Population Growth, the Natural Rate of Interest, and Inflation[*]

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Abstract

Recent decades have seen considerable variations in population growth rates across developed countries. In this paper, I include fertility shocks in a dynamic stochastic general equilibrium model. The impulse responses, in particular that of the natural rate of interest, to a fertility shock crucially depend on how the representative household weights generations of different size across time. Matching the model-based and the empirical impulse responses to a fertility shock, I find that a one percentage point increase in the population growth rate raises the natural rate of interest by 0.59 percentage points. This has two implications. First, the optimal monetary policy response to a positive fertility shock is to raise the nominal interest rate. Second, the decline in the US inflation rate during the 1980s and 1990s may be partially attributable to negative fertility shocks.

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1 Introduction

Over the last decades population growth[1] has fallen considerably across the world's largest economies (figure 1). True, demographic variables evolve more gradually than other factors driving business cycles, such as technology or interest rates. It seems thus reasonable to conjecture that their influence on the macroeconomy and on monetary policy should be modest (Bean, 2004, 449). On the other hand, the baby boom and bust cycle caused sizable fluctuations in the US population growth rate, which fell by one percentage point alone during the early 1980s. And growth rates are projected to further decline throughout this century.

Using monthly data on live births in the United States, starting in January 1941, I calculate the "natural" population growth rate of the US working age population and include it in a standard VAR model of the US economy. An impulse response analysis shows that fertility shocks have only small effects on per-capita output, consumption, investment, hours worked, and the real wage. Inflation, on the other hand, increases by 0.2 percentage points (pp) in response to a one standard deviation fertility shock, corresponding to an annualized increase in the population growth rate of 0.1 pp. Moreover, the positive effect is significant for ten years. Interestingly, there is a negative response of the nominal interest rate on impact. Only after five years the nominal interest rate turns significantly positive.

In order to better asses the importance of fertility shocks, I calculate a forecast error variance decomposition of the different variables. As one may have expected, fertility shocks are not a driving force of business cycles. They account for less than 5% of the variation in output, consumption, investment, hours worked, and the real wage at business cycle horizons of less than 10 years. Fertility shocks, on the other hand, account for 10-15% of the variation in inflation at horizons of more than five years. A historical decomposition analysis suggests that lower population growth may have helped reducing inflation and nominal interest rates by 1.5 to 2 pp during the 1980s and 1990s.

I incorporate stochastic population growth in a standard dynamic stochastic general equilibrium (DSGE) model and analyze the impulse responses to a fertility shock that temporarily raises the population growth rate. With a varying population size, the dynamics of the model depend on how the representative household weights present and future generations. There are two polar cases. In the first one, total utility, i.e. per-capita utility[2] multiplied by the size of the household, is maximized (*Benthamite* preferences). In the second one, per-capita utility is maximized, irrespective of the size of the different generations (*Millian* preferences). The natural rate of

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[1]In the remaining paper, population refers to the population 16 years and older.
[2]That is to say, the utility from per-capita consumption and leisure, respectively.
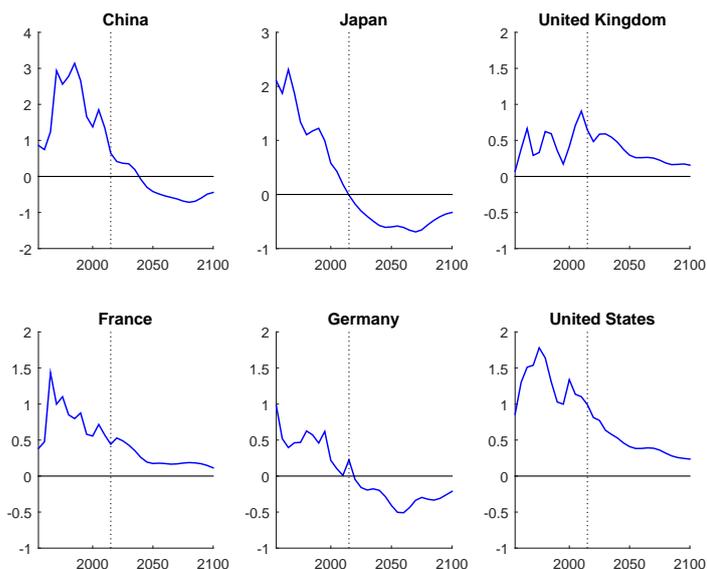
Population 16 years and older. Growth rates in percent. Five-year averages. 1950-2015: estimates. 2015-2100: projections (medium variant). Source: UN Population Prospects 2015.

Figure 1: Population Growth Estimates and Projections, Selected Countries, 1950-2100

interest falls, at least in the short-run, in the Benthamite case, but increases both in the short- and in the long-run in the Millian case. The mechanism is as follows. A fertility shock increases the depreciation rate of the per-capita capital stock[3] in both cases. Under Benthamite preferences, the household treats each generation identically, notwithstanding their size. As a consequence, investment increases in order to offset the fall in capital. Under Millian preferences, on the other hand, a positive fertility shock does affect the intertemporal optimality condition similar to a time preference shock, leaving the household more impatient. The household is not willing to invest more, given that the capital stock decreases faster with a higher population growth rate, and given that the changing size of the population does not appear in the objective function of the representative household. As a consequence, investment falls. The different saving behavior of the household is mirrored in the different responses of the natural rate of interest.

> In the end it is clear that the tools of modern growth theory lead to an ambiguous answer about how population growth affects the return to capital. One can write down textbook models in which the two variables move together (the Solow model), and one can write down models in which they do not (the Ramsey model). The natural response to this theoretical

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[3]In the remaining paper, unless otherwise noted, all quantities are in per-capita units.

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ambiguity is to muster evidence, either from time-series data or from the international cross section, about the actual effect of population growth. (Mankiw, 2005, 317-18)

Following Becker and Barro (1988), I allow for a more general population weighting function that incorporates the aforementioned preferences as special cases. Matching the theoretical impulse responses of the model to the estimated impulse responses of the VAR following a fertility shock, I estimate the parameter that governs the curvature of this function. This parameter represents the steady state, percentage point increase in the interest rate due to a one percentage point increase in the population growth rate.[4] The estimates vary between 0.48 and 0.66, depending on the specification of the VAR and the theoretical model. The baseline estimate is 0.59, which suggest that there is indeed a positive link between population growth and the natural rate of interest.

In the presence of price stickiness only, it is optimal for the central bank to increase the nominal interest rate in accordance with the higher natural rate, following a positive fertility shock. A failure to do so, on the other hand, keeps real interest rates too low, leading to inflation and a positive output gap. As an alternative to tracking the natural rate, the central bank may use more inertial monetary policy rules that mitigate the mismeasurement of the natural rate (Orphanides and Williams, 2002). Simulating the dynamic responses to the post-war fertility shocks in the DSGE model, I find that both the natural rate of interest and inflation have fallen by about half a percentage point due to the decline in population growth over the last decades.

Related literature   Recent papers studying the economic causes of the US postwar baby boom include Greenwood et al. (2005), Zhao (2014), Jones and Schoonbroodt (2014), and Doepke et al. (2015). This paper, on the other hand, analyzes the impact of the postwar baby boom and the subsequent baby bust on the macroeconomy. Jaimovich and Siu (2009) employ panel-data methods to investigate the relationship between the age composition of the labor force and business cycle volatility. They find that demographics account for about 30 percent of the decline in US macroeconomic volatility since the 1980s. This paper investigates the effects of fertility shocks on

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[4]The parameter is usually considered to capture the degree of "imperfect familial altruism" (Baker et al., 2005, 299). It is not obvious, however, that a household with Benthamite preferences is perfectly altruistic. With Benthamite preferences the steady state per-capita capital stock is independent of the population growth rate. With declining population growth, the household would consume parts of the aggregate capital stock to sustain the same per-capital capital stock. This conflicts with common definitions of altruism. Altruism: unselfish regard for or devotion to the welfare of others (Merriam-Webster). Altruism: term coined by Comte for the disinterested concern for the welfare of another, as an end in itself (The Oxford Dictionary of Philosophy). Ethical Altruism: an action is morally right if the consequences of that action are more favorable than unfavorable to everyone except the agent (Internet Encyclopedia of Philosophy). Benthamite preferences imply that the household does not discriminate (neither positively nor negatively) with respect to the size of a generation.

the macroeconomy, rather than their impact on the transmission mechanism of other shocks. Jaimovich et al. (2013) rationalize the age differences in the volatility of hours worked using a neoclassical growth model featuring capital-experience complementarity in production. In contrast to their paper, I do not consider any changes in the age-composition of the labor force due to a higher or lower population growth.

