand-side of equation (16) is the direct quantity-of-debt effect of debt forgiveness. Because of this effect, marginal debt forgiveness $d\tilde{B}_0 < 0$ contributes negatively to the value $p_1(s;\tilde{B}_0)\tilde{B}_0$ of the claims of legacy creditors. The second term on the left-hand-side of equation (16) is the indirect price-of-debt effect of forgiveness. Because of this effect, debt forgiveness $d\tilde{B}_0 < 0$ contributes positively to the value $p_1(s;\tilde{B}_0)\tilde{B}_0$ of the claims of legacy creditors. The net effect of debt forgiveness depends on the relative strength of these two effects.

The indirect price-of-debt effect of debt forgiveness is stronger, the more elastic is the price $p_1(s;\tilde{B}_0)$ to the amount of legacy debt $\tilde{B}_0$. And the feedback loop between sovereign and financial balance sheets that we have characterized in Section 3 works precisely to increase this elasticity. Indeed, debt forgiveness increases the date-1 price of debt, which improves the balance sheets of bankers, reducing the size of the bailout, and hence reducing the need for the government to engage in additional borrowing at date 1, which reduces the probability of default and further increases the date-1 price of debt, etc. ad infinitum. The feedback loop therefore makes the price-of-debt effect more potent, without affecting the quantity-of-debt effect, therefore pushing the economy towards the decreasing part of the legacy Laffer curve $p_1(s;\tilde{B}_0)\tilde{B}_0$.

We denote by $\bar{B}_0(s)$ the peak of the legacy Laffer curve

$$\frac{d[p_1(s;\tilde{B}_0)\tilde{B}_0]}{d\tilde{B}_0}|_{\tilde{B}_0=\bar{B}_0(s)} = 0. \tag{17}$$

[19]We assume that banks do not free-ride on the renegotiation. All our results would go through if we assumed instead that banks could perfectly free-ride on the renegotiation.

[20]The possibility of debt forgiveness does not rely on the presence of banks or the existence of bailouts. See Hatchondo-Martinez-Sosa Padilla (2014) for a recent analysis.

We also denote by $\bar{B}_1(s)$ the peak of the issuance Laffer curve

$$\frac{d[[1 - F(\tilde{B}_1|s)]\tilde{B}_1]}{d\tilde{B}_1}|_{\tilde{B}_1=\bar{B}_1(s)} = 0. \tag{18}$$

It is characterized by the condition $\bar{B}_1(s)\frac{f(\bar{B}_1(s)|s)}{1-F(\bar{B}_1(s)|s)} = 1$.

When some debt forgiveness can improve the outcome of the legacy creditors, a mutually beneficial negotiation can take place between legacy creditors and the domestic government. The outcome of the negotiation depends on the ability of legacy creditors to coordinate and on the distribution of bargaining power between legacy creditors and the domestic government. We assume that legacy creditors are able to coordinate, and have all the bargaining power: They collectively make a take-it-or-leave-it offer to the domestic government. The best that they can achieve is to forgive debt up to the peak $\bar{B}_0(s)$ of the legacy Laffer curve.

We denote by $B_0(s)$ the amount of date-1 debt after debt forgiveness but before issuance. There is debt forgiveness if $\bar{B}_0(s) \leq B_0$, in which case $B_0(s) = \bar{B}_0(s)$. There is no debt forgiveness if $\bar{B}_0(s) > B_0$, in which case $B_0(s) = B_0$. This can be summarized compactly as

$$B_0(s) = \min\{\bar{B}_0(s), B_0\},$$

$$B_1(s) = B_1(s; B_0(s)).$$

From now on, to lighten the notation, we write $p_1(s)$ for $p_1(s; B_0(s))$.

Proposition 6 (Legacy Laffer Curve and Debt Forgiveness)**.** *Consider the equilibrium for a given fixed diversification requirement $\underline{b}_0^{**} \in [0, \underline{b}_0^*]$.*[21] *Suppose that there are states s where debt forgiveness takes place so that $B_0(s) = \bar{B}_0(s)$ and focus on these states. Then the amount of date-1 debt after debt forgiveness and issuance associated with the peak $\bar{B}_0(s)$ of the legacy Laffer curve is at the peak $\bar{B}_1(s)$ of the issuance Laffer curve so that $B_1(s) = B_1(s; \bar{B}_0(s)) = \bar{B}_1(s)$. In addition, $\bar{B}_0(s)$ is increasing in s so that worse states are associated with more debt forgiveness.*

Proof. Suppose that there is debt forgiveness but no bailout in state s. Then we have $\bar{B}_0(s) = B_1(s; \bar{B}_0(s)) = \bar{B}_1(s)$, proving the first statement in the proposition. We also have

$$\bar{B}_1(s)\frac{f(\bar{B}_1(s)|s)}{1 - F(\bar{B}_1(s)|s)} = 1.$$

This shows that $\bar{B}_0(s) = \bar{B}_1(s)$ is increasing in s, proving the second statement in the

[21]This means that we consider an equilibrium of a modified policy game where the supervisor is forced to choose the diversification requirement $\underline{b}_0^{**}$.

proposition.

Suppose now that there is debt forgiveness and a bailout in state s. The bailout equation

$$[1-F(B_1(s;\tilde{B}_0)|s)][B_1(s;\tilde{B}_0)-\tilde{B}_0] = I(s) - \underline{b}_0^{**} - \frac{A-\underline{b}_0^{**}}{p_0 B_0}\tilde{B}_0[1-F(B_1(s;\tilde{B}_0)|s)],$$

can be manipulated into

$$\left[1-\frac{A-\underline{b}_0^{**}}{p_0 B_0}\right]\tilde{B}_0[1-F(B_1(s;\tilde{B}_0)|s)] = [1-F(B_1(s;\tilde{B}_0)|s)]B_1(s;\tilde{B}_0) - [I(s)-\underline{b}_0^{**}].$$

This immediately implies that the date-1 debt after debt forgiveness and issuance corresponds to the peak of the issuance Laffer curve $B_1(s;\bar{B}_0(s)) = \bar{B}_1(s)$, proving the first statement in the proposition. We therefore have the following characterization of $\bar{B}_0(s)$ and $\bar{B}_1(s)$:

$$\bar{B}_1(s)\frac{f(\bar{B}_1(s)|s)}{1-F(\bar{B}_1(s)|s)} = 1,$$

$$\bar{B}_1(s) = \bar{B}_0(s) - (A-\underline{b}_0^{**})\frac{\bar{B}_0(s)}{p_0 B_0} + \frac{I(s)-\underline{b}_0^{**}}{1-F(\bar{B}_1(s)|s)}.$$

The first equation shows that $\bar{B}_1(s)$ is increasing in s. The second equation shows that $\bar{B}_0(s)$ is increasing in $\bar{B}_1(s)$ and in s. This immediately implies that $\bar{B}_0(s)$ is increasing in s, proving the second statement in the proposition. □

4.2 Strategic Supervisory Leniency

The possibility of a legacy Laffer curve can make it optimal for the government to engage in supervisory leniency $\underline{b}_0^{**} < \underline{b}_0^{*}$ so as to extract larger concessions from legacy creditors. Another way to put this is that the government might have incentives to let its domestic banks load up on domestic sovereign debt in order to extract concessions from legacy creditors.

Consider the equilibrium for a given fixed diversification requirement $\underline{b}_0^{**} \in [0, \underline{b}_0^{*}]$. All equilibrium variables are indexed by $\underline{b}_0^{**}$ but we leave this dependence implicit to make the notation lighter. Suppose first that there is no bailout, then there is debt forgiveness if and only $s \leq \bar{s}$, where the threshold $\bar{s}$ is defined implicitly by the equation

$$1 = B_0 \frac{f(B_0|\bar{s})}{1-F(B_0|\bar{s})}. \tag{19}$$

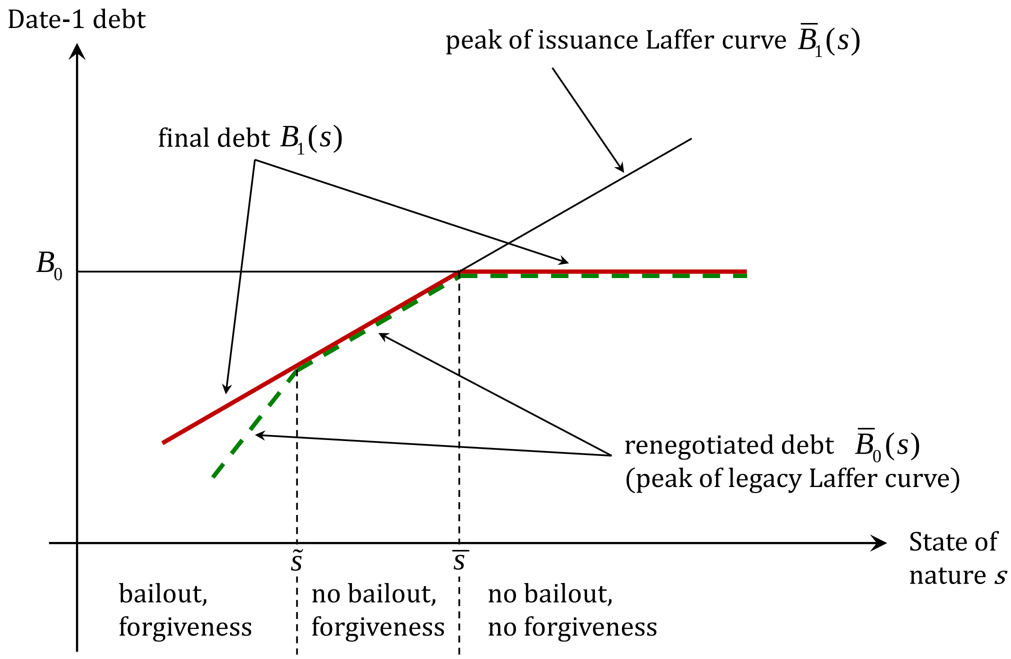

Figure 2: Debt Forgiveness when the Equilibrium is Bailout-Shifting.

Suppose now that there is debt forgiveness and assume that $I(s)$ is independent of s and that $A = \bar{I}$. Then there is a bailout if and only if $s \leq \tilde{s}$, where the threshold $\tilde{s}$ is defined implicitly by the equation

$$\frac{\bar{B}_1(\tilde{s})}{B_0}\frac{1 - F(\bar{B}_1(\tilde{s})|\tilde{s})}{p_0} = 1, \tag{20}$$

where $\bar{B}_1(s)$ is the peak of the issuance Laffer curve with $\bar{B}_1(s)\frac{f(\bar{B}_1(s)|s)}{1-F(\bar{B}_1(s)|s)} = 1$. If these two thresholds are ordered $\tilde{s} < \bar{s}$, then it is possible to prove that there is a bailout if and only if $s \leq \tilde{s}$, and debt forgiveness if and only if $s \leq \bar{s}$.[22] This configuration is guaranteed to arise when B_0 is large.[23] We then say that the equilibrium is *bailout-shifting* to capture the idea that under this configuration, the cost of bailouts is entirely shifted to foreigners

[22]We only need to verify that the condition for no bailout if there is no debt forgiveness $\frac{1-F(B_0|s)}{p_0} \geq 1$ holds for $s > \bar{s}$, and that the condition for debt forgiveness if there is a bailout $\frac{f(B_1(s)|s)}{1-F(B_1(s)|s)}B_0[1 + \frac{A-b_0^{**}}{p_1(s)B_0}(1 - \frac{p_1(s)}{p_0})] \geq 1$ holds for $s \leq \tilde{s}$. The first part follows immediately from the observation that $s > \bar{s}$ implies that $s > \tilde{s}$ and hence that $\frac{1-F(B_0|s)}{p_0} \geq \frac{1-F(B_0|\tilde{s})}{p_0}$, together with the observation that $\frac{1-F(B_0|\tilde{s})}{p_0} \geq \frac{1-F(B_0(\tilde{s})|\tilde{s})}{p_0} = \frac{1-F(B_1(\tilde{s})|\tilde{s})}{p_0} = \frac{B_0}{B_1(\tilde{s})} = \frac{B_0}{B_0(\tilde{s})} \geq 1$. The second part follows immediately from the observation that $s \leq \tilde{s}$ implies that $s < \bar{s}$ and hence that $B_0 \geq B_1(s)$ and that $\frac{f(B_1(s)|s)}{1-F(B_1(s)|s)}B_0 \geq \frac{f(B_1(s)|s)}{1-F(B_1(s)|s)}B_1(s) = 1$.

[23]That $\tilde{s} < \bar{s}$ for B_0 large enough can be seen as follows. As B_0 grows, $\bar{s}$ converges to the upper bound $s^{max} \in \bar{\mathbb{R}}$ of the support of the s distribution. At the same time, p_0B_0 converges to the constant $\int \bar{B}_0(s)[1 - F(\bar{B}_1(s)|s)]\pi(s)ds$, which implies that $\tilde{s}$ converges to the interior point of the support of the s distribution defined implicitly by $\frac{\bar{B}_1(\tilde{s})[1-F(\bar{B}_1(\tilde{s})|\tilde{s})]}{\int \bar{B}_0(s)[1-F(\bar{B}_1(s)|s)]\pi(s)ds} = 1$.

through debt forgiveness. This is illustrated in Figure 2.

Proposition 7 (Debt Forgiveness and Strategic Supervisory Leniency). *Suppose that $I(s)$ is independent of s, that $A = \bar{I}$, and that the distribution $\pi(s)$ admits a continuous density $d\pi(s) = \pi(s)ds$. Consider the equilibrium with no supervisory leniency $\underline{b}_0^{**} = \underline{b}_0^*$. Assume that for this maximal diversification requirement, the equilibrium is bailout-shifting. Then for any lower diversification requirement $\underline{b}_0^{**} \leq \underline{b}_0^*$, the equilibrium with a fixed diversification requirement $\underline{b}_0^{**}$ is bailout shifting. Moreover, the only full equilibrium with endogenous supervisory leniency features maximal supervisory leniency $\underline{b}_0^{**} = 0$.*

Proof. Differentiating

$$p_0 = \int \frac{B_0(s)}{B_0}[1 - F(B_1(s)|s)]\pi(s)ds,$$

and using the fact that $\frac{dB_1(s)}{d\underline{b}_0^{**}} = \frac{d\bar{B}_1(s)}{d\underline{b}_0^{**}} = 0$ for $s \leq \tilde{s}$ and $\frac{dB_1(s)}{d\underline{b}_0^{**}} = \frac{dB_0(s)}{d\underline{b}_0^{**}} = \frac{dB_0}{d\underline{b}_0^{**}} = 0$ for $s > \tilde{s}$, we find that

$$\frac{dp_0}{d\underline{b}_0^{**}} = \int_{\{s \leq \tilde{s}\}} \frac{1}{B_0} \frac{dB_0(s)}{d\underline{b}_0^{**}}[1 - F(B_1(s)|s)]\pi(s)ds. \tag{21}$$

For $s \leq \tilde{s}$, we have

$$B_1(s) = B_0(s) - (A - \underline{b}_0^{**})\frac{B_0(s)}{p_0 B_0} + \frac{A - \underline{b}_0^{**}}{1 - F(B_1(s)|s)}.$$

Differentiating this equality and using the fact that $\frac{dB_1(s)}{d\underline{b}_0^{**}} = 0$ for $s \leq \tilde{s}$, we get

$$\frac{dB_0(s)}{d\underline{b}_0^{**}}[1 - \frac{A - \underline{b}_0^{**}}{p_0 B_0}] + (A - \underline{b}_0^{**})\frac{B_0(s)}{p_0^2 B_0}\frac{dp_0}{d\underline{b}_0^{**}} = \frac{1}{1 - F(B_1(s)|s)}[1 - \frac{B_0(s)}{B_0}\frac{p_1(s)}{p_0}].$$

Integrating this equation over $s \leq \tilde{s}$ with measure $d\pi(s)$, and combining with equation (21) and the fact that $1 - \frac{A - \underline{b}_0^{**}}{p_0 B_0} > 0$, $A - \underline{b}_0^{**} > 0$, and $1 - \frac{B_0(s)}{B_0}\frac{p_1(s)}{p_0} > 0$, this implies that $\frac{dp_0}{d\underline{b}_0^{**}} > 0$. Going back to the definition (20) of $\tilde{s}$, this in turn implies that $\frac{d\tilde{s}}{d\underline{b}_0^{**}} \geq 0$. Together with the implication from the definition (19) of $\bar{s}$ that $\frac{d\bar{s}}{d\underline{b}_0^{**}} = 0$, this proves the first statement in the proposition.