Beginning with the Samuelson-Lerner debate, economists have been divided on how to maximize social welfare in the presence of population growth.[5] Classical utilitarianism, following Bentham, suggests maximizing the total sum of individual utility. Average utilitarianism, following Mill, advocates maximizing the utility of the average individual. Depending on the formulation of the welfare function, the optimal interest rate may be equal to the (biological) population growth rate, (Samuelson, 1958, 1959), or not, (Lerner, 1959a,b). Arrow and Kurz (1970, 13) reject Samuelson's Millian preference formulation, arguing "that the social felicity is better measured by the sum of all the individual felicity in a given generation; if more people benefit, so much the better." For Rawls (1999, 252-53), by contrast, "maximizing total utility may lead to an excessive rate of accumulation (at least in the near future),"[6] assuming that a higher technology level in the future is associated with a larger population. Standard textbooks on growth theory differ on this as well. Blanchard and Fisher (1989) employ Millian preferences, while Barro and Sala-i-Martin (1995) advocate a Benthamite formulation of the welfare function. In the context of optimal fertility choice, Becker and Barro (1988) were the first to allow for a more general formulation of preferences.[7]

Besides its normative implications, the specification of the intertemporal utility function is particularly important for assessing the consequences of lower population growth on capital accumulation, as highlighted by Canton and Meijdam (1997). Millian preference formulations imply a higher capital intensity with lower population growth in the future, as in Yoo (1994), while there is no change in the capital-output ratio with Benthamite preferences, as in Cutler et al. (1990). To the best of my knowledge, this paper is the first to address this theoretical ambiguity empirically.

Recently, papers have discussed the threat of a period of "secular stagnation", re-

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[5]See Lane (1977) and Nerlove et al. (1987).

[6]"Thus it seems evident, for example, that the classical principle of utility leads in the wrong direction for questions of justice between generations. For if one takes the size of the population as variable, and postulates a high marginal productivity of capital and a very distant time horizon, *maximizing total utility may lead to an excessive rate of accumulation (at least in the near future)* [emphasis added]. Since from a moral point of view there are no grounds for discounting future well-being on the basis of pure time preference, the conclusion is all the more likely that the greater advantages of future generations will be sufficiently large to outweigh most any present sacrifices. This may prove true if only because with more capital and better technology it will be possible to support a sufficiently large population. Thus the utilitarian doctrine may direct us to demand heavy sacrifices of the poorer generations for the sake of greater advantages for later ones that are fare better off" (Rawls, 1999, 252-53).

[7]See also Maußner and Klump (1996) or Baker et al. (2005).

ferring to a term coined by Hansen (1939). Most prominently, Summers (2014) and Eggertsson and Mehrotra (2014) argue that declining, and possibly negative equilibrium real interest rates in combination with a zero lower bound on nominal interest rates may create difficulties in achieving full employment. Lower population growth is considered to be a major trigger of a lower natural rate of interest.[8] In this paper I find some evidence for a positive link between population growth and the natural rate of interest, although the implied fall in the natural rate due to lower population growth is presumably too modest to trigger a "secular stagnation".

Using a new Keynesian overlapping generations model, Carvalho and Ferrero (2014) argue that Japan's deflation of the last two decades is a result of the central bank's failure to account for the secular decline in the natural rate of interest due to both lower population growth and higher life expectancy. In this paper I find that this natural rate channel is also present in the model which is commonly used for monetary policy analysis, namely the infinitely-lived, representative agent model.

Using cross-country panel data, Juselius and Takats (2014) find a U-shaped pattern between cohort shares and inflation (i.e. deflation when middle- and old-aged workers are overrepresented). Yoon et al. (2014) detect a positive relation between inflation and population growth across OECD countries, albeit significant only in the case of Japan. This paper confirms their findings for the US using time series data.

Section 2 estimates the macroeconomic dynamics following fertility shocks in a VAR model of the US economy. Section 3 analyses the responses to fertility shocks within a DSGE model and discusses how they depend on the specification of the household's preferences. In section 4, I estimate the preference parameter governing the population weights by matching the VAR-based and the model-based impulse responses. Section 5 discusses the implications for monetary policy. Section 6 concludes.

2 VAR Evidence: Fertility Shocks

2.1 Population Growth in the US

Using monthly data on live births from the *National Center for Health Statistics (NCHS)*, I construct a quarterly time series for the "natural" growth rate of the US working age population

$$\nu_t \equiv \frac{b_{t-16y,t} \times Births_{t-16y} - Deaths_t}{N_t}.$$

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[8]"Second, it is well known, going back to Alvin Hansen and way before, that a declining rate of population growth, ..., means a declining natural rate of interest" (Summers, 2014, 69). "And the equilibrium real interest rate may easily be permanently negative. Forces that work in this direction include a slowdown in population growth, which increases relative supply of savings" (Eggertsson and Mehrotra, 2014, 2).
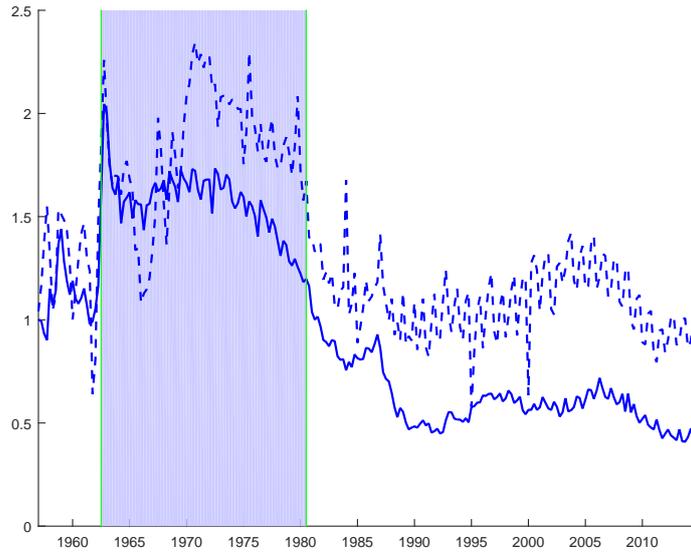
Solid line: natural population growth rate, 16 years and older. Dashed line: population growth rate, 16 years and older. Shaded area: baby boom cohorts. Annualized percentage points.

Figure 2: Civilian Noninstitutional Population, 16 Years and Older, US 1957Q1-2014Q4

$Births_{t-16y}$ is the number of live births 16 years ago, $b_{t-16y,t}$ is the fraction of persons surviving to age 16, and $Deaths_t$ is the total number of deaths, 16 years and older. See appendix A.1. Figure 2 presents the total and the natural growth rate of the US working age population between 1957Q1 and 2014Q4. The baby boomer[9] cohorts joined the working age population between 1962Q3 and 1980Q2, creating an increase in the growth rate from one pp per year in the early 1960s to an average rate of 1.5 pp during the 1960s and 1970s. The following baby bust lead to a decline in the growth rate to 0.5 pp in 1990. The growth rate increased by about 0.2 - 0.3 pp in the late 1990s and the early 2000s, due to the arrival of the so called "echo boomer", i.e. the children of the baby boomer, to the working age population. Since the mid-2000s the growth rate has declined to 0.4 pp in 2014.

2.2   VAR

Consider the following VAR(p) model

$$z_t = c + \sum_{s=1}^{p} B_s z_{t-s} + u_t, \quad t = 1, ..., T,$$ (2.1)

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[9]Baby boomer are the cohorts from mid-1946 to mid-1964.

with $\mathbb{E}[u_t] = 0$, $\mathbb{E}[u_t u_t'] = \Sigma_u$, and $\mathbb{E}[u_t u_s'] = 0$, for $s \neq t$. The endogenous variables are collected in the $M \times 1$ vector

$$z_t \equiv \begin{bmatrix} v_t & y_t & c_t & x_t & h_t & w_t & \pi_t & r_t \end{bmatrix}'.$$

The marcoeconomic variables included in the VAR are: real gross domestic product, y_t, real private consumption expenditures, c_t, real private non-residential investment, x_t, total hours worked, h_t, real wages, w_t, the inflation rate (based on the GDP deflator), π_t, and the short-term nominal interest rate (i.e. the federal funds rate), r_t.[10] Output, consumption, investment, and hours worked are in per-capita terms. All variables enter the VAR in levels. Finally, u_t is a $M \times 1$ vector of white noise processes. The sample period goes from 1957Q1 to 2014Q4. The Bayesian VAR is estimated using a noninformative prior. The optimal lag order is $p = 2$, according to the Aikaike information criterion.[11] Except for population growth, the variables are the same as in Dedola and Neri (2007).

Identification of population shocks   In order to create a link between the forecast errors u_t and the structural shocks ε_t

$$u_t = A\varepsilon_t, \quad \mathbb{E}[\varepsilon_t \varepsilon_t'] = I_M, \tag{2.2}$$

I consider a recursive identification

$$A = \text{chol}(\Sigma_u), \quad \Sigma_u = AA', \tag{2.3}$$

implying that forecast errors in v_t are due to fertility shocks only. It is quite natural to assume, as did Jaimovich et al. (2013), that fertility decisions 16 years ago are unaffected by today's business cycle conditions.
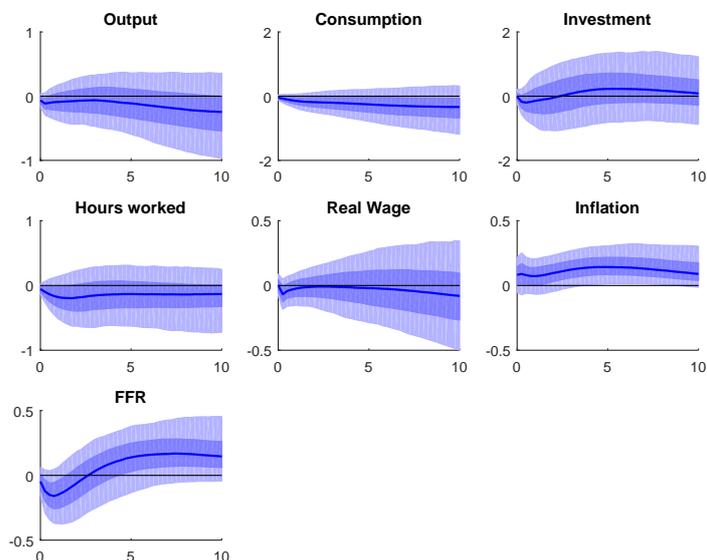
Impulse responses   [12] Figure 3 presents the impulse responses to a fertility shock. The results are based on 1,000 draws from the posterior distribution of the reduced-form VAR. Fertility shocks have no clear effects on the dynamics of most macroeconomic variables. Consumption and hours worked fall at a 68%-significance level, while showing no effect at a 95%-significance level. Investment declines on impact before turning positive after some years. Yet none of the responses is significant, neither are the responses of output and the real wage. The striking exception is inflation,

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[10]See appendix A.1.