We now turn to the second statement in the proposition. We consider the equilibrium for a given fixed diversification requirement $0 < \underline{b}_0^{**} \leq \underline{b}_0^*$. We fix the value of p_0 and characterize the dependence of welfare $\mathcal{W}_0$ on the choice of $\tilde{\underline{b}}_0^{**}$ by the supervisor, given p_0, in the corresponding subgame. In what follows, all the derivatives should therefore be understood as keeping p_0 fixed. Once again, we leave the dependence of all variables on

p_0 and $\tilde{\underline{b}}_0^{**}$ implicit to make the notation lighter. We next prove that $\frac{d\mathcal{W}_0}{d\tilde{\underline{b}}_0^{**}}|_{\tilde{\underline{b}}_0^{**}=\underline{b}_0^{**}} < 0$. This implies that $0 < \underline{b}_0^{**}$ cannot be a full equilibrium with endogenous supervisory leniency.

We have

$$\mathcal{W}_0 = \int \mathcal{W}_0(s)\pi(s)ds,$$

with

$$\mathcal{W}_0(s) = \int_{B_1(s)}^{\infty} [E - B_1(s)]\, f(E|s)dE + \int_0^{B_1(s)} [E - \Phi]\, f(E|s)dE + \beta^I(s)A$$
$$+ \beta^B \rho_1(s)A + \beta^B \max\{\tilde{\underline{b}}_0^{**} + (A - \tilde{\underline{b}}_0^{**})\frac{B_0(s)}{B_0}\frac{p_1(s)}{p_0} - A, 0\}.$$

For $s \leq \tilde{s}$, there is debt forgiveness and a bailout, and we have $\frac{dB_1(s)}{d\underline{b}_0^{**}} = \frac{d\bar{B}_1(s)}{d\underline{b}_0^{**}} = 0$ and $\tilde{\underline{b}}_0^{**} + (A - \tilde{\underline{b}}_0^{**})\frac{B_0(s)}{B_0}\frac{p_1(s)}{p_0} - A < 0$, so that

$$\frac{d\mathcal{W}_0(s)}{d\tilde{\underline{b}}_0^{**}} = 0.$$

For $\tilde{s} < s \leq \bar{s}$, there is debt forgiveness and no bailout, and we have $\frac{dB_1(s)}{d\underline{b}_0^{**}} = \frac{d\bar{B}_1(s)}{d\underline{b}_0^{**}} = 0$ and $\tilde{\underline{b}}_0^{**} + (A - \tilde{\underline{b}}_0^{**})\frac{B_0(s)}{B_0}\frac{p_1(s)}{p_0} - A > 0$, so that

$$\frac{d\mathcal{W}_0(s)}{d\tilde{\underline{b}}_0^{**}} = \beta^B[1 - \frac{B_0(s)}{B_0}\frac{p_1(s)}{p_0}] < 0.$$

For $s > \bar{s}$, there is no debt forgiveness and no bailout, and we have $\frac{dB_1(s)}{d\underline{b}_0^{**}} = \frac{dB_0(s)}{d\underline{b}_0^{**}} = \frac{dB_0}{d\underline{b}_0^{**}} = 0$ and $\tilde{\underline{b}}_0^{**} + (A - \tilde{\underline{b}}_0^{**})\frac{B_0(s)}{B_0}\frac{p_1(s)}{p_0} - A > 0$, so that

$$\frac{d\mathcal{W}_0(s)}{d\tilde{\underline{b}}_0^{**}} = \beta^B[1 - \frac{p_1(s)}{p_0}] < 0.$$

Together with the fact that $\mathcal{W}_0(s)$ is continuous (so that infinitesimal changes in the thresholds $\tilde{s}$ and $\bar{s}$ do not change welfare), this implies that

$$\frac{d\mathcal{W}_0}{d\tilde{\underline{b}}_0^{**}} = \int \frac{d\mathcal{W}_0(s)}{d\tilde{\underline{b}}_0^{**}}\pi(s)ds < 0.$$

□

Proposition 7 shows that it can be optimal for the government to engage in supervisory leniency $\underline{b}_0^{**} < \underline{b}_0^{*}$ and allow domestic banks to take on more domestic debt, and risk needing a bailout when the government experiences a bad fiscal shock. Supervisory leniency allows the government to extract more concessions from legacy creditors.[24] Domestic consumers are left as well off because they are entirely shielded from the extra cost of bailouts, which are completely covered by extra debt forgiveness by foreigners, and because expected default costs are unchanged. Bankers are strictly better off in the no bailout states because domestic sovereign debt has a higher return, and they are as well off in the bailout states because they are bailed out.

In terms of the decomposition $\mathcal{W}_0 = \mathcal{E}_0 - \mathcal{R}_0 + \mathcal{C}_0$ given in (8) and adjusted for debt forgiveness, supervisory leniency has the following effects.[25,26] First, it has an efficiency benefit because it leads to more debt forgiveness but not more defaults. This increases $\mathcal{E}_0$. Second, is has a distributive cost because it increases the rents extracted by bankers because of bailouts. This increases $\mathcal{R}_0$. Third, it has a distributive cost because bankers now purchase debt from legacy creditors at a price which is higher than the expected repayment (net of debt forgiveness). This decreases $\mathcal{C}_0$. Proposition 7 shows that the benefit outweighs the costs.

Proposition 7 offers a possible explanation for the well-known fact that a re-nationalization of sovereign debt was observed in Europe as the recent crisis intensified. In Section 3.5, we proposed a different mechanism based on the imperfect ability of the government to limit the exposure of banks to domestic sovereign default risk through supervision.

[24]We refer the reader to Appendix B.3 for an illustration of Proposition 7 in the context of a simple example which can be solved in closed form.

[25]We use the following adaptations of the decompositions $\mathcal{W}_0 = \mathcal{E}_0 - \mathcal{R}_0$ and $\mathcal{W}_0 = \mathcal{E}_0 - \mathcal{R}_0 + \mathcal{C}_0$ to the case of debt forgiveness:

$$\mathcal{E}_0 = \int \left[\int_{B_1(s)}^{\infty} [E - B_0(s)]\, f(E|s) dE + \int_0^{B_1(s)} [E - \Phi]\, f(E|s) dE \right] d\pi(s) + \int \left[\left[\beta^I(s) + \beta^B(\rho_1(s) - 1) \right] A + \beta^B A \right] d\pi(s),$$

$$\mathcal{R}_0 = -(1-\beta^B) \int \min\{\underline{\tilde{b}}_0^{**} + (A - \underline{\tilde{b}}_0^{**}) \frac{B_0(s)}{B_0} \frac{p_1(s)}{p_0} - A, 0\} d\pi(s),$$

$$\mathcal{C}_0 = \beta^B \int \left[\underline{\tilde{b}}_0^{**} + (A - \underline{\tilde{b}}_0^{**}) \frac{B_0(s)}{B_0} \frac{p_1(s)}{p_0} - A \right] d\pi(s).$$

[26]We consider the equilibrium for a given fixed diversification requirement $0 < \underline{b}_0^{**} \leq \underline{b}_0^{*}$. We fix the value of p_0 and characterize the dependence of welfare $\mathcal{W}_0$ on the choice of $\underline{\tilde{b}}_0^{**}$ by the supervisor, given p_0, in the corresponding subgame. The results above show that

$$\frac{d\mathcal{W}_0}{d\underline{\tilde{b}}_0^{**}} = \frac{d\mathcal{E}_0}{d\underline{\tilde{b}}_0^{**}} - \frac{d\mathcal{R}_0}{d\underline{\tilde{b}}_0^{**}} + \frac{d\mathcal{C}_0}{d\underline{\tilde{b}}_0^{**}} < 0.$$

Because $\frac{dB_0(s)}{d\underline{\tilde{b}}_0^{**}} < 0$, we have $\frac{d\mathcal{E}_0}{d\underline{\tilde{b}}_0^{**}} < 0$. Because in addition $\frac{dp_1(s)}{d\underline{\tilde{b}}_0^{**}} = 0$, we have $\frac{d\mathcal{R}_0}{d\underline{\tilde{b}}_0^{**}} < 0$ and $\frac{d\mathcal{C}_0}{d\underline{\tilde{b}}_0^{**}} > 0$.

Remark. Excessively lax supervision can of course pre-date the crisis. Indeed, our formalism allows for agency costs within government, as the supervisor may put excessive (relative to the population) weight on bankers' welfare or too much weight on real estate lending for instance. Proposition 7 then means that the prospect of debt forgiveness may make supervisors even more lenient than they would be otherwise.

4.3 A Rationale for a Banking Union

We build on Section 4.2 and put ourselves under the hypotheses of Proposition 7. We show that foreign investors are made worse off by the relaxation of supervision of domestic banks by the domestic government: Once they have lent, their welfare is maximized by a tough supervision $\underline{b}_0^{**} = \underline{b}_0^{*}$. Of course their welfare is adversely impacted only if this relaxation of supervision is not anticipated at the time of the debt issuance, otherwise it is fully priced in. Interestingly, in this latter case, domestic welfare can be increased by a tough supervision $\underline{b}_0^{**} = \underline{b}_0^{*}$. But this requires commitment on the part of the domestic government not to relax supervision after the debt is issued. A banking (in the sense of shared supervision) union can help deliver the commitment outcome.

Consider the debt level B_0' that generates the same amount of revenue at date 0 when the effective diversification requirement is $\underline{b}_0^{**\prime} = 0$ as the debt level B_0 when the effective diversification requirement is $\underline{b}_0^{**} = \underline{b}_0^{*}$. And compare the equilibrium with a fixed diversification requirement $\underline{b}_0^{**} = \underline{b}_0^{*}$ and debt level B_0 to the full equilibrium with endogenous supervisory leniency $\underline{b}_0^{**\prime} = 0$ and debt level B_0'. We denote all variables associated with the latter equilibrium with prime superscripts. We make the additional assumption that the former equilibrium is "bailout-shifting" with $\tilde{s} < \bar{s}$.[27]

We use the decomposition $\mathcal{W}_0 = \mathcal{E}_0 - \mathcal{R}_0$ given in (4) adjusted for debt forgiveness. We show in Proposition 8 below that $\mathcal{W}_0' = \mathcal{E}_0' - \mathcal{R}_0' < \mathcal{W}_0 = \mathcal{E}_0 - \mathcal{R}_0$. This occurs for two reasons. First, with supervisory leniency anticipated at the issuance stage, we have $p_0' < p_0$ and hence $B_0' > B_0$ since $p_0' B_0' = p_0 B_0$. The increase in legacy debt increases default occurrences, increases expected default costs, and reduces welfare as captured by $\mathcal{E}_0' < \mathcal{E}_0$. Second, bankers collect bigger rents. The rents of bankers only come at the expense of consumers because supervisory leniency is fully priced in by foreign investors at the issuance stage. The increase in the rents of bankers therefore decreases the welfare of consumers one for one. Since consumers carry a higher welfare weight than banks, this also reduces welfare as captured by $\mathcal{R}_0' < \mathcal{R}_0$.

[27] Under this assumption, one can show that $B_0' > B_0$, $\tilde{s}' = \tilde{s}$ and that $\bar{s}' > \bar{s}$, so that $\tilde{s}' < \bar{s}'$ the full equilibrium with endogenous supervisory leniency $\underline{b}_0^{**\prime} = 0$ and debt B_0' is "bailout-shifting".

Of course achieving $\mathcal{W}_0$ requires commitment on the part of the domestic government since once the date-0 debt B_0 has been issued at price p_0, the government faces the temptation to renege by relaxing supervision and lowering the effective diversification requirement $\underline{b}_0^{**}$ below $\underline{b}_0^{*}$.[28] Foreigners are powerless to resist the re-nationalization of domestic debt unless they are able to coordinate not to sell their domestic sovereign bonds to domestic banks, which unlike debt relief negotiations, seems to have few real world counterparts.[29]

One of the important aspects of banking unions is the transfer of banking supervision from the national to the supranational level. Such a transfer weakens or removes the temptation of domestic governments to strategically allow their banks to load up on domestic sovereign bonds to extract larger concessions from legacy creditors. It can therefore facilitate the implementation of the commitment solution with a high diversification requirement $\underline{b}_0^{**} = \underline{b}_0^{*}$. This is because the international supervisor's objective function naturally puts more weight on international investors than the domestic government, making it less tempting to relax supervision ex post.[30] To make this point starkly, we study the limit where the supranational supervisor puts full weight on international investors and no weight on domestic agents. In this limit, the commitment solution is implemented.[31]

Proposition 8 (Banking Union)**.** *Consider the same hypotheses as in Proposition 7. If the relaxation of supervision is fully priced in by international investors at the time of the issuance of date-*0 *debt, then the domestic government faces a time-inconsistency problem. It is made better off by promising not to engage in supervisory leniency and to set a high effective diversification requirement* $\underline{b}_0^{**} = \underline{b}_0^{*}$ *before issuing debt at date* 0*, but it is tempted to relax this requirement after the issuance and lower* $\underline{b}_0^{**}$ *below* $\underline{b}_0^{*}$*. A banking union removes this temptation and improves welfare.*

Proof. We need to show that $\mathcal{W}_0' < \mathcal{W}_0$.

We have $p_0' B_0' = p_0 B_0$. It is easy to verify that $p_0' < p_0$, $B_0' > B_0$, and $\tilde{s}' = \tilde{s} < \bar{s} < \bar{s}'$. And moreover: for all $s < \tilde{s} = \tilde{s}'$, $B_1'(s) = B_1(s)$, and $B_0'(s) < B_0(s)$; for all $\tilde{s} = \tilde{s}' \leq s \leq \bar{s}$, $B_0'(s) = B_0(s) = B_1(s) = B_1'(s)$; for all $\bar{s} < s \leq \bar{s}'$, $B_1(s) = B_0(s) = B_0 < B_1'(s) = B_0'(s) <$

[28] That this temptation arises is guaranteed by Proposition 7 since the equilibrium is "bailout-shifting" with $\tilde{s} < \bar{s}$.

[29] To the extent that foreign investors are located in different countries, foreign national supervisors would also need to coordinate in order to facilitate this outcome.

[30] Another possibility is that the international supervisor has a better ability to commit to regulation than the domestic government.

[31] We refer the reader to Appendix B.4 for an illustration of Proposition 8 in the context of a simple example which can be solved in closed form.

B_0'; and for all $\bar{s} < s$, $B_1(s) = B_0(s) = B_0 < B_1'(s) = B_0'(s) = B_0'$.[32]

We write $\mathcal{W}_0 = \mathcal{E}_0 - \mathcal{R}_0$ and $\mathcal{W}_0' = \mathcal{E}_0' - \mathcal{R}_0'$. Because $B_1'(s) \geq B_1(s)$ for all s with a strict inequality with positive probability, and because

$$\int \int_{B_1'(s)}^{\infty} B_0'(s) f(E|s) dE \pi(s) ds = \int \int_{B_1(s)}^{\infty} B_0(s) f(E|s) dE \pi(s) ds = p_0' B_0' = p_0 B_0,$$

we have $\mathcal{E}_0' < \mathcal{E}_0$.

We will show shortly that we also have $\mathcal{R}_0' > \mathcal{R}_0$, which proves the proposition. We have

$$\mathcal{R}_0 = -(1-\beta^B) \int_{\{s \leq \tilde{s}\}} \min\{\underline{b}_0^* + (A - \underline{b}_0^*) \frac{B_0(s)}{B_0} \frac{p_1(s)}{p_0} - A, 0\} \pi(s) ds,$$

$$\mathcal{R}_0' = -(1-\beta^B) \int_{\{s \leq \tilde{s}'\}} \min\{A \frac{B_0'(s)}{B_0'} \frac{p_1'(s)}{p_0'} - A, 0\} \pi(s) ds.$$

We use $\tilde{s} = \tilde{s}'$. For all $s < \tilde{s} = \tilde{s}'$, we have $B_1'(s) = B_1(s)$ and $B_0'(s) < B_0(s)$. Together with the fact that $p_0 B_0 = p_0' B_0'$, this implies that for all $s \leq \tilde{s} = \tilde{s}'$ we have $A \frac{B_0'(s)}{B_0'} \frac{p_1'(s)}{p_0'} < A \frac{B_0(s)}{B_0} \frac{p_1(s)}{p_0} < \underline{b}_0^* + (A - \underline{b}_0^*) \frac{B_0(s)}{B_0} \frac{p_1(s)}{p_0} < A$. The result follows. □

5 Extensions

The analysis so far has relied on strong assumptions: Bankers cannot pledge income and therefore cannot borrow; the government can always finance bailouts; sovereign debt maturity follows an ALM precept of matching maturity and fiscal receipts; foreign investors are never bailed out; and finally there is a single risky country. We now relax all of these assumptions. In the main text, we focus on two extensions: sovereign debt maturity (Section 5.2), and multiple risky countries (Section 5.1). In the appendix, we consider three additional extensions: We allow for and investigate the role of leverage (Section A.2), limited bailouts (Section A.3), and foreign banks in the foreign (safe) country (Section A.4).