[11]The optimal lag length according to Hannan-Quinn and the Schwarz information criterion is $p = 2$ and $p = 1$, respectively. Results are robust to a higher lag order (see below).

[12]See appendix B.1.

increasing by 0.2 pp to a one standard deviation fertility shock.[13] Moreover, this positive effect lasts for ten years. At the same time, the FFR falls on impact, remains negative for two years, before turning positive after four years.
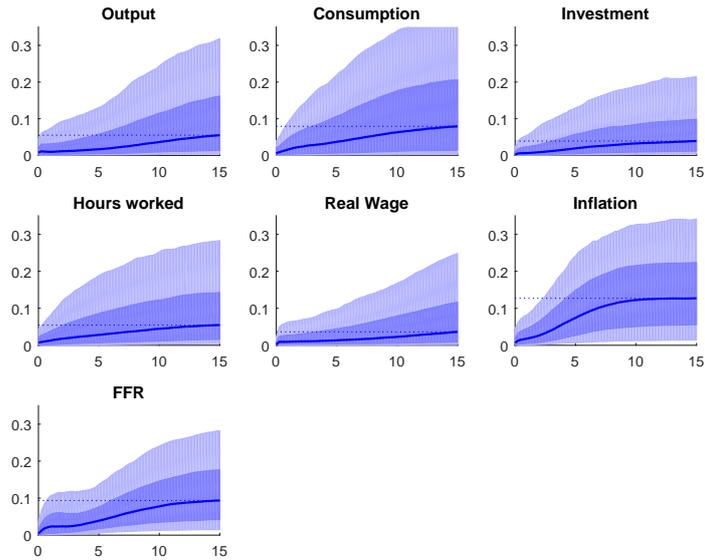

Median (solid lines), 68% and 95% probability bands (shaded areas) of posterior distribution. Years on x-axis.

Figure 3: VAR-based Impulse Response to a Fertility Shock

Forecast error variance decomposition Figure 4 presents the contribution of fertility shocks to the variance of the *k*-step ahead forecast error of the different variables. As expected, population growth is not a major source of business cycle movements. It is almost negligible at commonly considered frequencies of five years, or less. Even at longer horizons, the importance of fertility shocks is minor (5 - 10%). Population growth is an important factor for inflation, however, confirming findings of Juselius and Takats (2014) or Yoon et al. (2014). Fertility shocks account for 10-15% of the variation in inflation at horizons of five years, and longer. Finally, fertility shocks account for roughly 10% of the forecast error variance of the FFR at horizons of more than 10 years.

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[13]A one standard deviation fertility shock implies a rise in the annualized population growth rate of about 0.1 pp.
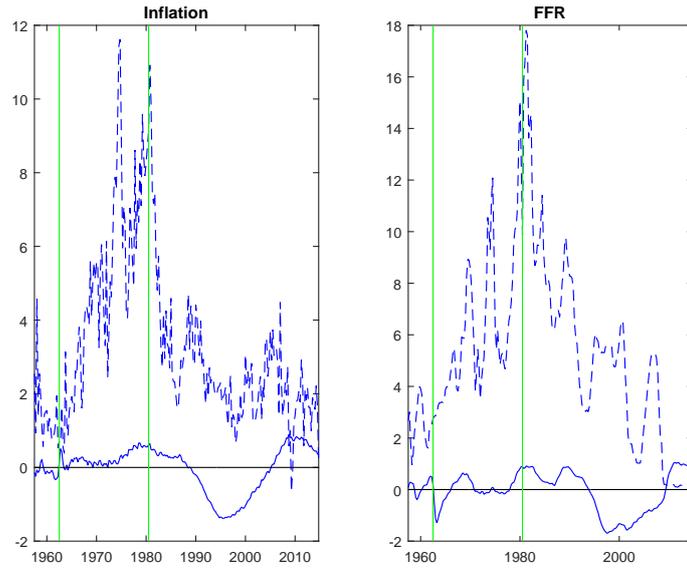
Median (solid lines), 68% and 95% probability bands (shaded areas) of posterior distribution. Years on x-axis.

Figure 4: Contribution of Population Shocks to the Variance of the Forecast Error

Historical decomposition   Figure 5 indicates that population growth explains a substantial portion of the secular movements of inflation and the nominal interest rate. Although fertility shocks seem to have been of minor importance during the high inflation period of the 1970s, they account for two percentage points of the decline in inflation rates during the 1980s and 1990s. Their contribution to the inflation rate reached a historical low of -1.5 pp in 1995, before it increased to 1 pp in 2008. Regarding the FFR, fertiltiy shocks account for about 2.5 pp of its decline during the 1990s and about of the same amount of the increase thereafter.

2.3   Sensitivity Analysis

Figure 6 shows the responses to a fertility shock when total hours divided by persons in the labor force, instead of hours per-capita enter the VAR. Hours worked respond negatively to a fertility shock in the baseline estimation (Figure 3). This is not surprising, as total hours worked (per-capita) were higher in decades (1980s and 1990s) with lower population growth. At the same time, however, labor force participation increased significantly, mainly because of more women working. With this alternative specification of hours worked, investment increases after two years (figure 6). The response of inflation is almost unchanged.

Solid lines: only fertility shocks. Dashed lines: all shocks.

Figure 5: Historical Decomposition of Inflation and the FFR
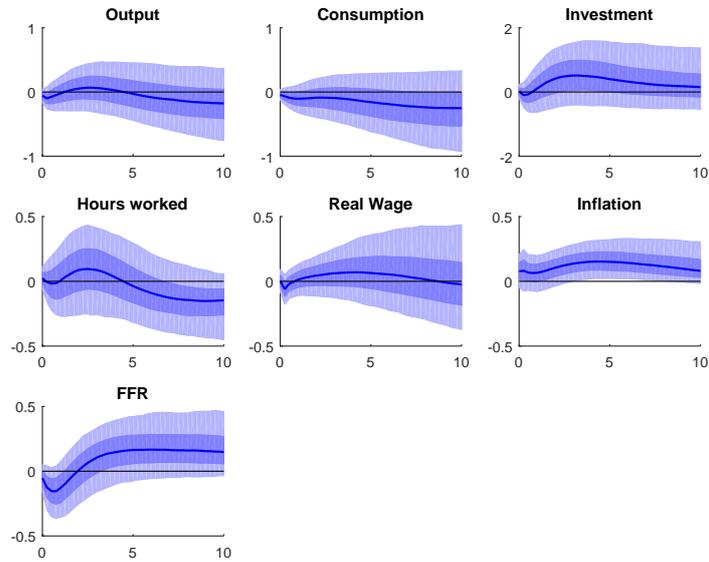
Median (solid lines), 68% and 95% probability bands (shaded areas) of posterior distribution. Years on x-axis.

Figure 6: VAR-based Impulse Response to a Fertility Shock - Total Hours / Labor Force

Figure 7 shows the impulse responses to a fertility shock when investment is defined as total private fixed investment, i.e. including residential investment. The

response of investment is now positive after two years, albeit significant only at a 68% level. Figures 15 (Inflation based on the CPI) and 16 (four lags) confirm the robustness of the results, in particular the positive response of inflation. Both figures can be found in appendix A.2.
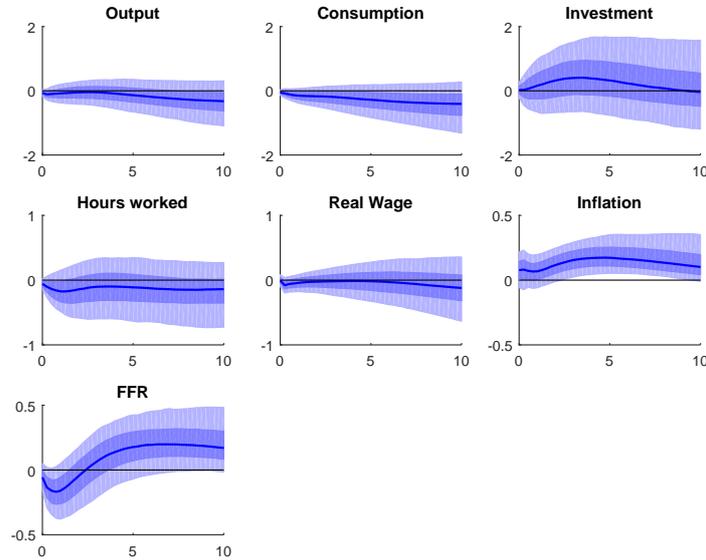

Median (solid lines), 68% and 95% probability bands (shaded areas) of posterior distribution. Years on x-axis.