[32]The only nontrivial part is to show that for all $s \leq \tilde{s}' = \tilde{s}$, $B_0'(s) < B_0(s)$. This can be seen as follows. For $s < \tilde{s}' = \tilde{s}$, we have $B_1'(s) = B_1(s)$ and $p_1'(s) = p_1(s)$, which together $p_0' B_0' = p_0 B_0$ and the fact that $A = I(s)$ can be used to rewrite the bailout equations as

$$B_0(s) = B_1(s) \frac{1 - \frac{A - \underline{b}_0^*}{B_1(s)[1 - F(B_1(s)|s)]}}{1 - \frac{A - \underline{b}_0^*}{p_0 B_0}} \quad \text{and} \quad B_0'(s) = B_1(s) \frac{1 - \frac{A}{B_1(s)[1 - F(B_1(s)|s)]}}{1 - \frac{A}{p_0 B_0}}.$$

Since $B_1(s)[1 - F(B_1(s)|s)] < p_0 B_0$ for $s < \tilde{s} = \tilde{s}'$, this implies that $B_0'(s) < B_0(s)$.

5.1 Sovereign Debt Maturity

In this section, we investigate the role of sovereign debt maturity. More specifically, we compare our economy with long-term sovereign bonds which are claims to coupons accruing at date 2 with an economy where sovereign bonds are short-term one-period bonds which are rolled over at date 1.

We assume that there is no debt forgiveness (either because debt is on the right side of the legacy Laffer curve, or because debt is on the wrong side of the legacy Laffer curve but investors have difficulties coordinating on a debt relief package). It is easy to see that the absence of supervisory leniency extends to the setting of this section. We therefore make use of the fact that $\underline{b}_0^{**} = \underline{b}_0^*$ throughout.

With long-term bonds, welfare is given by the decomposition (4) so that

$$\mathcal{W}_0 = \mathcal{E}_0 - \mathcal{R}_0$$

with

$$\mathcal{E}_0 = \int \left[\int_{B_1(s)}^{\infty} [E - B_0] f(E|s) dE + \int_0^{B_1(s)} [E - \Phi] f(E|s) dE \right] d\pi(s) + \int \left[\left[\beta^I(s) + \beta^B(\rho_1(s) - 1) \right] A + \beta^B A \right] d\pi(s),$$

$$\mathcal{R}_0 = -\int (1 - \beta^B) \min\{\underline{b}_0^* + (A - \underline{b}_0^*) \frac{p_1(s)}{p_0} - I(s), 0\} d\pi(s).$$

We now consider the economy with short-term bonds. We denote all variables with a tilde. To make the comparison with the economy with short-term bonds meaningful, we impose that the government must raise the same amount of revenues G_0 in period 0, i.e.

$$\tilde{B}_0 = B_0 \int [1 - F(B_1(s)|s)] = G_0. \tag{22}$$

In addition, the government must raise exactly enough revenues at date 1 to repay the date-0 debt that is coming due, i.e. we must have for all s[33]

$$\tilde{B}_1(s)[1 - F(\tilde{B}_1(s)|s)] = \tilde{B}_0. \tag{23}$$

[33]To give short-term debt a good shot, we assume that the government is always able to roll over its short-term debt. This is indeed the case if negative shocks s are not too catastrophic, so that the debt can be rolled over by pledging income in the good realizations at date 2.

Because $\bar{I} \leq A$, no bailouts are required. Welfare is given by

$$\tilde{\mathcal{W}}_0 = \tilde{\mathcal{E}}_0 - \tilde{\mathcal{R}}_0$$

with

$$\tilde{\mathcal{E}}_0 = \int \left[\int_{\tilde{B}_1(s)}^{\infty} \left[E - \tilde{B}_1(s) \right] f(E|s) dE + \int_0^{\tilde{B}_1(s)} \left[E - \Phi \right] f(E|s) dE \right] d\pi(s)$$
$$+ \int \left[\left[\beta^I(s) + \beta^B(\rho_1(s) - 1) \right] A + \beta^B A \right] d\pi(s),$$
$$\tilde{\mathcal{R}}_0 = 0.$$

Using (22) and (23), we can write

$$\mathcal{W}_0 - \tilde{\mathcal{W}}_0 = (\mathcal{E}_0 - \tilde{\mathcal{E}}_0) - (\mathcal{R}_0 - \tilde{\mathcal{R}}_0),$$

with

$$\mathcal{E}_0 - \tilde{\mathcal{E}}_0 = \int \Phi[F(\tilde{B}_1(s)|s) - F(B_1(s)|s)] d\pi(s),$$
$$\mathcal{R}_0 - \tilde{\mathcal{R}}_0 = -(1 - \beta^B) \int \min\{\underline{b}_0^* + (A - \underline{b}_0^*) \frac{p_1(s)}{p_0} - I(s), 0\} d\pi(s).$$

The term $(\mathcal{E}_0 - \tilde{\mathcal{E}}_0)$ is given by the difference in default costs, and the second term $-(\mathcal{R}_0 - \tilde{\mathcal{R}}_0)$ represents the welfare impact of the rents extracted by bankers at the expense of consumers because of bailouts when domestic sovereign bonds are long term. In the proof of the proposition below, we show that under some additional assumptions on the distributions of E, the first term is positive.[34] The second term is always negative and arises because by issuing short-term bonds, the government reduces the risk-taking possibilities of banks and insulates the banks from fiscal developments—there is no feedback loop between banks and the sovereign. As a result, the intuitive asset-liability management (ALM) principle of matching maturities of incomes and payments holds when the minimum diversification requirement $\underline{b}_0^*$ is high enough: Long-term debt then leads to a strictly lower expected probability of default than short-term debt.

Intuitively, a short maturity has both benefits and costs. The cost is that a short maturity is bad for fiscal hedging. The benefit is that a short maturity reduces the risk-shifting

[34] The additional assumptions are that $F(E|s) = F(E - s)$ where F is increasing and convex. These assumptions, which imply the monotone hazard rate properties $\frac{\partial(f(E|s)/(1-F(E|s)))}{\partial s} \leq 0$ and $\frac{\partial(f(E|s)/(1-F(E|s)))}{\partial E} > 0$, are sufficient but not necessary to prove the result.

possibilities of banks. A short maturity is therefore a costly substitute to supervision. As a result, a long maturity is preferable when supervision is efficient enough ($\underline{b}_0^*$ is high enough) or when the banking sector is small enough (A and $\bar{I}$ are small enough).

Proposition 9 (Optimal Debt Maturity)**.** *Suppose that $F(E|s) = F(E-s)$ where F is increasing and convex. Then for $\underline{b}_0^*$ high enough or for A and $\bar{I}$ low enough, welfare is higher with long-term sovereign bonds than with short-term sovereign bonds $\mathcal{W}_0 > \tilde{\mathcal{W}}_0$.*

Proof. We prove the result for $\underline{b}_0^*$ high enough. The proof of the result for for A and $\bar{I}$ low enough is similar. Note that $B_1(s) \geq B_0$ and that $B_1(s)$ converges to B_0 for all s as $\underline{b}_0^*$ goes to $\bar{I}$, while $\tilde{B}_1(s)$ is independent of $\underline{b}_0^*$. Hence the result follows if we can show that $\int \Phi[F(\tilde{B}_1(s)|s) - F(B_0|s)]d\pi(s) < 0$. We now proceed to prove this result, which we refer to as result A. The result is a direct consequence of the following related (dual) result, which we refer to as result B. Let B_0 be defined by

$$\int B_0[1 - F(B_0|s)]d\pi(s) = G_0$$

as above, and let $\tilde{\tilde{B}}_1(s)$ and $\tilde{\tilde{G}}_0$ be defined by the system of equations

$$\tilde{\tilde{B}}_1(s)[1 - F(\tilde{\tilde{B}}_1(s)|s)] = \tilde{\tilde{G}}_0 \quad \text{for all } s,$$

$$\int F(\tilde{\tilde{B}}_1(s)|s)d\pi(s) = \int F(B_0|s)d\pi(s).$$

Result B is that $\tilde{\tilde{G}}_0 < G_0$. We now prove result B, which in turn directly implies result A.

Since

$$\tilde{\tilde{G}}_0 \int \frac{B_0}{\tilde{\tilde{B}}_1(s)} d\pi(s) = G_0,$$

we need to show that

$$\int \frac{B_0}{\tilde{\tilde{B}}_1(s)} d\pi(s) > 1.$$

By Jensen's inequality, result B is implied by the following result, which we refer to as result C:

$$\frac{B_0}{\int \tilde{\tilde{B}}_1(s) d\pi(s)} > 1.$$

Since $\int F(\tilde{\tilde{B}}_1(s) - s)d\pi(s) = \int F(B_0 - s)d\pi(s)$ and F is increasing, result C is equivalent to

$$\int F(\int \tilde{\tilde{B}}_1(s')d\pi(s') - s)d\pi(s) < \int F(\tilde{\tilde{B}}_1(s) - s)d\pi(s).$$

Define

$$g(\lambda) = \int F[\int \tilde{\tilde{B}}_1(s')d\pi(s') + \lambda(\tilde{\tilde{B}}_1(s) - \int \tilde{\tilde{B}}_1(s')d\pi(s')) - s]d\pi(s).$$

We have

$$g'(\lambda) = \int f[\int \tilde{\tilde{B}}_1(s')d\pi(s') + \lambda(\tilde{\tilde{B}}_1(s) - \int \tilde{\tilde{B}}_1(s')d\pi(s')) - s][\tilde{\tilde{B}}_1(s) - \int \tilde{\tilde{B}}_1(s')d\pi(s')]d\pi(s).$$

Because $f[\int \tilde{\tilde{B}}_1(s')d\pi(s') + \lambda(\tilde{\tilde{B}}_1(s) - \int \tilde{\tilde{B}}_1(s')d\pi(s')) - s]$ and $\tilde{\tilde{B}}_1(s) - \int \tilde{\tilde{B}}_1(s')d\pi(s')$ are decreasing in s for all $\lambda \geq 0$, the right-hand side is the covariance of two decreasing functions of the random variable s and is therefore positive. It follows that $g'(\lambda) > 0$ for all $\lambda \geq 0$. Since $g(0) = \int F(\int \tilde{\tilde{B}}_1(s')d\pi(s') - s)d\pi(s)$ and $g(1) = \int F(\tilde{\tilde{B}}_1(s) - s)d\pi(s)$, we get result C. Results B and A follow, concluding the proof of the Proposition.

□

There are obvious extensions of our setup that would reinforce the conclusion of Proposition 9. For example, the desirable features of short-term sovereign debt in terms of limiting the risk-taking possibilities of banks, bailouts and feedback loops between banks and sovereigns, would be mitigated in a model with a richer set of risk taking possibilities apart from domestic sovereign debt, or in an infinite horizon version of our model with overlapping generations of bankers, consumers and investors, where some banks hold domestic sovereign debt for liquidity in all periods.

Connection to the sovereign debt literature on debt maturity. The result in Proposition 9 is related to a set of arguments in the sovereign debt literature that seek to rationalize the empirical observation that governments tend to shorten the maturity of their debt when the probability of default increases.[35,36] This literature has two branches. The first one focuses on the different hedging properties of short-term bonds and long-term bonds. The second one focuses on the different incentive effects (for the sovereign) of short-term bonds and long term bonds: Short-term bonds limit the possibilities of dilution, increase

[35] See e.g. Angeletos (2002), Buera-Nicolini (2004), Jeanne (2009), Bolton-Jeanne (2009), Hatchondo-Martinez (2009), Chaterjee-Eyigungor (2012), Arellano-Ramanarayanan (2012), Dovis (2012), Broner-Lorenzoni-Schmukler (2013), Aguiar-Amador (2014), and Fernandez-Martin (2015).

[36] There is also a corporate finance literature on debt maturity. For example, Calomiris-Kahn (1991) and Diamond-Rajan (2001) show that the threat of not rolling over short-term debt can be used to discipline a manager. The fact that existing long-term bondholders hold a claim on liquidated assets which makes them vulnerable to dilution has been the focus of a large literature since Fama-Miller (1972). Brunnermeier-Oehmke (2013) show how this can lead to a maturity rat race towards short maturities. Sovereign default differs from bankruptcy in that there is no liquidation after default.

the incentives to pay down debt and implement pro-market reforms in order to lower interest rates for future borrowing. Proposition 9 identifies a novel tradeoff in debt maturity choice that requires balancing hedging effects and a different form of incentive effects. On the one hand, a long maturity structure might be desirable for fiscal hedging. On the other hand, a short maturity structure might be desirable to reduce the risk-shifting opportunities of banks when the government cannot commit not to bail them out ex post and cannot adequately supervise them ex ante. This novel tradeoff makes the new prediction is that countries with effective supervision or with a small banking sector or are less likely to shift towards short maturities when the probability of default increases.

5.2 Multiple Risky countries

As discussed in the introduction, risk shifting by a troubled European country's banks could have occurred through the purchase of another troubled country's bonds rather than through a re-nationalization of the domestic financial market.

In this section, we therefore considers multiple risky countries. We show that, provided that balance sheet shocks and fiscal shocks within a country are at least slightly positively correlated and that fiscal shocks across countries are not perfectly correlated (a reasonable assumption), risk shifting solely through domestic bond holding is a strict equilibrium. We also show that, with multiple risky countries, our "double-decker bailout" theory predicts that when the fiscal outlook is bad, governments in risky countries have an incentive to relax supervision and let their banks load up on risky domestic debt (and not risky foreign debt).

All in all, this extension shows that the multiple forces that we have identified for risk shifting in the baseline model occur through the purchase of risky domestic debt rather than risky foreign debt, implying that the re-nationalization results of the baseline model are robust to multiple risky countries.

The structure of the model is the same as in the basic model, but there are now two symmetric risky countries A and B, together with the foreign (safe) country. We consider banks in countries A and B, but for simplicity, we abstract from banks in the foreign (safe) country. We denote by s^i the state of the world at date 1 in country i, and we denote by π the joint distribution of (s^A, s^B). We focus on symmetric equilibria throughout.

In each country $i \in \{A, B\}$, banks have an endowment A at date 0 and some investment opportunities $I(s^i)$ with private and social returns (for country i) given by $\rho_1(s^i)$ and $\beta^I(s^i)$. Banks invest their net worth at date 0 in a portfolio of safe foreign bonds, risky domestic bonds and risky foreign bonds. The return on their portfolio at date 1 determines their net worth at date 1. If it falls short of their investment need, then they are bailed out by their country's government.

We denote by B_0 the quantity of debt at date 0 and by p_0 the price of debt at date 0 in both risky countries. We assume that in each country $i \in \{A, B\}$, supervision forces banks of country i to hold a portfolio with holdings of safe foreign sovereign bonds of at least $\underline{b}_0^{**} \leq \underline{b}_0^*$, but does not place constraints on the relative holding of risky sovereign debt of countries A and B. We start by assuming that there is no debt forgiveness (we reintroduce this possibility later). Each government chooses not to engage in supervisory leniency and sets $\underline{b}_0^{**} = \underline{b}_0^*$, which we assume from now on.

In each country i, we denote by $b_0^* \geq \underline{b}_0^*$, $b_0^d \geq 0$ and $b_0^f \geq 0$ the holdings of foreign safe debt, risky domestic debt (of country i) and risky foreign debt (of country $-i$) with $b_0^* + p_0(b_0^d + b_0^f) = A$. We denote by $B_1(s^d, s^f)$ the quantity of debt at date 1 and by $p_1(s^d, s^f)$ the price of debt at date 1 when the domestic when country i is in state $s^i = s^d$ and country $-i$ is in state $s^{-i} = s^f$.