Figure 7: VAR-based Impulse Response to a Fertility Shock - Private Fixed Investment

3 Population Growth in a Standard DSGE Model

This section presents a standard DSGE model including stochastic fertility shocks. The model features four departures from the basic neoclassical growth model: (i) convex investment adjustment costs, (ii) variable capital utilization, (iii) monopolistic competition in the goods and in the labor market, and (iv) nominal price and wage rigidities.

Households There is a continuum of households indexed by $j \in [0,1]$, and each of size N_t. Each households is a monopoly supplier of a differentiated labor service (Erceg et al., 2000). The existence of state-contingent securities guarantees that households are homogeneous with respect to consumption, investment and asset holdings but heterogeneous with respect to the real wage $W_t(j)$ they earn and the amount of

hours they work $h_t(j)$.[14] The preferences of a household j are given by

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t N_t^{1-\theta}u(c_t,h_t(j))\right], \quad \beta \in (0,1),$$

where c_t denotes consumption, and where the instantaneous utility function is compatible with a balanced growth path (King et al., 1988)

$$u(c_t,h_t(j)) = \ln(c_t) - v\frac{h_t(j)^{1+\varphi}}{1+\varphi}, \quad \varphi \geq 0.$$

The size of the household N_t is subject to stochastic shocks ε_t^v and evolves according to

$$v_t \equiv \ln(N_{t+1}) - \ln(N_t) = \rho_v v_{t-1} + \varepsilon_t^v, \quad \varepsilon_t^v \overset{i.i.d.}{\sim} \mathcal{N}(0,\sigma_v^2).$$

Following to Becker and Barro (1988), the parameter θ represents the weighting factor with respect to the household size N_t. With $\theta = 0$, the per-capita utility of each generation is weighted by its size (Benthamite preferences). With $\theta = 1$ the per-capita utility of each generation is weighted equally, regardless of its size (Millian preferences).[15] The household's flow budget constraint is given by

$$P_t(c_t + x_t + \Psi(u_t)k_t) + \frac{N_{t+1}}{N_t}b_t \geq R_{t-1}b_{t-1} + R_t^K k_t + W_t(j)h_t(j) + d_t.$$

P_t is the aggregate price level, x_t is investment, b_t is the quantity of one-period, nominally riskless bonds purchased in period t and maturing in period $t+1$, R_t is the gross nominal return of the bond between t and $t+1$, R_t^K is the rental price of capital services $u_t k_t$ sold to firms, u_t is the capital utilization rate, k_t is the capital shock owned by the households, and d_t are dividends that the households receives from firm ownership. Increasing capital utilization is subject to convex adjustment costs $\Psi(u_t)$ with $\Psi'(u_t) > 0$ and $\Psi''(u_t) > 0$. The capital stock that it owns evolves according to

$$\frac{N_{t+1}}{N_t}k_{t+1} \leq (1-\delta)k_t + (1-S(x_t/x_{t-1}))x_t, \quad \delta \in (0,1),$$

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[14]All quantities are expressed in per-capita terms.

[15]The parameter θ is usually considered to capture the degree of "imperfect familial altruism" (Baker et al., 2005, 299). It is not obvious, however, that a household with $\theta = 0$ is perfectly altruistic. With Benthamite preferences the steady state *per-capita* capital stock is independent of the population growth rate. With declining population growth, the household would consume parts of the aggregate capital stock - the saving rate would decline - to sustain the same per-capital capital stock. This conflicts with common definitions of altruism. *Altruism*: unselfish regard for or devotion to the welfare of others (Merriam-Webster). *Altruism*: term coined by Comte for the disinterested concern for the welfare of another, as an end in itself (The Oxford Dictionary of Philosophy). *Ethical Altruism*: an action is morally right if the consequences of that action are more favorable than unfavorable to everyone except the agent (Internet Encyclopedia of Philosophy). Benthamite preferences imply that the household does not discriminate (neither positively nor negatively) with respect to the size of a generation.

where $S(x_t/x_{t-1})$ captures convex adjustment costs to investment. In particular, $S(1) = S'(1) = 0$ and $S''(1) > 0$.

The household sells its labor service $h_t(j)$ to a representative, competitive firm that transforms it into aggregate labor services h_t using the following technology

$$h_t \equiv \left(\int_0^1 h_t(j)^{\frac{\epsilon_w-1}{\epsilon_w}} dj \right)^{\frac{\epsilon_w}{\epsilon_w-1}},$$

where $\epsilon_w > 1$ is the elasticity of substitution between the different varieties of labor. The aggregate nominal wage rate is

$$W_t \equiv \left(\int_0^1 W_t(j)^{1-\epsilon_w} dj \right)^{\frac{1}{1-\epsilon_w}},$$

and the optimal demand for labor service j is given by

$$h_t(j) = \left(\frac{W_t(j)}{P_t} \right)^{-\epsilon_w} h_t.$$

In each period, a random fraction of households $1 - \lambda_w$, with $\lambda_w \in [0,1)$, are able to reoptimize their nominal wage. The optimality condition for a household j that can adjust its wage in period t is

$$\mathbb{E}_t \left[\sum_{s=t}^{\infty} (\beta\lambda_w)^{s-t} N_s^{1-\theta} \left\{ h_s(j) u_{c_s} \left(\frac{W_t(j)}{P_s} - \frac{\epsilon_w}{\epsilon_w - 1} MRS_s(j) \right) \right\} \right] = 0,$$

where the marginal rate of substitution between consumption and labor in periods s for a household resetting its wage in t is defined as

$$MRS_s(j) \equiv -\frac{u_{h_s(j)}}{u_{c_s}}.$$

Final-good firms There is a perfectly competitive final-good sector. The final good that households use for consumption and investment is produced using the following technology

$$y_t \equiv \left(\int_0^1 y_t(i)^{\frac{\epsilon_p-1}{\epsilon_p}} di \right)^{\frac{\epsilon_p}{\epsilon_p-1}},$$

where $y_t(i)$ denotes the quantity of intermediate good of type i that is used in final goods production, and where $\epsilon_p > 1$ is the elasticity of substitution between intermediate goods i. The aggregate price index is

$$P_t \equiv \left(\int_0^1 P_t(i)^{1-\epsilon_p} di \right)^{\frac{1}{1-\epsilon_p}},$$

where $P_t(i)$ denotes the price of good i. The optimal demand for good i is given by

$$y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_p} y_t.$$

Intermediate-goods firms There is a continuum of firms indexed by $i \in [0,1]$. Firm i is the monopoly supplier of good i. All firms use the same technology, represented by the production function

$$y_t(i) = k_t(i)^\alpha h_t(i)^{1-\alpha},$$

where $k_t(j)$ and $h_t(i)$ are the capital and labor services demanded by firm i. Cost minimization implies that real marginal costs mc_t are identical across firms and given by

$$mc_t = \frac{(R_t^K k_t(i))^\alpha (W_t h_t(i))^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha}}.$$

Each period, a random fraction $1 - \lambda_p$ of firms adjust their price, with $\lambda_p \in [0,1)$. The optimality condition for a firm i that can adjust its price in period t is

$$\mathbb{E}_t\left[\sum_{s=t}^{\infty} \lambda_p^{s-t} Q_{t,s} \left(\frac{P_{s,t}(i)}{P_s} - \frac{\epsilon_p}{\epsilon_p - 1} mc_s\right) y_s\right] = 0,$$

where the stochastic discount factor for real profits is defined as

$$Q_{t,s} \equiv \beta^{s-t} \left(\frac{N_s}{N_t}\right)^{-\theta} \frac{u_{c_s}}{u_{c_t}}, \quad s \geq t.$$

Monetary policy The central bank sets the nominal interest rate according to the following Taylor (1993) rule

$$R_t = R^{\rho_r} \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{(1-\rho_r)\phi_\pi} \left(\frac{y_t}{y_t^n}\right)^{(1-\rho_r)\phi_y}, \quad \rho_r \in [0,1], \quad \phi_\pi, \phi_y > 0.$$

Here, $R = 1/\beta$ denotes the steady state nominal interest rate, $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ the gross inflation rate, $\bar{\Pi}$ the inflation target of the central bank, and y_t^n the natural level of output, i.e. the level of output under flexible prices and wages.

Market clearing and equilibrium The market clearing conditions for capital and labor services are

$$\int_0^1 k_t(i)di = k_t u_t,$$
$$\int_0^1 h_t(i)di = h_t,$$

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and the aggregate resource constraint reads

$$y_t = c_t + x_t + \Psi(u_t).$$

The model equations are linearized (appendix B.2) around the zero-inflation, non-stochastic steady state and then solved using standard solution methods for linear rational expectations models.

3.1 Real Business Cycle (RBC) Model

Table 1 lists the parameters that are used in the RBC model. All parameters are standard. With regard to investment adjustment costs I choose a value of 2.5 for $\eta \equiv S''(1)$, using the estimate of Christiano et al. (2005). The capital utilization cost elasticity $\omega \equiv \frac{\Psi''(1)}{\Psi'(1)}$ is set equal to 1, which is the point estimate of Basu and Kimball (1997). Given the imprecise estimates[16] of ω, I conduct robustness checks with respect to ω in section 4. Figure 8 shows the impulse responses to a fertility shock in a RBC model, i.e. when both prices and wages are flexible ($\lambda_p = \lambda_w = 0$). Let $\widehat{x}_t$ denote the log-deviation of x_t from its nonstochastic steady state value x. The population growth rate ν_t affects both the accumulation of capital

$$\widehat{k}_{t+1} + \nu_t = (1 - \delta)\widehat{k}_t + \delta\widehat{x}_t, \tag{3.1}$$

and the optimal consumption-saving decision of households between two periods

$$\widehat{c}_t = \mathbb{E}_t[\widehat{c}_{t+1}] - (\widehat{r}_t - \mathbb{E}_t[\widehat{\pi}_{t+1}] - \theta\nu_t). \tag{3.2}$$

It is evident from equations (3.1) and (3.2), that a positive fertility shock is a combination of depreciation and time preference shocks.