A symmetric equilibrium is characterized by the following pricing equations

$$p_1(s^d, s^f) = 1 - F(B_1(s^d, s^f)|s^d),$$

$$p_0 = \int p_1(s^d, s^f) d\pi(s^d, s^f),$$

with

$$B_1(s^d, s^f) = B_0 + \max\{\frac{I(s^d) - b_0^*}{p_1(s^d, s^f)} - \frac{b_0^d}{p_0} - \frac{b_0^f}{p_0}\frac{p_1(s^f, s^d)}{p_1(s^d, s^f)}, 0\},$$

together with the requirement that bank portfolios (b_0^*, b_0^d, b_0^f) solve the following maximization problem

$$\max_{(\tilde{b}_0^*, \tilde{b}_0^d, \tilde{b}_0^f)} \int \left[\rho_1(s^d)I(s^d) + \max\{\tilde{b}_0^* + p_1(s^d, s^f)\tilde{b}_0^d + p_1(s^f, s^d)\tilde{b}_0^f - I(s^d), 0\}\right] d\pi(s^d, s^f),$$

subject to

$$\tilde{b}_0^* + p_0(\tilde{b}_0^d + \tilde{b}_0^f) = A \quad \text{and} \quad \tilde{b}_0^* \geq \underline{b}_0^*.$$

Home bias with multiple risky countries. We show that in each country, there exists a symmetric equilibrium where domestic banks choose to hold as little safe foreign bonds and as much risky domestic bonds as allowed by supervision, but no risky foreign bonds. The reason is that their financing needs are co-monotone with the state of the domestic economy. As a result, domestic banks maximize the value of their put on the taxpayer's money by risk-shifting into risky domestic sovereign rather in risky foreign sovereign debt.

Proposition 10 (Home Bias with Multiple Risky Countries)**.** *Suppose that s^d and s^f are not co-monotone and that $I(s)$ is strictly decreasing in s. Then there exists a symmetric equilibrium where banks in each country $i \in \{A, B\}$ choose to hold as little safe foreign bonds and as much risky domestic bonds as allowed by supervision, but no risky foreign bonds: $b_0^* = \underline{b}_0^*$, $b_0^d = \frac{A-\underline{b}_0^*}{p_0}$ and $b_0^f = 0$. This equilibrium is strict.*

Proof. Assume that the equilibrium is of the form given by the proposition. Under the assumed equilibrium, we have $p_1(s^d, s^f) = p_1(s^d)$ and $p_1(s^f, s^d) = p_1(s^f)$, for some increasing function $p_1(s)$. We now show that banks then indeed choose portfolios $b_0^* = \underline{b}_0^*$, $b_0^d = \frac{A-\underline{b}_0^*}{p_0}$, and $b_0^f = 0$, and that this preference is strict. This proves that the assumed equilibrium is indeed an equilibrium, and that it is strict.

We use the following result. Take a function $g(x_1, x_2)$ with $\frac{\partial^2 g(x_1,x_2)}{\partial x_1 \partial x_2} \geq 0$ and take two marginals F_1 and F_2. Consider the problem of maximizing $E[g(X_1, X_2)]$ subject to the marginals of (X_1, X_2) being given by F_1 and F_2. Then the maximum is reached when the two variables are co-monotone. This is a well-known solution of the Monge-Kantorovich optimal transport problem in the case of supermodular objective functions.

We apply this result with $g(x_1, x_2) = \max\{x_1 + x_2, 0\}$, with F_1 given by the distribution of $\tilde{b}_0^* + p_1(s^d)(A - \tilde{b}_0^*)$ and F_2 given by the distribution of $-I(s^d)$. The random variables $X_1 = p_1(s^d)\tilde{b}_0^d + p_1(s^f)(A - \tilde{b}_0^* - \tilde{b}_0^d)$ and $X_2 = -I(s^d)$ are such that the marginals of (X_1, X_2) are given by F_1 and F_2. They are co-monotone if and only if $\tilde{b}_0^d = A - \tilde{b}_0^*$.

This shows given $\tilde{b}_0^* \geq \underline{b}_0^*$, banks strictly prefer $\tilde{b}_0^d = A - \tilde{b}_0^*$ and $\tilde{b}_0^f = 0$. It is then immediate to see that banks strictly prefer $\tilde{b}_0^* = \underline{b}_0^*$, $\tilde{b}_0^d = A - \tilde{b}_0^*$, and $\tilde{b}_0^f = 0$. □

The intuition for Proposition 10 is simple. Because balance sheet shocks in a given country are perfectly correlated with fiscal shocks in this country, but imperfectly correlated with fiscal shocks in other countries, bankers maximize the bailout that they extract from the government by investing in risky domestic sovereign bonds rather than risky foreign sovereign bonds.[37]

In this setting, because as shown in Proposition 10, banks in country $i \in \{A, B\}$ choose to hold no debt from country $-i$, there is no interaction between the two risky countries A and B: Debt prices and quantities in each risky country are determined independently exactly as in Section 2. All of our results in Section 3, properly specialized to the setting

[37] We refer the reader to Appendix B.5 for an illustration of Proposition 10 in the context of a simple example which can be solved in closed form.

of this illustrating example, carry through with no modification.[38] The results in Section A.4 also readily extend. Our other extensions in Sections A.2-5.1 would require further adaptation.

The results in Section 3.5 are particularly interesting in this context. Propositions 15 and 16 apply with no modification. In response to a bad fiscal shock, there is a re-nationalization of sovereign debt markets: Banks in country i increase their holdings of risky domestic sovereign debt from country i but not their holdings of risky foreign sovereign debt from country $-i$.

Strategic supervisory leniency with multiple risky countries. We now introduce the possibility of debt forgiveness and examine how the results in Sections 4.2 and 4.3 regarding strategic supervisory leniency generalize to a setting with multiple risky countries. In particular, we want to show that governments have an incentive to let their own banks load up on domestic risky bonds but not on foreign risky bonds. To make that point in the starkest possible way, we assume that supervision is perfect: In each country i, the government can exactly control the portfolio (b_0^*, b_0^d, b_0^f) of its banks through supervision. This means that we not only assume that supervisory capability as we have defined it so far is perfect so that $\underline{b}_0^* = A$, but also in addition that the government can now perfectly determine the relative holding of risky sovereign debt of countries A and B.

We can derive the following counterpart to Proposition 7 and show that governments in risky countries have an incentive to let their banks load up on risky domestic sovereign debt as opposed to risky foreign sovereign debt, in order to maximize the concessions from legacy creditors. This maximizes the rents of domestic banks, and at the same time entirely shifts the cost of domestic bailouts from domestic consumers to foreign investors by ensuring that all domestic bailouts take place in states with domestic sovereign debt forgiveness.

Proposition 11 (Strategic Supervisory Leniency with Multiple Risky Countries)**.** *Suppose that $I(s)$ is independent of s and that $A = \bar{I}$, and that the distribution $\pi(s^d, s^f)$ admits a continuous density $d\pi(s^d, s^f) = \pi(s^d, s^f)ds^d ds^f$. In addition, suppose that in the limit where A and $\bar{I}$ are small, for the equilibrium with a single risky country and no supervisory leniency $\underline{b}_0^{**} = \underline{b}_0^*$, the equilibrium is "bailout-shifting" with $\tilde{s} < \bar{s}$, and that s^d and s^f are not co-monotone. Then in the limit where A and $\bar{I}$ are small, it is optimal for the government in country i to force its banks to invest all their net worth A in risky domestic sovereign bonds, and to invest zero in safe foreign*

[38] Some of our results were proved in the case where $I(s)$ is independent of s whereas here we require here that $I(s)$ is strictly decreasing in s. In these cases, we can simply take the limit when the range of $I(s)$ collapses to a point.

sovereign bonds and zero in risky foreign sovereign bonds: $b_0^* = 0$, $b_0^d = \frac{A}{p_0}$ *and* $b_0^f = 0$.[39]

Proof. When choosing the portfolio of banks, the domestic government takes p_0 as given and maximizes the following welfare function in (b_0^*, b_0^d, b_0^f) for a given p_0:

$$\mathcal{W}_0 = \int \mathcal{W}_0(s^d, s^f)\pi(s^d, s^f)ds^d ds^f,$$

with

$$\mathcal{W}_0(s^d, s^f) = \int_{B_1(s^d,s^f)}^{\infty} \left[E - B_1(s^d, s^f)\right] f(E|s^d)dE + \int_0^{B_1(s^d,s^f)} [E - \Phi] f(E|s^d)dE + \beta^I(s^d)I(s^d)$$
$$+ \beta^B \rho_1(s^d)I(s^d) + \beta^B \max\{b_0^* + b_0^d \frac{B_0(s^d, s^f)}{B_0} p_1(s^d, s^f) + b_0^f \frac{B_0(s^f, s^d)}{B_0} p_1(s^f, s^d) - I(s^d), 0\}.$$

It is clear that $b_0^* = 0$, $b_0^d = \frac{A}{p_0}$, and $b_0^f = 0$ is a maximizer of the default term

$$\int \int_{B_1(s^d,s^f)}^{\infty} \left[E - B_1(s^d, s^f)\right] f(E|s^d)dE\pi(s^d, s^f)ds^d ds^f + \int_0^{B_1(s^d,s^f)} [E - \Phi] f(E|s^d)dE\pi(s^d, s^f)ds^d ds^f.$$

This is because with this portfolio, all bank bailouts occur in states where there is debt forgiveness, and in these states, the amount of post-debt forgiveness debt is independent of the banks' portfolios. This implies that for all (s^d, s^f), post-debt forgiveness $B_1(s^d, s^f)$ is greater for any other portfolio for banks.

It remains to show that $b_0^* = 0$, $b_0^d = \frac{A}{p_0}$, and $b_0^f = 0$ is a maximizer of the the banks' rents term

$$\int \beta^B \max\{b_0^* + b_0^d \frac{B_0(s^d, s^f)}{B_0} p_1(s^d, s^f) + b_0^f \frac{B_0(s^f, s^d)}{B_0} p_1(s^f, s^d) - I(s^d), 0\}\pi(s^d, s^f)ds^d ds^f.$$

The first-order term in $A = \bar{I}$ in this expression is given by

$$\int \beta^B \max\{b_0^* + b_0^d \frac{\hat{B}_0(s^d)}{B_0} \hat{p}_1(s^d) + b_0^f \frac{\hat{B}_0(s^f)}{B_0} \hat{p}_1(s^f) - \bar{I}, 0\}\pi(s^d, s^f)ds^d ds^f,$$

where $\hat{B}_0(s^d)$ and $\hat{p}_1(s^d)$ are the equilibrium values for$A = \bar{I} = 0$ (and the value of p_0 under consideration). Using the same result for the Monge-Kantorovich optimal transport problem in the case of supermodular objective functions as in the proof of Proposition 10, it follows this expression is maximized when either $b_0^d = 0$ or $b_0^f = 0$ and in addition

[39] Assuming that A and $\bar{I}$ are small considerably simplifies the analysis by allowing to neglect the change in debt prices induced by changes in portfolios in the calculation of the rents of bankers.

$b_0^* = 0$. □

Obviously, if the government could not determine the relative holdings of domestic and foreign risky bond holdings, but only impose an effective requirement $b_0^* \geq \underline{b}_0^{**}$ with $\underline{b}_0^{**} \leq b_0^*$, then we would obtain (in the limit where $\underline{I}$ tends to $\bar{I} = A$ from below) that it is optimal to set $\underline{b}_0^{**} = 0$. Banks would then by themselves load up on domestic risky bonds, choosing $b_0^d = \frac{A}{p_0}$ and $b_0^f = 0$. Proposition 11 shows the perhaps more interesting result that even if the supervisor could perfectly control the portfolios of banks, it would choose to encourage them to load up on domestic risky bonds.

6 Summing Up

This paper's framework describes a feedback loop between banks and their Sovereign. Fiscal shocks reduce the value of sovereign debt, generate bailouts if bank portfolios exhibit a home bias, leading to further debt sustainability problems, etc. Bank balance sheet shocks similarly contribute to the feedback loop. Relative to the earlier literature, we have uncovered the following insights:

i. In the absence of debt forgiveness, a tougher supervision reduces both bailouts and therefore the social cost of bankers' rents and the occurrence of default. Welfare increases with supervisory discipline, both ex ante (when discipline is priced in when Sovereign bonds are issued) and ex post (once bonds have been issued).

ii. While bank purchases of domestic bonds amounts to a debt buyback by the country, this result differs from the classic Bulow-Rogoff argument against buybacks, obtained in an economy with no cost of default and no financial intermediaries. Costs of defaults work against the Bulow-Rogoff result, but in net we show that maximum supervisory discipline (minimum buybacks) is optimal for the country.

iii. As long as the country has the capability to bail out banks, the latter's exposures to their domestic government's debt are strategic complements.

iv. Introducing debt forgiveness, we showed that the amount of date-1 debt after debt forgiveness and issuance associated with the peak of the legacy Laffer curve is at the peak of the issuance Laffer curve.

v. With debt forgiveness, the only full equilibrium with endogenous supervisory leniency features maximal supervisory leniency.

Overall, there are two distinct rationales for re-nationalization. First, the banks invest in opacity and try to evade prudential diversification rules when fiscal prospects are bleak and balance sheets degraded. Second, even when the government can perfectly monitor its banks, the government may strategically turn a blind eye to their lack of country diversification and count on legacy debt forgiveness to finance the rescue of its banking sector in case of difficulties. In either case, re-nationalization occurs when the legacy debt increases or prospects about the country's fiscal capability and/or bank balance sheets worsen. Furthermore, re-nationalization is robust to the distress of multiple risky countries and therefore to the co-existence of multiple ways of shifting risk through the holding of sovereign bonds.

We also supplied a rationale for a banking union. If the ex-post leniency of domestic supervisors is anticipated ex ante at the time of sovereign debt issuance, then it is priced in the form of higher spreads. The government is better off committing ex ante to a tough ex-post supervisory stance, but is tempted to relax it ex post. If the government lacks commitment, then it benefits from relinquishing its supervisory powers to a supranational supervisor by joining a banking union.

Finally, we provided a number of worthwhile extensions. We showed that the feedback loop paradoxically stems from a prudent matching of debt maturity with the country's fiscal capability, and provided conditions under which this matching is nonetheless optimal. As noted above, we then showed that the insights were robust to multiple risky countries. Two other extensions concerned the impact of (endogenous) leverage and limited bailout funds. The feedback loop is stronger in case of joint default on private and sovereign debt. And when push comes to shove and the government may run out of money to finance bailouts, banks may by contrast engage in a diversification rat race.

Mapping the theory with the European experience is complex. The blanket guarantees granted by the ECB in the middle of the crisis (akin to the debt forgiveness in the model) have so far prevented it from unfolding. Nonetheless the existing empirical evidence is encouraging. Acharya et al. (2013) looks at the price of European sovereign and bank CDSs over the period 2007-2011. These became negatively correlated after the first bank bailouts—pointing at a perception of risk transfer—and then exhibited a significant positive correlation, suggesting that the market was concerned about a feedback loop. Gennaioli et al. (2014a,b) among other things find that bank holdings increase during crises, when the expected return is high, and that much of this increase is due to the large, "too-big- too-fail" banks, which fits well with our banking bailout story. At the bank level, the fact that bond holdings correlate negatively with subsequent lending during sovereign defaults demonstrates the link between bank balance sheets and sovereign distress.

Our research leaves open a number of fascinating questions. First, we have assumed that the bailouts take the fiscal route. As observed recently in many countries, the Central Bank may participate in the bailout, perhaps risking inflation and devaluation. Second, we have assumed that sovereign defaults are not strategic (the government defaults only if it cannot repay). If defaults are strategic, domestic exposure choices by domestic banks influence the incentives to default (the government is less likely to default if its debt is held domestically), opening up the possibility of complex strategic interactions between banks and sovereigns, and conferring a benefit (disciplining the government) upon debt re-nationalization. Finally, further research should be devoted to the governance of the banking union, and in particular to the interactions between prudential and fiscal integrations: Should the union be committed to solidarity? Should a country bear the first losses when one of its banks defaults? We leave these and other questions for future research.