When $\theta = 0$, a higher population growth rate leads to a surge of investment, given that more workers need more capital. At the same time, consumption expenditures fall, while hours worked rise due to the negative wealth effect. As the capital-labor ratio falls, the rental price of capital increases, while the real wage decreases. Movements in factor prices, however, are dampened by a higher utilization rate of capital following a positive population growth shock. The rise in labor supply, investment and capital utilization results in a positive reaction of output per-capita.

When $\theta = 1$, investment, hours worked and output fall in response to positive fertility shock. With Benthamite preferences ($\theta = 0$), a higher population growth reduces both the effective return on saving, i.e. in per-capita units of capital or bonds, for a given rental price of capital, and the effective time preference rate, as the household

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[16]The confidence interval of ω is $[-0.2, 2]$ (Basu and Kimball, 1997).

Table 1: Calibration - RBC Model

|		
Discount factor	β	0.99
Frisch elasticity of labor supply (inverse)	φ	1
Capital income share	α	0.36
Capital depreciation rate	δ	0.025
Investment adjustment costs	η	2.5
Capital utilization	ω	1
Persistence of fertility shock	ρ_v	**0.990**
Std. of fertility shock (in %)	σ_v	**0.019**
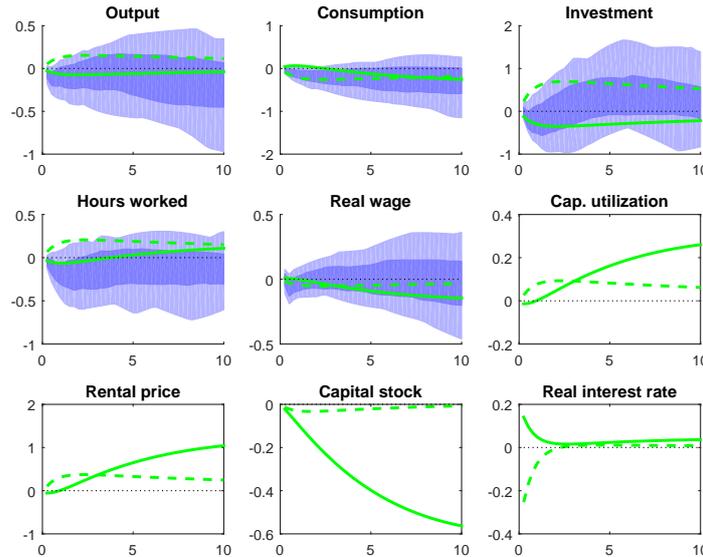
Solid lines: model-based IRFs (Millian preferences). Dashed lines: model-based IRFs (Benthamite preferences). Shaded areas: 68% and 95% probability bands of VAR-based IRFs. Years on x-axis.

Figure 8: Model-based IRF to a Fertility Shock - RBC Model

puts more weight on future, more populous generations (Canton and Meijdam, 1997, 321). The intertemporal optimality condition of the household (3.2) is thus not affected. Or differently, as the household weights the per-capita of each person equally, notwithstanding whether he or she belongs to a large or small generation, investment increases to equip each generation with the same (at least in steady state) per-capita capital stock. With Millian preferences ($\theta = 1$), a higher population growth leads to a larger fall in the per-capital capital stock as the decline in the effective return on saving (for a given rental price of capital) is not offset by a simultaneous decline in the time preference rate.

Table 2: Calibration - NK Model

Elasticity of substitution (goods)	ϵ_p	10
Elasticity of substitution (labor services)	ϵ_w	10
Calvo (1983) parameter for wages	λ_w	2/3
Calvo (1983) parameter for prices	λ_p	2/3
Interest rate inertia	ρ_r	0.8
Inflation coefficient	ϕ_π	2
Inflation target	$\bar{\Pi}$	1
Output gap coefficient	ϕ_y	0.1
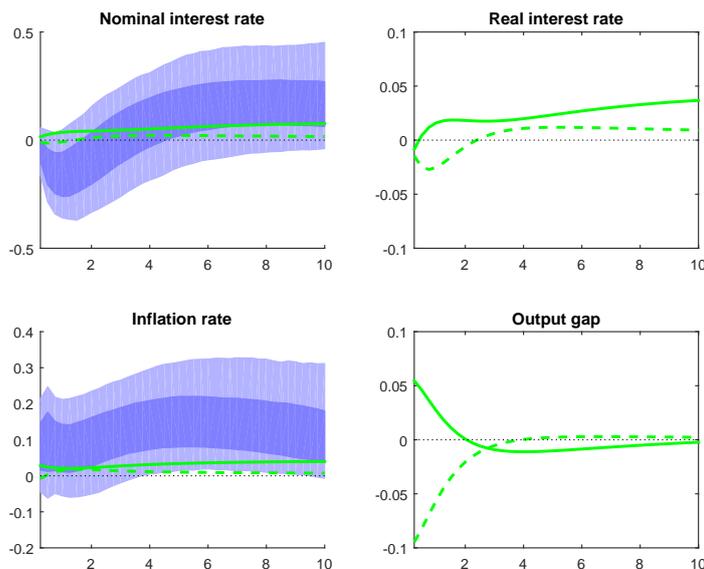
Solid lines: model-based IRFs (Millian preferences). Dashed lines: model-based IRFs (Benthamite preferences). Shaded areas: 68% and 95% probability bands of VAR-based IRFs. Years on x-axis.

Figure 9: Model-based IRF to a Fertility Shock - NK Model

3.2 New Keynesian (NK) Model

Table 2 lists the parameters that are used in the NK model. All parameters are standard. Figure 9 shows the impulse responses to a fertility shock under sticky prices and wages. With $\theta = 0$, marginal costs fall together with the natural rate of interest. Because of the inadequate response of the nominal interest rate, real rates are negative, as is the output gap. With $\theta = 1$, the reverse is true. Marginal costs, the natural rate of interest, and the output gap increase. Moreover, inflation remains positive for several years. The model, however, only partly captures the observed positive response of inflation. Further, the central bank *raises* the interest rate in response to inflation, while the VAR suggests that it actually *lowers* the rate following a positive population growth shock. I come back to this point in section 5.

4 The Natural Rate of Interest

Following Christiano et al. (2005), the parameter θ is estimated using the following minimum distance estimator

$$\widehat{\theta} = \arg\min_{\theta}[IR(\theta) - \widehat{IR}]'W^{-1}[IR(\theta) - \widehat{IR}].$$

$IR(\theta)$ are the theoretical impulse responses from the DSGE model, which depend on the parameter θ, while $\widehat{IR}$ are the estimated impulse responses from the VAR. The time horizon for the matches is ten years. The weighting matrix W is a diagonal matrix with the sample variances of $\widehat{IR}$ along the diagonal. Table 3 presents the estimates for θ. The median estimate of θ is 0.59 with 80% of estimates lying between 0.24 and 0.96. When total hours relative to the labor force instead of hours per capita are included in the VAR, estimates are slightly lower. This is because output, investment, and hours worked now fall by less or even slightly increase in the VAR (figure 6), which is more in line with lower values of θ (figure 8). As said before, allowing for variable capital utilization in the model is crucial, as capital is used more intensively as more workers are entering the economy. Unfortunately, the estimates for the elasticity of capital utilization costs are rather imprecise. To asses the implications of different short-run elasticity of capital, I estimate θ varying ω. Figures 17 and 18 in section B.3 of the appendix compare the VAR-based impulse responses to the model-based impulse reposes for $\omega = 0.5$ and $\omega = 2$, respectively.[17] First, notice that varying the cost of variable utilization plays a much more important role for a household with Millian preferences ($\theta = 1$). For $\omega = 0.5$ utilization cost slowly rise with the level of utilization. Under Millian preferences, households use the existing capital stock more intensively instead of investing more. Investment, output, and hours worked fall by more than the VAR-responses suggest. As a result I obtain a lower estimate of θ at 0.49. For $\omega = 2$, varying the capital utilization rate is rather costly and thus less intensively used. In order to offset the dilution of the capital stock following higher population growth, investment has to be increased now. Consequently, the estimate of θ increases to 0.66. Changing the investment adjustment costs elasticity or the Frisch elasticity of labor supply has negligible effects on $\widehat{\theta}$.

Table 4 compares these estimates with the implied relationship between the population growth rate and the natural rate of interest across different models: the neoclassical growth model of Solow (1956), the overlapping families model of Weil (1989), the overlapping generations model of Auerbach and Kotlikoff (1987), and the overlapping generations model of Gertler (1999). In accordance with the empirical evidence of this

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[17]King and Rebelo (1999) calibrate ω to 0.1 but also consider higher values (0.5, 1, ∞). Burnside and Eichenbaum (1996) obtain an estimate for ω of 0.54. The estimate of Smets and Wouters (2007) is $1.17 = \frac{0.54}{1-0.54}$.