References

[1] Acharya, V., Drechsler, I., and P. Schnabl (2015) "A Pyrrhic Victory? Bank Bailouts and Sovereign Credit Risk," *Journal of Finance*, 69(6): 2689-2739.

[2] Acharya, V. and S. Steffen (2015) "The Greatest Carry Trade Ever? Understanding Eurozone Bank Risks," *Journal of Financial Economics*, 115: 215-236.

[3] Acharya, V. and T. Yorulmazer (2007), "Too many to fail - an analysis of time-inconsistency in bank closure policies," *Journal of Financial Intermediation*, 16(1): 1-31.

[4] Amador, M. and M. Aguiar (2014) "Take the Short Route: How to Repay and Restructure with Multiple Maturities," mimeo, Minneapolis Fed and Princeton.

[5] Angeletos, G.M. (2002) "Fiscal Policy With Noncontingent Debt And The Optimal Maturity Structure," *Quarterly Journal of Economics*, 117(3): 1105-1131.

[6] Arellano, C. and A.Ramanarayanan (2012), "Default and the Maturity Structure in Sovereign Bonds," *Journal of Political Economy*, 120: 187-232.

[7] Bianchi, J. (2013) "Efficient Bailouts?", mimeo University of Wisconsin.

[8] Bocola, L. (2014) "The Pass-Through of Sovereign Risk", mimeo Northwestern University and Minneapolis Fed.

[9] Bolton, P. and O. Jeanne (2009) "Structuring and Restructuring Sovereign Debt: The Role of Seniority," *Review of Economic Studies*, 76(3): 879-902.

[10] Bolton, P. and O. Jeanne (2011) "Sovereign Default Risk and Bank Fragility in Financially Integrated Economies," *IMF Economic Review*, 59: 162-194.

[11] Broner, F., Erce, A., Martin, A. and J. Ventura (2013) "Sovereign Debt Markets in Turbulent Times: Creditor Discrimination and Crowding-out Effects", mimeo, U. Pompeu Fabra.

[12] Broner, F. and J. Ventura (2011) "Globalization and Risk Sharing," *Review of Economic Studies*, 78: 49-82.

[13] Broner, F., Lorenzoni, G. and S. Schmukler (2013) "Why Do Emerging Economies Borrow Short Term?" *Journal of the European Economic Association*, 11: 67-100.

[14] Brunnermeier, M. and M. Oehmke (2013) "The Maturity Rat Race," *Journal of Finance*, 68: 483-521.

[15] Buera, F. and J.P. Nicolini (2004) "Optimal maturity of government debt without state contingent bonds," *Journal of Monetary Economics*, 51(3): 531-554.

[16] Bulow, J. and K. Rogoff (1988) "The Buyback Boondogle," *Brookings Papers on Economic Activity*, 675-698.

[17] Bulow, J. and K. Rogoff (1991) "Sovereign Debt Repurchases: No Cure for Overhang," *Quarterly Journal of Economics*, 106: 1219–1235.

[18] Calomiris, C. and C. Kahn (1991), "The Role of Demandable Debt in Structuring Optimal Banking Arrangements," *American Economic Review,* 81: 497-513.

[19] Chari, V.V. and P. Kehoe (2013) "Bailouts, Time Inconsistency, and Optimal Regulation," NBER Working Papers 19192.

[20] Chari, V.V, Dovis, A. and P. Kehoe (2014) "On the Optimality of Financial Repression," mimeo Minneapolis Fed.

[21] Chatterjee, S. and B. Eyigungor (2012), "Maturity, Indebtedness, and Default Risk," *American Economic Review,* 102(6): 2674-2699.

[22] Calvo, G. (1988) "Servicing the Public Debt: The Role of Expectations," *American Economic Review*, 78(4): 647-661.

[23] Cooper, R. and Nikolov (2013) "Government Debt and Banking Fragility: The Spreading of Strategic Uncertainty", mimeo, Penn State University and ECB.

[24] Diamond, D., and P. Dybvig (1983) "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy*, 91: 401-419.

[25] Diamond, D., and R. Rajan (2001) "Liquidity Risk, Liquidity Creation, and Financial Fragility: A Theory of Banking," *Journal of Political Economy*, 109: 283-327.

[26] Diamond, D., and R. Rajan (2012) "Illiquid Banks, Financial Stability, and Interest Rate Policy," *Journal of Political Economy*, 120(3), 552-591.

[27] Dovis, A. (2012) "Efficient Sovereign Default," mimeo.

[28] Farhi, E. and J. Tirole (2012) "Collective Moral Hazard, Maturity Mismatch, and Systemic Bailouts," *American Economic Review*, 102(1): 60-93.

[29] Fernandez, R. and F. Martin (2015), "The Long and Short of It: Sovereign Debt Crises and Debt Maturity," NBER working paper.

[30] Gennaioli, N., Martin, A. and S. Rossi (2014,a) "Sovereign Default, Domestic Banks and Financial Institutions," *Journal of Finance*, 69(2): 819:866.

[31] Gennaioli, N., Martin, A. and S. Rossi (2014,b) "Banks, Government Bonds and Defaults: What Do the Data Say?" mimeo.

[32] Gennaioli, N., Shleifer, A. and R. Vishny (2012) "Neglected Risk, Financial Innovation, and Financial Fragility," *Journal of Financial Economics*, 104: 452-468.

[33] Hatchondo, J. and L. Martinez (2009) "Long-duration bonds and sovereign defaults," *Journal of International Economics*, 79(1): 117-125.

[34] Hatchondo, J., Martinez, L., and C. Sosa Padilla (2014) "Voluntary Sovereign Debt Exchanges," *Journal of Monetary Economics*, 61: 32-50.

[35] Holmström, B. and J. Tirole (1997) "Financial Intermediation, Loanable Funds, and the Real Sector," *Quarterly Journal of Economics*, 112: 663–691.

[36] Jeanne, O. (2009) "Debt Maturity and the International Financial Architecture," *American Economic Review*, 99(5): 2135-2148

[37] Keister, T. (2014) "Bailouts and Financial Fragility," Departmental Working Papers 201401, Rutgers University, Department of Economics.

[38] Lagarde, C. (2012), *Opening Remarks to IMF/CFP Policy Roundtable on the Future of Financial Regulation*, International Monetary Fund, Washington DC, April 17.

[39] Mengus, E. (2013a) "Honoring Sovereign Debt or Bailing Out Domestic Residents? A Theory of Internal Cost of Default", mimeo, TSE.

[40] Mengus, E. (2013b) "International Bailouts: Why Did Banks' Collective Bet Lead Europe to Rescue Greece?" mimeo, TSE.

[41] Nosal, J. and G. Ordonez (2014) "Uncertainty as Commitment", mimeo, Columbia University and University of Pennsylvania.

[42] Philippon, T. and V. Skreta (2012) "Optimal Interventions in Markets with Adverse Selection," *American Economic Review*, 102(1): 1-28.

[43] Rancière, R., A. Tornell, and F. Westermann (2008) "Systemic Crises and Growth," *The Quarterly Journal of Economics*, 123(1): 359-406.

[44] Reinhart, C. and K. Rogoff (2009) *This Time Is Different: Eight Centuries of Financial Folly*, Princeton University Press.

[45] Schneider, M. and A. Tornell (2004) "Balance Sheet Effects, Bailout Guarantees and Financial Crises," *Review of Economic Studies*, 71: 883-913.

[46] Stavrakeva, V. (2013) "Optimal Bank Regulation and Fiscal Capacity", mimeo London Business School.

[47] Tirole, J. (2012) "Overcoming Adverse Selection: How Public Intervention Can Restore Market Functioning", *American Economic Review*, 102(1): 29-59.

[48] Uhlig, H. (2014) "Sovereign Default Risk and Banks in a Monetary Union", mimeo, U. of Chicago.

[49] Woodford, M. (1990) "Public Debt as Private Liquidity," *American Economic Review*, 80(2): 382-388.

A Appendix: Extensions

A.1 Credit Crunch Foundations of the Social Welfare Function

We sketch the foundations of the welfare function, following Homlström and Tirole (1997). At date 1, the bank can make an investment in knowledge/staff so as to be able to invest in a mass $I(s)$ of firms at date 1. These firms enter in a relationship with the bank at date 1; from then on, they share available resources in coalition with the banks. At date 2, firms succeed (return $r(s)$ per firm) or fail (return 0). Success is guaranteed if the firm managers as well as the workers in the firm do not shirk. Otherwise success accrues with probability 0. Shirking for a firm manager brings benefit $\phi^F(s)$, and shirking for a firm worker brings benefit $\phi^W(s)$. Therefore incentive payments $\rho_1^F(s) = \phi^F(s)$ and $\rho_1^W(s) = \phi^W(s)$ per firm are required to discipline the firm manager and the workers. For simplicity, we assume that workers and firms are cashless. Bankers can divert their share of the return $\rho_1(s) = r(s) - \phi^F(s) - \phi^W(s)$ on the project of each firm so that they cannot borrow.

In Section A.2, we relax the assumption that bankers cannot borrow. This can be modeled as follows. Instead of assuming that bankers can divert their share of the return on the project of each firm, we assume that banks need to monitor firms. A firm succeeds if not only workers and firms do not shirk, but also bankers. Shirking for a banker brings about a benefit ϕ^B. Banks are then able to borrow $\rho_0(s) = r(s) - (\phi^B(s) + \phi^F(s) + \phi^W(s))$

per firm that they finance, and receive a share $\rho_1(s) = r(s) - (\phi^F(s) + \phi^W(s))$ of the return of each firm.

A.2 The Role of Leverage

In this section, we introduce leverage into the model. We assume that a fraction $\rho_0(s)I(s)$ of the return $\rho_1(s)I(s)$ is pledgeable to outside international investors at date 1. Bankers can now raise $\rho_0(s) < 1$ units of funds per unit of investment at date 1. Consistent with our previous assumptions, we assume that financial claims on $\rho_0(s)$ are issued abroad. This can be accommodated by our formalization along the lines of Holmström-Tirole (1997) (see Appendix A.1). Because we assume away the possibility of debt forgiveness, there is no supervisory leniency. We make use throughout of the fact that $\underline{b}_0^{**} = \underline{b}_0^*$.

Leverage and financial shocks. Because bankers can lever up, they only need a net worth of $I(s)(1 - \rho_0(s))$ in order to invest $I(s)$. As a result, the required bailout is now

$$X(s) = \max\{I(s)(1 - \rho_0(s)) - (b_0^* + p_1(s)b_0), 0\}.$$

The pricing equation (10) is unchanged, leading to the following fixed-point for the date-1 price $p_1(s)$ of government bonds

$$p_1(s) = 1 - F(B_1(s)|s),$$

where

$$B_1(s) = B_0 + \max\{\frac{I(s)(1 - \rho_0(s)) - \underline{b}_0^*}{p_1(s)} - \frac{A - \underline{b}_0^*}{p_0}, 0\}.$$

Proposition 12 (Feedback Loop and Leverage)**.** *When a fraction $\rho_0(s)$ of the date-2 return of the investment project of bankers is pledgeable, the sensitivity of date-1 bond prices $p_1(s)$ to the state s when a bailout is required is given by*

$$\frac{dp_1(s)}{ds} = \frac{-\frac{\partial F(B_1(s)|s)}{\partial s} - \frac{1}{p_1(s)} f(B_1(s)|s) \frac{d[(1-\rho_0(s))I(s)]}{ds}}{1 - \frac{I(s)(1-\rho_0(s)) - \underline{b}_0^*}{p_1^2(s)} f(B_1(s)|s)}.$$

Proposition 12 extends Proposition 1 to the case where leverage is positive. The main difference is that the financing needs $I(s)$ are replaced by $I(s)(1 - \rho_0(s))$. This is simply because bankers can leverage every unit of bailout with private funds by borrowing $\rho_0(s)$ units of funds from international investors.

Joint defaults. So far we have ignored the possibility that private debt contracts of bankers might be defaulted upon. In other words, we have assumed that the enforcement of private debt contracts is perfect. In reality, whether or not to enforce private contracts is to a large extent a decision by the domestic government. And the decisions to enforce private debt contracts and to repay sovereign debt tend to be correlated. After all, not enforcing private debt contracts is another way for the government to default on the country's obligations.[40] We capture this idea by assuming that the costs of not enforcing debt contracts and to default on sovereign debt take the form of a single fixed cost. This feature builds in a complementarity between the two decisions. As a result, sovereign defaults come together with defaults on the private debt contracts issues by bankers, resulting in a positive correlation between bank and sovereign spreads.

Private debt contracts are priced fairly and reflect the probability that they will not be enforced. As a result, leverage becomes endogenous. Entrepreneurs can raise $\rho_0(s)p_1(s)$ units of funds per unit of investment. The fact that the debt that they raise bears enforcement risk limits their ability to raise funds at date 1, and increases the size of the required bailout to

$$X(s) = \max\{I(s)(1 - \rho_0(s)p_1(s)) - (b_0^* + p_1(s)b_0), 0\}.$$

The pricing equation (10) is unchanged, leading to the following fixed-point for the date-1 price $p_1(s)$ of government bonds

$$p_1(s) = 1 - F(B_1(s)|s),$$

where

$$B_1(s) = B_0 + \max\{\frac{I(s)(1 - \rho_0(s)p_1(s)) - \underline{b}_0^*}{p_1(s)} - \frac{A - \underline{b}_0^*}{p_0}, 0\}.$$

Proposition 13 (Feedback Loop and Joint Defaults)**.** *When a fraction $\rho_0(s)$ of the date-2 return of the investment project of bankers is pledgeable and private debt contracts are defaulted upon when there is a sovereign default, the sensitivity of date-1 bond prices $p_1(s)$ to the state s*

[40] In our model, private financial contracts are between domestic agents (bankers) and foreign agents (international investors). A more general model would also feature private financial contracts between domestic agents. To the extent that enforcement decisions cannot discriminate between contracts based on the identities of the parties to the contract, this introduces potential additional costs to the decision of not enforcing private contracts. These costs are both ex-post in the form of undesirable redistribution and ex-ante in the form of a reduction in private trade between domestic agents (see e.g. Broner and Ventura 2011). We purposefully stay away from these fascinating issues, which are not the focus of this paper.

when a bailout is required is given by

$$\frac{dp_1(s)}{ds} = \frac{-\frac{\partial F(B_1(s)|s)}{\partial s} - \frac{1}{p_1(s)} f(B_1(s)|s) \frac{d[(1-\rho_0(s)p_1(s))I(s)]}{ds}}{1 - \frac{I-\underline{b}_0^*}{p_1^2(s)} f(B_1(s)|s)}.$$

There are two key differences between Proposition 13 and Proposition 12. The first difference is that the second term in the numerator is now $-\frac{1}{p_1(s)} f(B_1(s)|s) \frac{d[(1-\rho_0(s)p_1(s))I(s)]}{ds}$ instead of $-\frac{1}{p_1(s)} f(B_1(s)|s) \frac{d[(1-\rho_0(s))I(s)]}{ds}$, reflecting the dependence of the liquidity needs of bankers on $p_1(s)$ through the pledgeability of returns and leverage. The second difference is in the denominator. For given values of the date-1 bond price $p_1(s)$, of the reinvestment need $I(s)$, of the bailout $X(s)$, and hence of date-1 debt $B_1(s)$, the denominator is now smaller at $1 - \frac{I(s)-\underline{b}_0^*}{p_1^2(s)} f(B_1(s)|s)$ instead of $1 - \frac{I(s)(1-\rho_0(s)p_1(s))-\underline{b}_0^*}{p_1^2(s)} f(B_1(s)|s)$. As a result the sensitivity $\frac{dp_1(s)}{ds}$ of the price $p_1(s)$ to the state s is larger.

The feedback loop is stronger, because of a new mechanism operating through the endogenous leverage of banks. As sovereign risk rises, banks have to reduce leverage. This is because banks' borrowing spreads increase, reflecting the increased probability of a default on the private debt that they issue. This requires a larger bailout, which puts further pressure on the government budget etc., ad infinitum.

A.3 Limited Bailouts and Endogenous Diversification

So far, we have maintained the assumption that no matter what portfolios banks hold, the government can always raise enough funds at date 1 to bail them out completely. We now relax this assumption.[41] We show that when the government's ability to bail out the banking system is limited, banks naturally limit their exposure to domestic sovereign default risk.