Table 3: Estimates for θ - Impulse Response Matching

	Mean	10%	50%	90%
Baseline	**0.59**	**0.24**	**0.58**	**0.96**
VAR specification				
Hours per worker	0.48	0.20	0.50	0.76
Private fixed investment (incl. res.)	0.53	0.14	0.54	0.90
CPI inflation	0.54	0.24	0.54	0.84
Number of lags = 4	0.57	0.24	0.58	0.90
Matching horizon = 5 years (Baseline: 10)	0.58	0.32	0.58	0.84
Model specification				
High elasticity of capital utilization $\omega = 0.5$	0.49	0.16	0.48	0.82
Low elasticity of capital utilization $\omega = 2$	0.66	0.28	0.66	1.06
Low elasticity of investment $\eta = 5$	0.58	0.22	0.58	0.94
Low elasticity of labor supply $\varphi = 2$	0.54	0.26	0.54	0.84

1,000 draws from posterior distribution. Minimum found by grid search over $-1 : 0.02 : 2$.

Table 4: Population Growth and the Natural Rate - Model Comparison

	Δr
Solow (1956)	1.1
Weil (1989)	0.3
Auerbach and Kotlikoff (1987)	
retires work	0.3
retires do not work	0.6
Gertler (1999)	0.7

Steady state comparative statics: $\Delta r = r(n = 1\%) - r(n = 0\%)$.

paper, all models suggest a positive link between population growth and the natural rate.

Figure 10 plots the estimated impulses to a fertility shock. The short-run sign of most responses is unclear, reflecting the impact of θ on the consumption-saving decision (section 3). Beyond a horizon of five years, however, consumption (hours worked) unambiguously falls (increase). Output and investment increase (investment in 68% of simulations). Inflation and the nominal interest increase after 2 years, albeit by less than the VAR-based responses.

Given theses estimates, I calculate the contribution of population growth shocks to interest rates and inflation (figure 11). The results suggest that the slowdown in population growth after the bust of the baby boom in the mid-1960s has led to a decline in the natural rate of interest and the inflation rate of about half a percentage point. The negative impact on the FFR is about one percentage point.
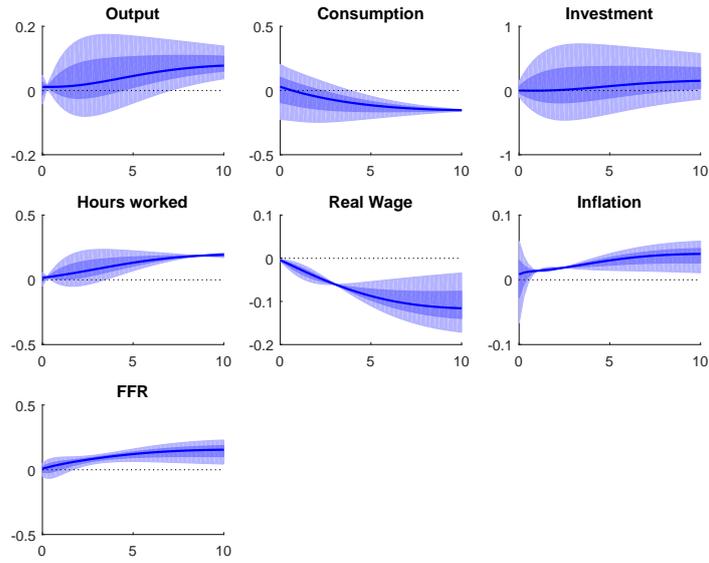
Median (solid lines), 68% and 95% probability bands (shaded areas) of posterior distribution. Years on x-axis. The probability bands take into account the uncertainty with regard to θ.

Figure 10: Estimated Impulse Responses to a Fertility Shock
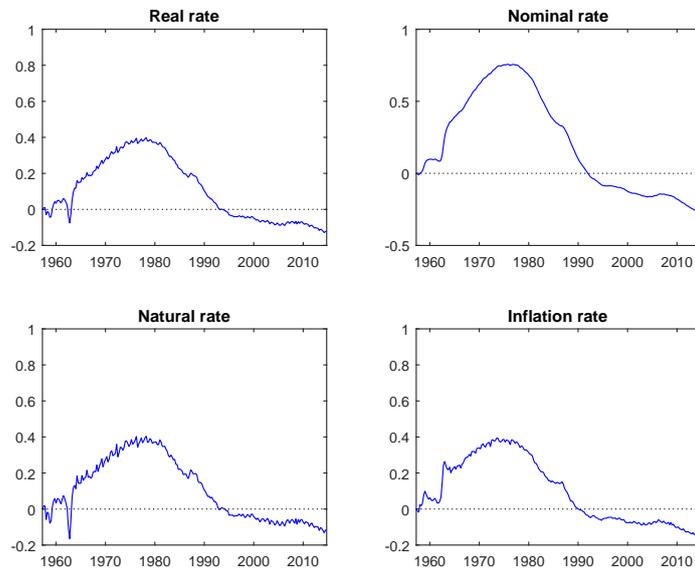
$$\theta = \widehat{\theta} = 0.59$$

Figure 11: Simulated Time Series for Interest Rates and Inflation

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5 Monetary Policy

Mismeasurement of the natural rate As mentioned in section 3, the model-based response of the nominal interest rate (figure 9) is at odds with the actual response of the FFR to fertility shocks (figure 3). Following Carvalho and Ferrero (2014), I assume in this paper that the central bank responds to inflation and the output gap only, and not to changes in the natural rate of interest. But what if the central bank does not simply ignore the effects of population growth on the natural rate, but misperceives them instead? Recall figure 8: depending on θ the natural rate may actually *decrease* following a positive fertility shock. Suppose the central bank believes θ to be zero, in other words it interprets increases in the population growth rate as *supply shocks*, brining with them higher investment, hours worked and output. Accordingly the log-linearized monetary policy rule

$$\widehat{r}_t = \rho_r \widehat{r}_{t-1} + (1 - \rho_r)(\widehat{r}_t^* + \phi_\pi \widehat{\pi}_t + \phi_y \widehat{y}_t),$$

now includes the estimate of the natural rate, $\widehat{r}_t^* = \widehat{r}_t^n(\theta = 0)$. Figure 12 shows the responses of interest rates and inflation under $\widehat{r}_t^* = \widehat{r}_t^n(\theta = 0)$ and $\widehat{r}_t^* = 0$. The central bank, believing that the natural rate has fallen - though it has been relatively constant (dash-dotted line) - lowers the nominal interest rate and thereby amplifies the response of inflation. After two years, however, the effect on both the natural rate and on its estimate has faded out and the responses in both scenarios converge. In the VAR, the FFR turns significantly positive only after 5 years. This might be either because the central bank believes that a positive shock to the population growth rate persistently lowers the natural rate, or because other factors linked to increases in the working age population force the central bank to lower the natural rate.

The role of nominal rigidities For a better understanding of the natural rate channel, I consider the same model as in section 3, this time with perfectly flexible prices and wages ($\lambda_p = \lambda_w = 0$). Interestingly, the responses of inflation and the nominal interest rate are little affected by the degree of real and nominal rigidities (figure 13). Even with both flexible prices and wages (dashed lines), inflation and the nominal interest rate are positive for several years after a population growth shock.

Consider the basic NK model, abstracting from capital accumulation ($\alpha = 0$), and focusing on price stickiness only ($\lambda_w = 0$). When prices are flexible, real marginal costs are constant, $\widehat{mc}_t = 0$. In the absence of other shocks, it follows that $\widehat{y}_t = \widehat{c}_t = \widehat{h}_t = 0$.[18] The consumption Euler equation then implies for the ex-ante real interest

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[18]Substitute for $\widehat{c}_t$ and $\widehat{h}_t$ using the aggregate resource constraint $\widehat{c}_t = \widehat{y}_t$ and the production function $\widehat{y}_t = \widehat{h}_t$. The real wage is $\widehat{w}_t = \widehat{mc}_t = 0$. From the labor supply condition $\widehat{w}_t = 0 = (1 + \varphi)\widehat{y}_t$ it follows immediately that $\widehat{y}_t = \widehat{c}_t = \widehat{h}_t = 0$.
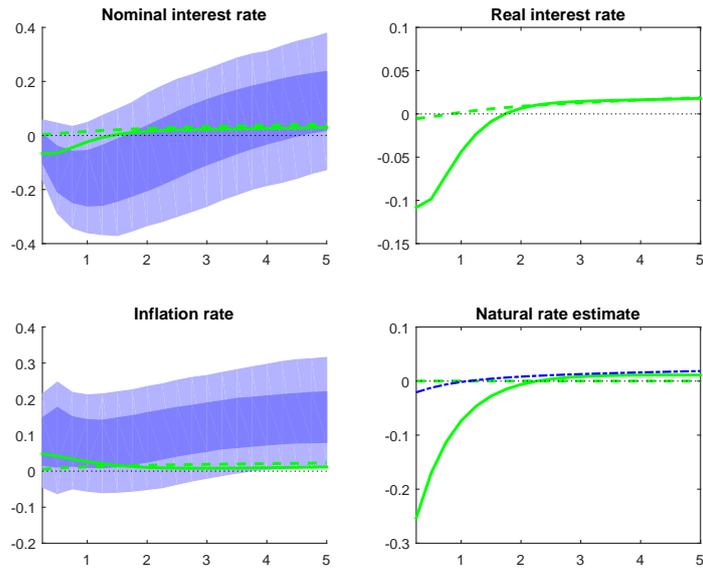
Solid lines: $\widehat{r}_t^* = \widehat{r}_t^n(\theta = 0)$. Dashed lines: $\widehat{r}_t^* = 0$. Dash-dotted line: natural rate of interest. Shaded areas: 68% and 95% probability bands of VAR-based IRFs. $\rho_r = 0.5$ and $\theta = \widehat{\theta} = 0.59$. Years on x-axis.