We assume that there is no debt forgiveness. It is easy to see that the absence of supervisory leniency extends to the setting of this section. We therefore make use of the fact that $\underline{b}_0^{**} = \underline{b}_0^*$ throughout.

To simplify, we assume that $I(s) = \bar{I}$ is independent of s so that there are no financial shocks but only fiscal shocks. Because $A \geq \bar{I}$, if banks choose $b_0^* = \bar{I}$, they do not need a bailout. But we assume that there are some states of the world where the government is not able to fully bail out banks if they choose $b_0^* = \underline{b}_0^*$. In states of the world s where

[41] Here it is important to restrict the concept of "banking union" to the notion of "shared supervision"; for, by providing a larger pool of bailout funds a banking union could increase incentives for risk shifting if the domestic government cannot raise enough funds itself.

funds are insufficient to bail out all the banks, the government optimally bails out as many banks as possible, saving first the banks with the highest pre-bailout net worth. This pecking order maximizes the number of banks that can be saved and hence ex-post welfare.

While banks are ex-ante identical, equilibria can be asymmetric. We therefore look for an equilibrium in which bankers invest different amounts in foreign bonds, according to a probability distribution with G with support contained in $[\underline{b}_0^*, \bar{I}]$. This probability distribution G is an endogenous object, to be solved for as part of the equilibrium. It might be a degenerate atom, in which case the equilibrium is symmetric.

In every state s, there is an endogenous threshold $b_0^*(s)$ such that bankers with $b_0^* \geq b_0^*(s)$ secure enough post-bailout funds to finance their investment. This threshold is monotonically decreasing in s. There is also an endogenous threshold $\tilde{b}_0^*(s) \geq b_0^*(s)$ such that bankers with $b_0^* \geq \tilde{b}_0^*(s)$ can finance their investment without any bailout. This threshold is defined by $\bar{I} - \tilde{b}_0^*(s) - (A - \tilde{b}_0^*(s))\frac{p_1(s)}{p_0} = 0$, and is also monotonically decreasing in s.

In states where $b_0^*(s) > \underline{b}_0^*$ so that bailouts are partial, the following bailout equations must hold

$$\int_{b_0^* \in [b_0^*(s), \tilde{b}_0^*(s))} (A - b_0^*)\frac{1}{p_0} dG(b_0^*) = \frac{p_1(s)}{f(B_1(s)|s)} - [B_1(s) - B_0], \tag{24}$$

$$\int_{b_0^* \in [b_0^*(s), \tilde{b}_0^*(s))} [\bar{I} - b_0^* - (A - b_0^*)\frac{p_1(s)}{p_0}] dG(b_0^*) = [B_1(s) - B_0] p_1(s),$$

where

$$B_1(s) = B_0 + \int_{b_0^* \in [b_0^*(s), \tilde{b}_0^*(s))} \max\{\frac{I - b_0^*}{p_1(s)} - \frac{A - b_0^*}{p_0}, 0\} dG(b_0^*). \tag{25}$$

This simply guarantees that the government determines how much debt to issue at date 1 in order to maximize the number of banks that can be saved.[42] Note that the government necessarily issues less debt than the amount that would maximize the revenues from this issuance. This is because at the peak of the issuance Laffer curve (the value of $B_1(s)$ which maximizes $[B_1(s) - B_0][1 - F(B_1(s)|s)]$, a marginal reduction in issuance $B_1(s) - B_0$

[42] Indeed equation (24) is the first-order condition for the following planning problem:

$$b_0^*(s) = \min_{\{\hat{b}_0^*(s), B_1(s)\}} \hat{b}_0^*(s)$$

s.t.

$$\int_{b_0^* \geq \hat{b}_0^*(s)} \max\{I - b_0^* - (A - b_0^*)\frac{1 - F(B_1(s)|s)}{p_0}, 0\} dG(b_0^*) = [B_1(s) - B_0][1 - F(B_1(s)|s)].$$

brings about a second-order reduction in issuance revenues $[B_1(s) - B_0][1 - F(B_1(s)|s)]$ but a first-order improvement in banks' pre-bailout net worth $b_0^* + (A - b_0^*)\frac{1-F(B_1(s)|s)}{p_0}$, and hence a first-order reduction in required bailouts and by implication a first-order increase in the number of banks that can be saved.

In addition, the following pricing equations must hold

$$p_1(s) = 1 - F(B_1(s)|s), \tag{26}$$

$$p_0 = \int p_1(s) d\pi(s). \tag{27}$$

An individual banker who invests b_0^* gets a bailout in states $s > s(b_0^*)$ but no bailout in states $s < s(b_0^*)$, where $s(b_0^*)$ is the inverse of $b_0^*(s)$ and is hence monotonically decreasing in b_0^*. There is another threshold $\tilde{s}(b_0^*)$ such that the entrepreneur does not need a bailout to finance his investment when $s > \tilde{s}(b_0^*)$, where $\tilde{s}(b_0^*)$ is the inverse of $\tilde{b}_0^*(s)$ and is hence monotonically decreasing in b_0^*. This banker now faces a meaningful tradeoff in his portfolio decision. By increasing his investment b_0^* in foreign bonds, he secures a bailout in some states of the world where he did not get a bailout by rising in the government bailout pecking order, but loses out in states where he does not need a bailout to fund his investment. The corresponding optimality conditions states that b_0^* maximizes his welfare[43]

$$b_0^* \in \arg\max_{b_0^*(i)} V_0^B(b_0^*(i)),$$

where

$$V_0^B(b_0^*(i)) = \int \rho_1(s)\bar{I} d\pi(s) + \int_{\{s \geq \tilde{s}(b_0^*(i))\}} [b_0^*(i) + (A - b_0^*(i))\frac{p_1(s)}{p_0} - \bar{I}] d\pi(s)$$
$$+ \int_{\{s < s(b_0^*(i))\}} [b_0^*(i) + (A - b_0^*(i))\frac{p_1(s)}{p_0} - \rho_1(s)\bar{I}] d\pi(s).$$

The determination of equilibrium resembles that of equilibria of full-information first-

[43]For b_0^* in the interior of the support of G, π must be absolutely continuous with respect to the Lebesgue measure in the neighborhood of $s(b_0^*)$ with Radon-Nikodym derivative $d\pi(s) = \pi(s)ds$, and the entrepreneur must be left indifferent by marginal changes in b_0^*, which requires that the following differential equation in $s(b_0^*)$ hold on the interior of the support of G:

$$-s'(b_0^*)\pi(s(b_0^*)) \left[\rho_1(s(b_0^*))\bar{I} - \left(b_0^* + (A - b_0^*)\frac{p_1(s(b_0^*))}{p_0}\right)\right] = \int_{\tilde{s}(b_0^*)}^{\infty} \left(\frac{p_1(s)}{p_0} - 1\right) \pi(s) ds.$$

The left-hand-side represents the marginal utility gain from securing bailouts in more states of the world, while the right-hand-side represents the utility loss in states where no bailout is required to fund the investment.

price auctions or wars of attrition. The complication here comes from the fact that the object that competitors vie for—here subsidies—is itself endogenous, as from equation (24), the pot of subsidies depends on the distribution of "bids", namely the holdings of foreign bonds.

An interesting feature of these equilibria is that they display a force for endogenous diversification. Bankers choose to hold foreign bonds even in the absence of regulation. This is because they cannot be certain to count on a government bailout. We illustrate this possibility with two simple examples in Section B.7. In the first example, the distribution G is a degenerate atom. In the second example, it is non-degenerate. In both cases, we abstract away from regulation and set $\underline{b}_0^* = 0$.[44]

A.4 Foreign Banks in the Foreign (Safe) Country

In our basic model, we abstracted from foreign banks in the foreign (safe) country. We can introduce such banks. These face a similar problem to domestic banks. They have some net worth A^F at date 0, and some investment opportunities $I^F(s)$ at date 1 with private and foreign social returns given by $\rho_1^F(s)$ and $\beta^{I,F}(s)$. We assume that with $A^F \geq \bar{I}^F$ where $\bar{I}^F = \max_{s \in S} I^F(s)$. Foreign banks invest their net worth at date 0 in a portfolio of risky "domestic" bonds (bonds of the domestic economy) and safe "foreign" bonds (bonds of the foreign economy).[45] The return on their portfolio at date 1 determines their net worth at date 1. If it falls short of their investment need, then they are bailed out by the foreign government. But these bailouts do not endanger the ability of the foreign government to repay its debt. The domestic and foreign countries differ only in the riskiness of their sovereign bonds. Domestic sovereign bonds are risky and foreign sovereign bonds are safe. We denote by $\underline{b}_0^{F*}$ the supervisory capability of the foreign government, and by $\underline{b}_0^{F**}$ the effective minimum diversification requirement.

[44]Another form of bailout rat race is developed in Nosal-Ordonez (2014). In their paper as in ours, the government ex ante dislikes bailing out banks, but cannot help doing so when faced with the fait accompli. The innovation of their paper is that a bank can be rescued either by the government or (more cheaply) by a healthy bank and the government prefers a private takeover to a public takeover. The government however does not know whether the first distressed bank's shock is idiosyncratic or aggregate (in which case there will be no healthy bank to rescue the distressed one). In a situation in which the conditional probability of an aggregate shock is not too large, the government waits, and therefore banks prefer not to be the first distressed institution. If they can sink resources to augment the probability of not being first, they will do so, a behavior akin to a rat race.

[45]Of course domestic bonds are foreign bonds from the perspective of foreign banks, and similarly foreign bonds are domestic bonds from the perspective of foreign banks. To avoid confusion, we always refer to domestic bonds as the sovereign bonds of the domestic economy, independently of whether they are held by domestic or foreign agents. Similarly, we refer to foreign bonds as the bonds of the foreign economy, independently of whether they are held by domestic of foreign agents.

Our analysis goes through in this extended model as long as risk-neutral international investors who do not benefit from bailout guarantees (not foreign banks) remain the marginal buyers of domestic and foreign sovereign bonds. In particular, Propositions 1-9 still hold without any modification. The key observation is that foreign banks' portfolio decisions are irrelevant for equilibrium prices, domestic bailouts and sovereign default probabilities, and domestic banks' portfolio decisions. Foreign banks' risk exposures do not give rise to any feedback loop, because the foreign government has enough fiscal capacity to bail them out without endangering its ability to repay its debt.

The extended model has additional predictions on the incentives of foreign banks and of the foreign government. Because of the bailout guarantees, foreign banks have an incentive to load up on risky domestic debt. The foreign government has an incentive to regulate foreign banks so that they do not take on too much domestic sovereign risk. We elaborate on these issues now.

Frictionless supervision of foreign banks in the foreign (safe) country. We first consider first optimal supervision in the foreign (safe) country as in Section 3.3. We can derive the following equivalent of Propositions 2 and 3.

Proposition 14 (Supervision in Foreign (Safe) Country)**.** *When the basic model in Section 3.3 is extended to include foreign banks in the foreign (safe) country, there is no supervisory leniency in the foreign (safe) country so that $\underline{b}_0^{F**} = \underline{b}_0^{F*}$. Moreover when the supervisory capability $\underline{b}_0^{F*}$ can be chosen at no cost by the foreign government (first-best frictionless supervision), it is optimal to set $\underline{b}_0^{F*} = \bar{I}^F$. All these statements are true whether or not debt forgiveness is allowed.*

Just like the domestic government in the absence of debt forgiveness, the foreign government has an incentive to prevent its banks from taking on (domestic) sovereign risk. This is because when foreign banks take on more risk, they receive a bailout from the foreign government following a bad shock, which has adverse distributional effects.

In the presence of debt forgiveness, Proposition 14 displays a sharp contrast between the supervisory incentives of the domestic government and those of the foreign government. Because foreign government debt is safe, the foreign government cannot extract any concessions from its creditors. As a result, the foreign government has no incentive to engage in strategic supervisory leniency. Instead it always seeks to strictly limit the exposure of foreign banks to domestic sovereign risk. We refer the reader to Section B.6 for additional results in the context of the simple example of Section B.1 which can be solved in closed form. In particular, we uncover an additional rationale for a banking union in the presence of costly or imperfect supervision: Domestic supervision has positive external effects for the foreign country, these effects are not internalized by the domestic government, and as a result, supervision is too lax in the domestic economy.

B Illustration of Some Results and Additional Results in a Simple Example

In this appendix, we construct a simple example of our general setup, which can be solved in closed form. We then put to it to use to illustrate some of the results in the paper, and to derive additional results.

B.1 Illustrative Example

We assume that $I(s) = \bar{I} = A$ and $\rho_1(s) = \rho_1$ for all s, and that $\underline{b}_0^* \leq A$. The structure of uncertainty is as follows. With probability π, the state s is H and the endowment is high enough at E that there is no default. With probability $1-\pi$, the state is L and the endowment is high enough at E so that there is no default with conditional probability x, intermediate e with conditional probability y, and 0 with conditional probability $1-x-y$. In addition, we assume that $e > B_0$.

For $E \geq B_1(L) > e$, we have $1 - F(B_1(L)|L) = x$ and so $p_1(L) = x$ and $p_0 = \pi + (1-\pi)x$. For $e \geq B_1(L) \geq 0$, we have $1 - F(B_1(L)|L) = x + y$ and so $p_1(L) = x + y$ and $p_0 = \pi + (1-\pi)(x+y)$. Depending on which of $(E - B_0)x$ and $(x+y)(e - B_0)$ is greater, the level of debt $B_1(L)$ that maximizes revenue in state L is either E or e.

B.2 Multiple Equilibria in a Simple Example for Section 3.5

In this section, se show that the strategic complementarities identified in Section 3.5 can lead to multiple equilibria. We demonstrate this possibility in the simple example introduced in Section B.1, which can be solved in closed form. We also provide global comparative statics results in the context of this example.

We assume that $(E - B_0)x > (x+y)(e - B_0)$ so that the revenue maximizing level of debt $B_1(L)$ in state L is E. We assume throughout that $(\Psi')^{-1}(\frac{\pi(1-\pi)(1-\theta)}{\pi+(1-\pi)\theta}) \in (0, \underline{b}_0^*)$ for $\theta \in \{x, x+y\}$. There are two possible equilibria depending on whether $B_1(L) \leq e$ or $B_1(L) > e$, which determines the probability θ of repayment in state L. When $B_1(L) \leq e$, we have $\theta = x + y$, and when $B_1(L) > e$, we have $\theta = x$. And prices are given by $p_1(L) = \theta$, $p_0 = \pi + (1-\pi)\theta$.

The welfare of a banker i who invests $b_0^*(i)$ is

$$\pi[\rho_1 A + (A - b_0^*(i))(\frac{1}{p_0} - 1)] + (1-\pi)\rho_1 A = \rho_1 A + \pi(A - b_0^*(i))\frac{(1-\pi)(1-\theta)}{\pi + (1-\pi)\theta} - \Psi(\underline{b}_0^* - b_0^*(i)).$$

In order for bankers to choose $b_0^* \in (0, \underline{b}_0^*)$, we must have

$$\Psi'(\underline{b}_0^* - b_0^*(i)) = \frac{\pi(1-\pi)(1-\theta)}{\pi + (1-\pi)\theta}.$$

The debt issuance condition is then

$$B_1(L) = B_0 + \Phi(\theta),$$

where Φ is a decreasing function defined by

$$\Phi(\theta) = \frac{1}{\theta}\frac{\pi(1-\theta)}{\pi + (1-\pi)\theta}[A - \underline{b}_0^* + (\Psi')^{-1}(\frac{\pi(1-\pi)(1-\theta)}{\pi + (1-\pi)\theta})].$$

We have an equilibrium with $B_1(L) \leq e$ if and only if

$$\Phi(x+y) \leq e - B_0. \tag{28}$$

Similarly, we have an equilibrium with $B_1(L) > e$ if and only if

$$e - B_0 < \Phi(x) \leq E - B_0. \tag{29}$$

The two equilibria coexist if and only if

$$E - B_0 \geq \Phi(x) > e - B_0 \geq \Phi(x+y). \tag{30}$$

Because the function Φ is decreasing, we can always find values of B_0, E and e such that condition (30) is verified so that there can be multiple equilibria for a range of parameter values. These multiple equilibria are a consequence of the strategic complementarities in the banks' individual exposures to domestic sovereign default risk.

Proposition 15 (Multiple Equilibria)**.** *In the illustrating example, there are two possible equilibria. There is an equilibrium with low diversification $b_0^* = \underline{b}_0^* - (\Psi')^{-1}(\frac{\pi(1-\pi)(1-x)}{\pi+(1-\pi)x})$ and a high probability of default $(1-\pi)(1-x)$, which exists if and only if condition (28) is verified. There is also an equilibrium with high diversification $b_0^* = \underline{b}_0^* - (\Psi')^{-1}(\frac{\pi(1-\pi)(1-x-y)}{\pi+(1-\pi)(x+y)})$ and a low probability of default $(1-\pi)(1-x-y)$, which exists if and only if condition (29) is verified.*

The two equilibria coexist if and only if condition (30) is verified.

This example also has other interesting implications.

Proposition 16 (Multiple Equilibria and Debt Renationalization)**.** *In the illustrating example, for $B_0 \in (0, E - \Phi(x))$, the equilibrium with low diversification and high probability of default is more likely to exist, the higher is legacy debt B_0 and the lower is fiscal capacity (proxied by the intermediate value of the endowment e). Conversely, the equilibrium with high diversification and low probability of default is more likely to exist, the lower is legacy debt and the higher is fiscal capacity.*

B.3 Strategic Supervisory Leniency in a Simple Example for Section 4.2

In this section, we provide an illustration of the results in Section 4.2 in the context of the simple example introduced in Section B.1, which can be solved in closed form.

Recall that in this example, $I(s) = \bar{I}$ is independent of s and $A = \bar{I}$. We assume that $e(1 + \frac{y}{x}) > B_0 > e$ and that $\beta^I(s) = \beta^I$ is independent of s. We assume that supervisory capability is not too low $A - \frac{\pi+(1-\pi)x}{\pi(1-x)}[e(x+y) - xB_0] \leq \underline{b}_0^*$. We now proceed to construct an equilibrium where it is optimal for the government $\underline{b}_0^{**} < \underline{b}_0^*$ so as to obtain concessions from legacy creditors.

There is no debt forgiveness in state H and no default. At date 1, in state L, legacy creditors either forgive no debt so that $B_0(L) = B_0$ or forgive debt $B_0(L) < B_0$ in the following amount

$$B_0(L) + \frac{[1 - \frac{B_0(L)}{B_0}\frac{x+y}{p_0}](A - \underline{b}_0^{**})}{x+y} = e, \tag{31}$$

in which case $B_1(L) = e$. There is debt forgiveness provided that when $B_0(L)$ is defined by equation (31), the following condition is verified:

$$(x+y)B_0(L) \geq xB_0. \tag{32}$$

To summarize, using equations (31) and (32), there is debt forgiveness in state L if

$$\frac{x}{x+y}B_0 + \frac{(1 - \frac{x}{p_0})(A - \underline{b}_0^{**})}{x+y} \leq e. \tag{33}$$

In order to maximize welfare

$$\mathcal{W}_0 = \pi(E - B_0) + (1-\pi)[x(E-e) - (1-x-y)\Phi] + \beta^I A + \beta^B[\rho_1 A + \pi(A - \underline{b}_0^{**})(\frac{1}{p_0} - 1)]. \quad (34)$$

It is then always optimal for the government to choose at date 0 the lowest value of $\underline{b}_0^{**}$ that satisfies equation (33). This defines an increasing function

$$\underline{b}_0^{**}(p_0) = A - \frac{x+y}{1-\frac{x}{p_0}}(e - \frac{x}{x+y}B_0).$$

The date-0 price is then given by $p_0 = \pi + (1-\pi)x$.[46] And the equilibrium effective diversification requirement is then $\underline{b}_0^{**} = \underline{b}_0^{**}(\pi + (1-\pi)x)$ which is guaranteed to be less than $\underline{b}_0^{*}$ by our assumption that supervisory capability is not too low.

Proposition 17 (Strategic Supervisory Leniency)**.** *In the illustrating example, it is optimal for the government to engage in strategic supervisory leniency by setting $\underline{b}_0^{**} = \underline{b}_0^{**}(\pi + (1-\pi)x) < \underline{b}_0^{*}$. The equilibrium effective diversification requirement $\underline{b}_0^{**}$ is decreasing in the probability $1-\pi$ of the occurrence of the bad fiscal shock (state L).*

The government reduces the effective diversification requirement (lowers $\underline{b}_0^{**}$) when the probability $1-\pi$ of a bad fiscal shock where a debt renegotiation takes place increases, because it makes it more attractive to extract concessions from legacy creditors.

B.4 Rationale for a Banking Union in a Simple Example for Section 4.3

In this section, we provide an illustration of the results in Section 4.3 in the context of the simple example introduced in Section B.1, which can be solved in closed form.

We build on Section B.3. Consider the debt level $B_0' > B_0$ that generates the same amount of revenue at date 0 when the effective diversification requirement is $\underline{b}_0^{**\prime} = \underline{b}_0^{**}(\pi + (1-\pi)x)$ as the debt level B_0 when the effective diversification requirement is $\underline{b}_0^{**} = \underline{b}_0^{*}$. This debt level is defined implicitly by the equation

$$[\pi + (1-\pi)x]B_0' = p_0(\underline{b}_0^{*}; \pi, B_0)B_0,$$

where we assume that the solution of this equation satisfies $e(1+\frac{t}{r}) > B_0' > e$, and where $p_0(\underline{b}_0^{*}; \pi, B_0)$ denotes the date-0 price when the effective diversification requirement is

[46] We have $p_0 = \pi + (1-\pi)(x+y)\frac{B_0(L)}{B_0}$, which using $(x+y)B_0(L) = xB_0$ implies $p_0 = \pi + (1-\pi)x$.

$\underline{b}_0^{**} = \underline{b}_0^*$.[47,48,49,50]

We necessarily have $\mathcal{W}_0 > \mathcal{W}_0'$.[51]

Proposition 18 (Banking Union). *If the relaxation of supervision is fully priced in by international investors at the time of the issuance of date-0 debt, then the domestic government faces a time-inconsistency problem. It is made better off by promising not to engage in supervisory leniency and to set a high effective diversification requirement $\underline{b}_0^{**} = \underline{b}_0^*$ before issuing debt at date 0, but it is tempted to relax this requirement after the issuance and lower $\underline{b}_0^{**}$ below $\underline{b}_0^*$. A banking union removes this temptation and improves welfare.*

B.5 Multiple Risky Countries in a Simple Example for Section 5.2

In this section, we illustrate the results of Section 5.2 in the context of the simple example introduced in Section B.1, which can be solved in closed form.

For simplicity, we carry out our multiple-country extension in the context of our illustrating example. There are two states H and L at date 1 for each country with probability π and $1-\pi$. Let k^H be the probability that country B is in state H if country A is in state

[47]The function $p_0(\underline{b}_0^*; \pi, B_0)$ is itself defined implicitly by the following equation

$$p_0 = \pi + (1-\pi)(x+y)\frac{e - \frac{A-\underline{b}_0^*}{x+y}}{B_0 - \frac{A-\underline{b}_0^*}{p_0}}. \tag{35}$$

This equation has a unique solution (the left-hand side is increasing in p_0 while the right-hand side is decreasing in p_0), which defines a function $p_0(\underline{b}_0^*; \pi, B_0)$ which is increasing in $\underline{b}_0^*$, decreasing in B_0 and increasing in π.

[48]The function $p_0(\underline{b}_0^*; \pi, B_0)$ is locally increasing in $\underline{b}_0^*$ if and only if $e(x+y) < p_0(\underline{b}_0^*; \pi, B_0)B_0$. It is easy to see that this inequality automatically holds when $\underline{b}_0^* = A$. This implies that it holds for all $\underline{b}_0^*$. Indeed, suppose that there exists $\underline{b}_0^* < A$ such that $e(x+y) > p_0(\underline{b}_0^*; \pi, B_0)B_0$. Then as we increase $\underline{b}_0^*$ from that point towards A, $p_0(\underline{b}_0^*; \pi, B_0)$ keeps decreasing and hence $e(x+y) > p_0(\underline{b}_0^*; \pi, B_0)B_0$ keeps being verified, a contradiction. Therefore $e(x+y) \leq p_0(\underline{b}_0^*; \pi, B_0)B_0$ for all $\underline{b}_0^*$. This in turn implies that $p_0(\underline{b}_0^*; \pi, B_0)$ is increasing in $\underline{b}_0^*$.

[49]That the function is decreasing in B_0 follows from the fact that the left-hand side of equation (35) is increasing in p_0 and independent of B_0, while the right-hand side is decreasing in p_0 and decreasing in B_0.

[50]That the function is increasing in π follows from the fact that the left-hand side of equation (35) is increasing in p_0 and independent of π, while the right-hand side is decreasing in p_0 and increasing in π. To see that the right-hand side of equation (35) is increasing in π, rewrite the right-hand side using equation (35) as $\pi + (1-\pi)(x+y)\frac{B_0(L) - \frac{A-\underline{b}_0^*}{p_0}}{B_0 - \frac{A-\underline{b}_0^*}{p_0}}$ where $B_0(L) \leq B_0$.

[51]This is immediate since under commitment and no commitment, all investments are financed, defaults occur in the same states, and foreigners are as well off. As a result, the sum of consumer welfare and banker welfare is the same under commitment and no-commitment $V_0^{C\prime} + V_0^{B\prime} = V_0^C + V_0^B$. However the welfare of bankers is higher and that of consumers lower under no commitment $V_0^B < V_0^{B\prime}$ and $V_0^C > V_0^{C\prime}$. Because $\beta^B < 1$, this implies that $W_0 = W_0^C + \beta^B V_0^B + \beta^I A - (1-\pi)(1-x-y)\Phi$ is greater than $W_0' = V_0^{C\prime} + \beta^B V_0^{B\prime} + \beta^I A - (1-\pi)(1-x-y)\Phi$.

H. Let k^L be the probability that country B is in state L if country A is in state L. Symmetry imposes that

$$(1-\pi)(1-k^L) = \pi(1-k^H).$$

We assume that $k^H < 1$ and $k^L < 1$ so that shocks in the two countries are not perfectly correlated.

In state H in country $i \in \{A, B\}$, date-2 fiscal revenues are equal to E with probability 1, and investment needs are equal to $\underline{I}$ and the return on investment is equal to ρ_1^H. In state L in country $i \in \{A, B\}$, date-2 fiscal revenues are equal to E with probability x, e with probability y, and 0 with probability $1-x-y$, investment needs are equal to $\bar{I}$, and the return on investment is equal to ρ_1^H. We assume that $\underline{I} < \bar{I} = A$. Hence fiscal and balance sheet shocks are positively correlated in a given country. As will be clear below, it is important for our results that $\underline{I} < \bar{I}$, but the size of the gap between $\underline{I}$ and $\bar{I}$ is not important. In other words, it only matters that there be some positive correlation between balance sheet and fiscal shocks. Although this is not important, we assume that $\rho_1^H \underline{I} > \rho_1^L \bar{I}$ so that state H (respectively L) corresponds to a state with high (respectively low) future profits but low (respectively high) liquidity needs.

Home bias with multiple risky countries. We assume that E is large enough so that $p_1(H,H) = 1$ and $p_1(H,L) = 1$. But we have $p_1(L,L) < 1$ and $p_1(L,H) < 1$. We show that as long as $p_1(L,H)$ is not too high, then banks in country j choose to hold as little safe foreign bonds and as much risky domestic bonds as allowed by supervision, but no risky foreign bonds.

Proposition 19 (Home Bias with Multiple Risky Countries)**.** *Consider the illustrating example with two symmetric risky countries $i \in \{A, B\}$. Then there exists a symmetric equilibrium in which $p_1(L,H) \leq p_0$ and banks in each country $i \in \{A, B\}$ choose to hold as little safe foreign bonds and as much risky domestic bonds as allowed by supervision, but no risky foreign bonds: $b_0^* = \underline{b}_0^*$, $b_0^d = \frac{A-b_0^*}{p_0}$ and $b_0^f = 0$.*[52] *This equilibrium is strict. Moreover, there are no other symmetric equilibria with $p_1(L,H) \leq p_0$.*

Proof. We show that in any symmetric equilibrium as long as $p_1(L,H) \leq p_0$, banks in each country $i \in \{A, B\}$ prefer to choose the following portfolio: $b_0^* = \underline{b}_0^*$, $b_0^d = \frac{A-b_0^*}{p_0}$

[52] The condition that $p_1(L,H) \leq p_0$ is equivalent to the assumption that there are bailouts when state L occurs in country i and state H occurs in country $-i$ if banks of country i choose portfolio $b_0^* = \underline{b}_0^*$, $b_0^d = \frac{A-b_0^*}{p_0}$ and $b_0^f = 0$. Proposition 19 then shows that there exists a unique symmetric equilibrium that satisfies this condition, and that in this equilibrium, banks choose the aforementioned portfolio.

and $b_0^f = 0$. Together with the fact that when banks do indeed choose this portfolio, $p_1(L,H) = p_1(L,L) < p_0 < p_1(L,H) = p_1(H,H) = 1$, this proves the proposition.

Consider a country $i \in \{A, B\}$. For the same reasons as in the main model, banks in country i will choose holdings of safe sovereign bonds of exactly $\underline{b}_0^*$. The payoff of a banker in country i from holding portfolio (b_0^d, b_0^f) with $b_0^d + b_0^f = b_0$ where $b_0 = \frac{A - \underline{b}_0^*}{p_0}$ is

$$\begin{aligned}
&\pi k^H[\rho_1^H \underline{I} + \max\{A - \underline{I} + (1-p_0)b_0^d + (1-p_0)b_0^f, 0\}] \\
&\quad + (1-\pi)k^L[\rho_1^L \bar{I} + \max\{A - \bar{I} + (p_1(L,L) - p_0)b_0^d + (p_1(L,L) - p_0)b_0^f, 0\}] \\
&\quad\quad + \pi(1-k^H)[\rho_1^H \underline{I} + \max\{A - \underline{I} + (1-p_0)b_0^d + (p_1(L,H) - p_0)b_0^f, 0\}] \\
&\quad\quad\quad + (1-\pi)(1-k^L)[\rho_1^L \bar{I} + \max\{A - \bar{I} + (p_1(L,H) - p_0)b_0^d + (1-p_0)b_0^f, 0\}].
\end{aligned}$$

Only the last two terms of the expression above matters for portfolio choice of the banker. The sum of the last two terms is a convex function of b_0^d and b_0^f. The optimal portfolio is therefore necessarily a corner solution $(b_0, 0)$ or $(0, b_0)$. We now compute the value of the sum of the last two terms at these two corners.

For $b_0^d = b_0$, the value of the sum of the last two terms is

$$\begin{aligned}
&\pi(1-k^H)[\rho_1^H \underline{I} + \max\{A - \underline{I} + (1-p_0)b_0, 0\}] \\
&\qquad\qquad + (1-\pi)(1-k^L)[\rho_1^L \bar{I} + \max\{A - \bar{I} + (p_1(L,H) - p_0)b_0, 0\}].
\end{aligned}$$

Since $A - \bar{I} + (p_1(L,H) - p_0)b_0 \leq 0$ (recall that $A = \bar{I}$), this can be re-expressed as

$$\pi(1-k^H)[\rho_1^H \underline{I} + A - \underline{I} + (1-p_0)b_0] + (1-\pi)(1-k^L)\rho_1^L \bar{I}.$$

For $b_0^d = 0$, the value of the sum of the last two terms is

$$\begin{aligned}
&\pi(1-k^H)[\rho_1^H \underline{I} + \max\{A - \underline{I} + (p_1(L,H) - p_0)b_0, 0\}] \\
&\qquad\qquad + (1-\pi)(1-k^L)[\rho_1^L \bar{I} + \max\{A - \bar{I} + (1-p_0)b_0, 0\}].
\end{aligned}$$

If $A - \underline{I} + (p_1(L,H) - p_0)b_0 \leq 0$, this can be re-expressed as

$$\pi(1-k^H)\rho_1^H \underline{I} + (1-\pi)(1-k^L)[\rho_1^L \bar{I} + A - \bar{I} + (1-p_0)b_0],$$

which is less than the value with $b_0^d = b_0$. If $A - \underline{I} + (p_1(L,H) - p_0)b_0 > 0$, this can be re-expressed as

$$\pi(1-k^H)[\rho_1^H \underline{I} + A - \underline{I} + (p_1(L,H) - p_0)b_0] + (1-\pi)(1-k^L)[\rho_1^L \bar{I} + A - \bar{I} + (1-p_0)b_0]$$
$$= \pi(1-k^H)[\rho_1^H \underline{I} + A - \underline{I} + (1-p_0)b_0] + (1-\pi)(1-k^L)[\rho_1^L \bar{I} + A - \bar{I} + (p_1(L,H) - p_0)b_0],$$

which is again less than the value for $b_0^d = b_0$.