Figure 12: Model-based Impulse Responses to a Fertility Shock: Natural Rate Mismeasurement
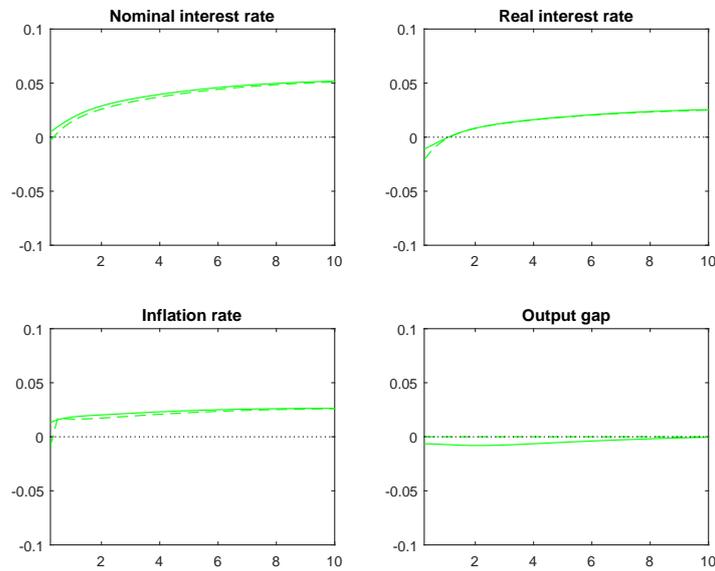
Solid lines: baseline. Dashed lines: flexible prices and wages. $\theta = \widehat{\theta} = 0.59$. Years on x-axis.

Figure 13: Model-based Impulse Responses to a Fertility Shock: Role of Nominal Rigidities

rate

$$\widehat{r}_t - \mathbb{E}_t[\widehat{\pi}_{t+1}] = \widehat{r}_t^n,$$

where the *natural rate of interest*, defined as the interest rate in the case of flexible prices, in log-deviation from its steady state value, is given by

$$\widehat{r}_t^n = \theta \nu_t.$$

Depending on θ, positive population growth shocks increase the natural rate of interest or not. Suppose there is no interest rate inertia ($\rho_r = 0$) and the coefficient on the output gap is zero ($\phi_y = 0$). The minimum state variable (MSV) solution for inflation under flexible prices is

$$\widehat{\pi}_t^{Flex} = \frac{\theta}{\phi_\pi - \rho_\nu} \nu_t. \tag{5.1}$$

With sticky prices ($\lambda_p > 0$), the MSV solution for the dynamics of output and inflation to a shock of ν_t is

$$\widetilde{y}_t = \frac{1 - \beta\rho_\nu}{\kappa} \Lambda \nu_t, \tag{5.2}$$

$$\widehat{\pi}_t = \Lambda \nu_t, \tag{5.3}$$

with $\Lambda \equiv \frac{\theta}{(1-\beta\rho_\nu)(1-\rho_\nu)/\kappa + (\phi_\pi - \rho_\nu)} > 0$. Thus,

$$\lim_{\rho_\nu \to 1} \widetilde{y}_t \approx 0, \tag{5.4}$$

$$\lim_{\rho_\nu \to 1} \widehat{\pi}_t = \widehat{\pi}_t^{Flex}. \tag{5.5}$$

Given the very high persistence of population growth shocks, the output gap is close to zero and the time paths of inflation under sticky and flexible prices are almost identical. Consider a modified monetary policy rule that includes population growth

$$\widehat{r}_t = \phi_\pi \widehat{\pi}_t + \chi \nu_t,$$

where χ denotes the responsiveness of the policy rate to actual changes in the population growth rate. The MSV solution for inflation is

$$\widehat{\pi}_t = \frac{\theta - \chi}{(1 - \beta\rho_\nu)(1 - \rho_\nu)/\kappa + (\phi_\pi - \rho_\nu)} \nu_t. \tag{5.6}$$

Thus, whenever the central bank underestimates ($\theta > \chi$) the impact of the population growth rate on the natural rate of interest, positive fertility shocks are inflationary.
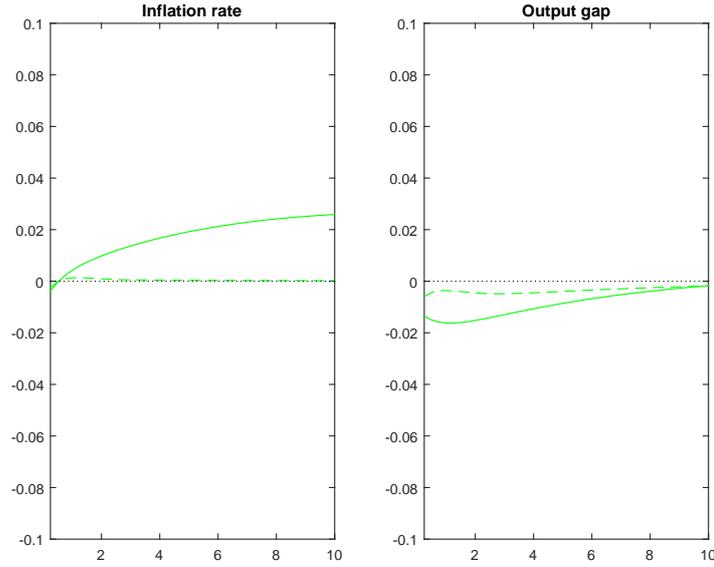
Solid lines: no policy inertia ($\rho_r = 0$). Dashed lines: difference rule ($\rho_r = 1$). $\theta = \widehat{\theta} = 0.59$

Figure 14: Model-based Impulse Responses to a Fertility Shock: Policy Inertia

Interest rate inertia Figure 14 shows the response to a fertility shock under the following monetary policy rule (Woodford, 1999; Orphanides and Williams, 2002, 2007)

$$\widehat{r}_t = \rho_r \widehat{r}_{t-1} + \phi_\pi \widehat{\pi}_t. \tag{5.7}$$

Holding the responsiveness of the policy rule to inflation fixed, more inertial policies are less exposed to a potential mismeasurement of the natural rate of interest, and help thus avoiding prolonged periods of inflation.

6 Conclusion

This paper addresses two questions. First, is there a positive link between population growth and the natural rate of interest? Second, is population growth inflationary, and what is the mechanism?

While overlapping generations models, as well as the Solow model indicate that there is positive relationship, the answer to the first question is ambiguous in the most commonly used model in macroeconomics: the infinitely-lived representative agent model. In particular, it depends on the weighting of the different generations in the utility function of the household. When generations are weighted by their size (Benthamite preferences), there is no link between population growth and the natural rate. When the future discounted utility of the household is maximized irrespective

of its future size (Millian preferences), there is a one-to-one link between population growth and the natural rate in steady state.

Carvalho and Ferrero (2014) build an overlapping generations model for the Japanese economy, arguing that the increase in life expectancy together with lower population growth led to a substantial decline in the natural rate and that due to the inactivity of the central bank deflation has emerged.

In this paper I estimate the effects of fertility shocks in a VAR model of the US economy over the period 1957Q1 to 2014Q4. Matching the empirical impulse responses to fertility shocks to the theoretical responses of a standard DSGE model, I estimate the parameter governing the weighting of different generations in the utility function of the representative household. The estimates range from 0.48 to 0.66, implying that a one percentage point increase in the steady state population growth rate leads to a 0.48-0.66 increase in the steady state natural rate of interest. Lower population growth in the aftermath of the baby boom and the subsequent baby bust has lead to a reduction of the natural rate of about 0.5 percentage points. Negative fertility shocks lowered inflation and the FFR by about 0.5 and 1 pp, respectively during the 1980s and 1990s.

In summary, this paper confirms the existence of a natural rate channel through which lower population growth exerts downward pressure on inflation and interest rates. The magnitude is moderate, however, at least compared to the simulation results of Carvalho and Ferrero (2014) for Japan. This might be partly due to the more pronounced demographic transition in Japan (figure 1). The VAR results indicate that fertility shocks have a larger impact on inflation than can be rationalized by the central bank's inadequate response to changes in the natural rate. In this paper I focus on the size of the working age population, not on its composition as in Jaimovich and Siu (2009) or Jaimovich et al. (2013). Since fertility shocks both affect the size and the composition of the labor force, future research in this direction may help better understanding the linkages between population growth and inflation.