Therefore the banker in country j chooses $b_0^d = b_0$ and $b_0^f = 0$. □

Strategic supervisory leniency with multiple risky countries. We assume that $e(1 + \frac{y}{x}) > B_0 > e$ and that $\beta^I(s) = \beta^I$ is independent of s. For simplicity, we consider the limit where $\underline{I} = \bar{I} = A$. We can derive the following counterpart to Proposition 17, which shows that governments in risky countries have an incentive to let their banks load up on risky domestic sovereign debt as opposed to risky foreign sovereign debt, in order to maximize the concessions from legacy creditors.

Proposition 20 (Strategic Supervisory Leniency with Multiple Risky Countries)**.** *Consider the illustrating example with two risky countries and assume that $B_0 < E$. In the limit where A is small compared to e and B_0, it is optimal for the government in country i to force its banks to invest all their net worth A in risky domestic sovereign bonds, and to invest zero in safe foreign sovereign bonds and zero in risky foreign sovereign bonds: $b_0^* = 0$, $b_0^d = \frac{A}{p_0}$ and $b_0^f = 0$.*

Proof. We consider a symmetric equilibrium, and denote with a tilde the equilibrium values, assuming, as we will verify below, that $\tilde{b}_0^f = 0$. And we look at the incentives of the government in country $i \in \{A, B\}$ to deviate from this equilibrium. Banks in country i invest in portfolio (b_0^d, b_0^f, b_0^*) with $b_0^d + b_0^f = b_0$ and $p_0 b_0^d + \tilde{p}_0 b_0^f + b_0^* = A$. The values of b_0^d, b_0^f and b_0^* are controlled by the government in country i.

We assume that E is large enough that the price of debt in a given country is always one when this country is in state H (this condition is guaranteed to hold in the limit where A is small compared to e and B_0).

If we have debt forgiveness in state (L, H), then the post-debt forgiveness amount of debt $B_0(L, H)$ satisfies

$$B_0(L,H) + \frac{p_0 - (x+y)\frac{B_0(L,H)}{B_0}}{x+y} b_0^d + \frac{\tilde{p}_0 - 1}{x+y} b_0^f = e,$$

and $B_1(L, H) = e$. If we have debt forgiveness in state (L, L), then the post-debt forgiveness amount of debt $B_0(L,L)$ satisfies

$$B_0(L,L) + \frac{p_0 - (x+y)\frac{B_0(L,L)}{B_0}}{x+y} b_0^d + \frac{\tilde{p}_0 - (x+y)\frac{\tilde{B}_0(L,L)}{B_0}}{x+y} b_0^f = e,$$

and $B_1(L,L) = e$.

There is debt forgiveness provided that the following conditions are verified:

$$(x+y)B_0(L,H) \geq xB_0,$$

and

$$(x+y)B_0(L,L) \geq xB_0.$$

These conditions are always verified when A is small enough compared to e and B_0,

It is then always optimal for the government to choose at date 0 the values of (b_0^d, b_0^f) that maximize welfare (taking p_0 as given)

$$\begin{aligned}
\mathcal{W}_0 = \pi(E - B_0) + (1-\pi)[x(E-e) - (1-x-y)\Phi] + \beta^I A + \beta^B \rho_1 A \\
+ \pi k^H G\{(1-p_0)b_0^d + (1-\tilde{p}_0)b_0^f\} \\
+ (1-\pi)k^L H\{((x+y)\frac{B_0(L,L)}{B_0} - p_0)b_0^d + ((x+y)\frac{\tilde{B}_0(L,L)}{B_0} - \tilde{p}_0)b_0^f\} \\
+ \pi(1-k^H)G\{(1-p_0)b_0^d + ((x+y)\frac{\tilde{B}_0(L,H)}{B_0} - \tilde{p}_0)b_0^f\} \\
+ (1-\pi)(1-k^L)H\{((x+y)\frac{B_0(L,H)}{B_0} - p_0)b_0^d + (1-\tilde{p}_0)b_0^f\},
\end{aligned}$$

where $G(x) = \beta^B \max\{x, 0\} + \min\{x, 0\}$ and $H(x) = \beta^B \max\{x, 0\}$. This defines two functions $b_0^d(p_0)$ and $b_0^f(p_0)$.

The date-0 price p_0 is then given by the fixed-point equation

$$p_0 = \pi + (1-\pi)(x+y)\frac{(1-k^L)B_0(L,H) + k^L B_0(L,L)}{B_0},$$

with $B_0(L,H)$ and $B_0(L,L)$ defined as above with $b_0^d = b_0^d(p_0)$ and $b_0^f = b_0^f(p_0)$.

In the limit where A is small compared to e and B_0, the last four terms of the expression for welfare W_0 can be rewritten (up to a first order approximation),

$$\begin{aligned}
&\pi k^H G\{(1-(\pi+(1-\pi)(x+y)\frac{e}{B_0}))b_0^d+(1-(\pi+(1-\pi)(x+y)\frac{e}{B_0}))b_0^f\}\\
&+(1-\pi)k^L H\{((x+y)\frac{e}{B_0}-(\pi+(1-\pi)(x+y)\frac{e}{B_0}))b_0^d+((x+y)\frac{e}{B_0}-(\pi+(1-\pi)(x+y)\frac{e}{B_0}))b_0^f\}\\
&+\pi(1-k^H)G\{(1-(\pi+(1-\pi)(x+y)\frac{e}{B_0}))b_0^d+((x+y)\frac{e}{B_0}-(\pi+(1-\pi)(x+y)\frac{e}{B_0}))b_0^f\}\\
&+(1-\pi)(1-k^L)H\{((x+y)\frac{e}{B_0}-(\pi+(1-\pi)(x+y)\frac{e}{B_0}))b_0^d+(1-(\pi+(1-\pi)(x+y)\frac{e}{B_0}))b_0^f\}.
\end{aligned}$$

The solution is clearly $b_0^d = \frac{A}{\pi+(1-\pi)(x+y)\frac{e}{B_0}}$ and $b_0^f = 0$. □

Obviously, if the government could not determine the relative holdings of domestic and foreign risky bond holdings, but only impose an effective requirement $b_0^* \geq \underline{b}_0^{**}$ with $\underline{b}_0^{**} \leq b_0^*$, then we would obtain (in the limit where $\underline{I}$ tends to $\bar{I} = A$ from below) that it is optimal to set $\underline{b}_0^{**} = 0$. Banks would then by themselves load up on domestic risky bonds, choosing $b_0^d = \frac{A}{\pi+(1-\pi)(x+y)\frac{e}{B_0}}$ and $b_0^f = 0$. Proposition 20 shows the more interesting result that even if the supervisor could perfectly control the portfolios of banks, it would choose to encourage them to load up on domestic risky bonds.

B.6 Foreign Banks in the Foreign (Safe) Country in a Simple Example for Section A.4

In this section, we illustrate the results of Section A.4 in the context of the simple example of Section B.1, which can be solved in closed form. We also provide additional results.

First, note that specializing the model to the illustrating example, Proposition 14 can be used to show that as the probability $1-\pi$ of a bad domestic fiscal shock increases, domestic supervision of domestic banks gets laxer, but foreign supervision of foreign banks does not, and as a result domestic banks tilt their portfolios towards risky domestic bonds and away from safe foreign bonds, but foreign banks do not.

Collective moral hazard and foreign banks in the foreign (safe) country. It is also interesting to investigate the portfolio decisions of foreign banks in the environment of Section 3.5, assuming that foreign banks face a cost of making their balance sheets opaque Ψ^F similar to that of domestic banks and that $I^F(s) = \bar{I}^F$ is independent of s and that $A^F = \bar{I}^F$. Using the fact that $\underline{b}_0^{F**} = \underline{b}_0^{F*}$, we can derive the following equivalent of Proposition 15.

Proposition 21 (Multiple Equilibria)**.** *When the illustrating example of Section 3.5 is extended to include foreign banks in the foreign (safe) country, the portfolio of foreign banks is given by* $b_0^{F*} = \underline{b}_0^{F*} - (\Psi^{F\prime})^{-1}(\frac{\pi(1-\pi)(1-\theta)}{\pi+(1-\pi)(\theta)})$ *with* $\theta = x$ *in the low (domestic) diversification equilibrium and* $\theta = x + y$ *in the high (domestic) diversification equilibrium.*

Foreign banks' exposure to domestic sovereign risk is higher in the low (domestic) diversification equilibrium than in the high (domestic) diversification equilibrium.[53]

The key observation that underlies these results is that there are strategic complementarities running from domestic banks' to foreign banks, but no strategic complementarities running in the other direction. Indeed, when domestic banks increase their exposure to domestic sovereign risk, the benefits of doing so also increases for foreign banks. But when foreign banks increase their exposure to domestic sovereign risk, the benefits of doing for domestic banks remains unchanged. This is because the riskiness of domestic debt increases in the former case but not in the latter.

This also implies that there are supervisory externalities running from the domestic country to the foreign (safe) country but not vice versa. Indeed, suppose that at some supervisory cost R (respectively R^F), the domestic (respectively foreign) government can achieve perfect supervision with supervisory capability $\underline{b}_0^* = A$ (respectively $\underline{b}_0^{*F} = A^F$), in which case, because our example assumes that there is no debt forgiveness, we also have $\underline{b}_0^{**} = \underline{b}_0^* = A$ (respectively $\underline{b}_0^{**F} = \underline{b}_0^{*F} = A^F$). Otherwise, supervision is inexistent (Ψ and Ψ^F are both zero), so that banks can perfectly evade regulation. Assume that $B_0 + \frac{1}{x}\frac{\pi(1-x)}{\pi+(1-\pi)x}A > e > B_0$.

If the domestic government chooses to incur the supervisory cost R, we have $B_1(L) = B_0$ and $\theta = x + y$. Otherwise $B_1(L) = B_0 + \frac{1}{x}\frac{\pi(1-x)}{\pi+(1-\pi)x}A$ and $\theta = x$. In both cases, we have $p_1(L) = \theta$ and $p_0 = \pi + (1-\pi)\theta$.

The net gain $(1-\pi)(1-\beta^B)A^F\frac{\pi(1-\theta)}{\pi+(1-\pi)\theta} - R^F$ from incurring the supervisory cost for the foreign government is lower ($\theta = x$) when the domestic government incurs the supervisory cost than when it doesn't ($\theta = x + y$). By contrast, the net gain from incurring the supervisory cost for the domestic government is independent of whether or not the domestic government incurs the supervisory cost. More interestingly, we have the following proposition.

Proposition 22 (Supervisory Externalities and Banking Union)**.** *In the illustrating example with either perfect or irrelevant supervision and foreign banks in the foreign (safe) country, foreign welfare increases with the supervisory effort (decreases with the supervisory cost) of the*

[53] Note that contrary to domestic and foreign banks, international investors have less exposure to domestic sovereign risk in the low (domestic) diversification equilibrium than in the high (domestic) diversification equilibrium.

domestic country, but domestic welfare is independent of the supervisory effort (independent of the supervisory cost) of the foreign country.

Proposition 22 uncovers an additional rationale for a banking union. Domestic supervision has positive external effects for the foreign country. These effects are not internalized by the domestic government, and as a result, supervision is too lax in the domestic economy. By transferring supervisory decisions from the national to the international level, a banking union allows these effects to be internalized, leading to a toughening of supervision in the domestic country and an improvement of welfare.

B.7 Limited Bailouts and Endogenous Diversification in a Simple Example

In this section, we provide two illustrations of limited bailouts and endogenous diversification as outlined in Section A.3, in the context of the simple example introduced in Section 2.1, which can be solved in closed form. In the first example, the distribution G is a degenerate atom. In the second example, it is non-degenerate. In both cases, we abstract away from regulation and set $\underline{b}_0^* = 0$.

Illustrating example 1. Our first example is a variant of the example in Section 2.1. We assume that $(E - B_0)x < (x + y)(e - B_0)$, so that the revenue maximizing level of $B_1(L)$ in state L is e.

Our candidate equilibrium is symmetric with $B_1(L) = e$, $p_1(L) = x + y$ and $p_0 = \pi + (1 - \pi)(x + y)$.[54] The limited-bailout condition is

$$\frac{\pi(1 - x - y)}{\pi + (1 - \pi)(x + y)}(A - b_0^*) = (e - B_0)(x + y). \tag{36}$$

In order for bankers to prefer b_0^* to 0, we must have

$$\pi[\frac{1}{\pi + (1 - \pi)(x + y)} - 1]b_0^* \leq (1 - \pi)A[\rho_1 - \frac{x + y}{\pi + (1 - \pi)(x + y)}]. \tag{37}$$

The solution b_0^* of equation (36) always (strictly) verifies equation (37). This guarantees that our candidate equilibrium is indeed an equilibrium as long as the solution of equation (36) verifies $0 < b_0^* < A$.

[54] It can be shown that there are no asymmetric equilibria in this example.

Illustrating example 2. We now consider a simple variant of the previous example. The structure of uncertainty is as follows. With probability π, the state s is H and the endowment is high enough at E that there is no default. With probability $(1-\pi)z$, the state is M, and the endowment is high enough at E so that there is no default with conditional probability x, intermediate e_M with conditional probability y, and 0 with conditional probability $1-x-y$. With probability $(1-\pi)(1-z)$, the state is L, and the endowment is high enough at E so that there is no default with conditional probability x, intermediate e_L with conditional probability y, and 0 with conditional probability $1-x-y$. What distinguishes states M and L is that $e_M > e_L$. We assume that $(x+y)(e_L - B_0) > (E - B_0)x$ so that the revenue maximizing level of debt is e_M in state M and e_L in state L.

Our candidate asymmetric equilibrium is such that there are full bailouts in the medium state, but limited bailouts in the low state. Bankers invest $\hat{b}_0^*(L)$ with probability ϕ and 0 with probability $1-\phi$. Prices are $p_0 = \pi + (1-\pi)(x+y)$, $p_1(L) = p_1(M) = x+y$.

The bailout conditions are

$$\phi \frac{\pi(1-x-y)}{\pi + (1-\pi)(x+y)} (A - \hat{b}_0^*(L)) = (e_L - B_0)(x+y), \tag{38}$$

$$\phi \frac{\pi(1-x-y)}{\pi + (1-\pi)(x+y)} (A - \hat{b}_0^*(L)) + (1-\phi) \frac{\pi(1-x-y)}{\pi + (1-\pi)(x+y)} A \le (e_M - B_0)(x+y). \tag{39}$$

In order for bankers to be indifferent between $b_0^* = \hat{b}_0^*(L)$ and $b_0^* = 0$, we must have

$$\pi[\frac{1}{\pi + (1-\pi)(x+y)} - 1]\hat{b}_0^*(L) = A(1-z)(1-\pi)[\rho_1 - \frac{x+y}{\pi + (1-\pi)(x+y)}]. \tag{40}$$

We can rewrite equation (40) as

$$\hat{b}_0^*(L) = A(1-z)[(\rho_1 - 1)\frac{\pi + (1-\pi)(x+y)}{\pi(1-x-y)} + 1].$$

Using equation (38), we find

$$\phi = \frac{e_L - B_0}{A} \frac{(x+y)\frac{\pi + (1-\pi)(x+y)}{\pi(1-x-y)}}{1 - (1-z)[(\rho_1 - 1)\frac{\pi + (1-\pi)(x+y)}{\pi(1-x-y)} + 1]}.$$

We have an equilibrium if $\hat{b}_0^*(L) < A$, $0 < \phi < 1$, and

$$(1-\phi)\frac{\pi(1-x-y)}{\pi + (1-\pi)(x+y)} A \le (e_M - e_L)(x+y),$$

which can always be ensured for appropriate parameter values.

Proposition 23 (Bailout Rat-Race and Incentives for Diversification)**.** *In the illustrating examples with limited bailouts and symmetric or asymmetric equilibria, it is optimal for banks to not fully load up on domestic sovereign default risk and instead choose a non-zero degree of diversification* $b_0^* > 0$ *with positive probability even when there is no regulation (*$\underline{b}_0^* = 0$*).*