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A Data Appendix

A.1 Data Sources

Table 5: US Population Data

	Description	Freq.	Code	Source
N_t	Civilian noninstitutional population, 16+	monthly	LNU00000000	BLS/CPS
LF_t	Civilian labor force, 16+	monthly	LNS11000000	BLS/CPS
$b_{t-16y,t}$	Fraction of persons surviving to age 16	decennial		NCHS
$Births_t$	Total number of live births	monthly		NCHS
$Deaths_t$	Total number of deaths, 15+	annual		NCHS

$b_{t-16y,t}$ interpolated to quarterly frequency. $Deaths_t = cN_t$, with $c = \frac{1}{T}\sum_{t=1}^{T}\frac{Deaths_t}{N_t}$. The monthly data for live births is seasonally adjusted using a stable seasonal filter.

Table 6: US Macroeconomic Data

Variable	Description	Code	Source
y_t	Gross Domestic Product	GDP	BEA
c_t	Personal Consumption Expenditures	PCE	BEA
x_t	Private Nonresidential Fixed Investment	PNFI	BEA
	Fixed Private Investment	FPI	BEA
h_t	Nonfarm Business Sector: Hours of All Persons	HOANBS	BLS
w_t	Nonfarm Business Sector: Compensation Per Hour	COMPNFB	BLS
π_t	Gross Domestic Product: Implicit Price Deflator	GDPDEF	BEA
	Consumer Price Index for All Urban Consumers: All Items	CPIAUCSL	BEA
r_t	Effective Federal Funds Rate	FEDFUNDS	FED

y_t, c_t, x_t, and h_t divided by N_t. y_t, c_t, x_t, and w_t in 2009 US Dollars. $\pi_t \equiv \ln(\te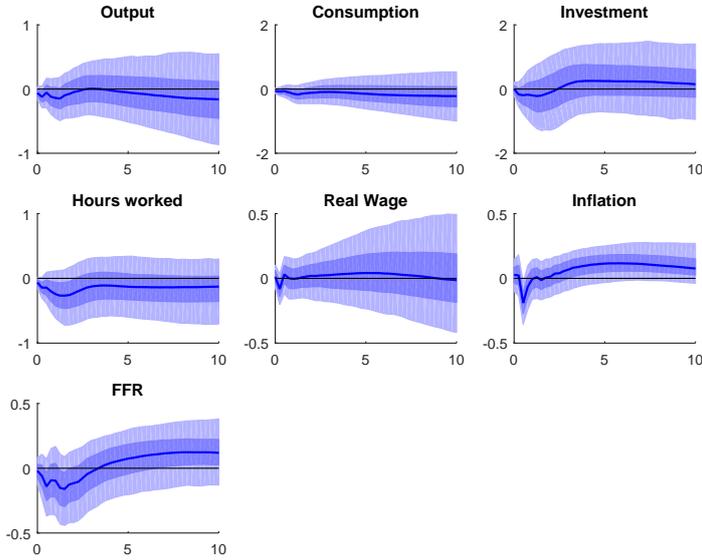xt{GDPDEF})_t - \ln(\text{GDPDEF})_{t-1}$. π_t and r_t annualized. All other variables in logs. Variables enter the VAR in levels. Data retrieved from FRED, Federal Reserve Bank of St. Louis.

A.2 VAR Robustness
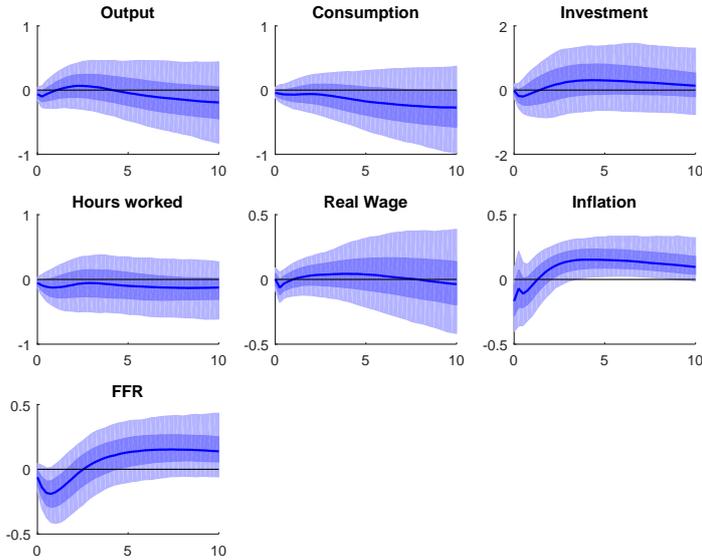
Median (solid lines), 68% and 95% probability bands (shaded areas) of posterior distribution. Years on x-axis.

Figure 15: VAR-based Impulse Response to a Fertility Shock - CPI Inflation

Median (solid lines), 68% and 95% probability bands (shaded areas) of posterior distribution. Years on x-axis.

Figure 16: VAR-based Impulse Response to a Fertility Shock - 4 Lags

B Technical Appendix

B.1 VAR

Consider the following VAR(p) model

$$z_t = c + \sum_{s=1}^{p} B_s z_{t-s} + u_t, \quad t = 1, ..., T, \tag{B.1}$$

with $\mathbb{E}[u_t] = 0$, $\mathbb{E}[u_t u_t'] = \Sigma_u$, and $\mathbb{E}[u_t u_s'] = 0$ for $s \neq t$. The endogenous variables are collected in the $M \times 1$ vector z_t whereases u_t is a $M \times 1$ vector of white noise processes. The VAR(p) can be written in compact form

$$Y = XB + U, \tag{B.2}$$

where

$$\underset{(T \times M)}{Y} \equiv \begin{bmatrix} y_1 & \cdots & y_T \end{bmatrix}', \quad \underset{(T \times K)}{X} \equiv \begin{bmatrix} 1 & y_0' & y_{-1}' & \cdots & y_{-p+1}' \\ 1 & y_1' & y_0' & \cdots & y_{-p+2}' \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & y_{T-1}' & y_{T-2}' & \cdots & y_{T-p}' \end{bmatrix},$$

$$\underset{(K \times M)}{B} \equiv \begin{bmatrix} c & B_1 & \cdots & B_p \end{bmatrix}', \quad \underset{(T \times M)}{U} \equiv \begin{bmatrix} u_1 & \cdots & u_T \end{bmatrix}', \quad K = Mp + 1.$$

With a noninformative (diffuse) prior

$$p(b, \Sigma_u) \propto |\Sigma_u|^{-0.5(K+1)}, \tag{B.3}$$

the posterior distribution is given by (Kadiyala and Karlsson, 1997)

$$p(b|\Sigma_u, y) \sim \mathcal{N}(\widehat{b}, \Sigma_u \otimes (X'X)^{-1}), \tag{B.4}$$

$$p(\Sigma_u|y) \sim iW((y - X\widehat{B})'(y - X\widehat{B}), T - K), \tag{B.5}$$

where $\widehat{B} \equiv (X'X)^{-1}(X'y)$ and $\underset{(MK \times 1)}{b} \equiv \text{vec}(B)$.

Impulse responses The companion form of (B.1) is

$$\widetilde{z}_t = \widetilde{c} + B\widetilde{z}_{t-1} + \widetilde{u}_t, \tag{B.6}$$

where

$$
\underset{(Mp \times 1)}{\widetilde{z}_t} \equiv \begin{bmatrix} z_t \\ z_{t-1} \\ \vdots \\ z_{t-p+1} \end{bmatrix}, \quad \underset{(Mp \times 1)}{\widetilde{c}} \equiv \begin{bmatrix} \widetilde{c} \\ 0 \\ \cdots \\ 0 \end{bmatrix},
$$

$$
\underset{(Mp \times Mp)}{\widetilde{B}} \equiv \begin{bmatrix} B_1 & B_2 & \cdots & B_{p-1} & B_p \\ I_M & 0 & \cdots & 0 & 0 \\ 0 & I_M & & 0 & 0 \\ \vdots & & \ddots & \vdots & 0 \\ 0 & 0 & \cdots & I_M & 0 \end{bmatrix}, \quad \underset{(Mp \times 1)}{\widetilde{u}_t} \equiv \begin{bmatrix} u_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}.
$$

Next, the moving average (MA) representation of the VAR is

$$
z_t = J\widetilde{z}_t = c + \sum_{i=0}^{\infty} \Phi_i u_{t-i} = c + \sum_{i=0}^{\infty} \Phi_i A \varepsilon_{t-i}, \tag{B.7}
$$

where $\underset{(M \times Mp)}{J} \equiv \begin{bmatrix} I_M & 0 & \cdots & 0 \end{bmatrix}$ and $\underset{(M \times M)}{\Phi_i} \equiv J\widetilde{B}^i J'$. Therefore, the impulse response to a structural shock at horizon h is given by

$$
IR(h) = \Phi_i A. \tag{B.8}
$$

Forecast error variance decomposition   The error of the h-step forecast is

$$
z_{t+h} - z_t(h) = \sum_{i=0}^{h-1} \Phi_i A \varepsilon_{t+h-i}.
$$

The proportion of the h-step forecast error variance of variable j, accounted for an innovation of ε_{mt} is

$$
FEVD(jm, h) = \sum_{i=0}^{h-1} (e_j' \Phi_i A e_m)^2 / MSE[z_{j,t}(h)],
$$

with e_m representing the mth column of I_M. The denominator $MSE[z_{j,t}(h)]$ is the jth diagonal element of the mean squared error (MSE) matrix

$$
MSE[z_t(h)] = \sum_{i=0}^{h-1} \Phi_i A A' \Phi_i' = \sum_{i=0}^{h-1} \Phi_i \Sigma_u \Phi_i'.
$$

Historical decomposition   Given the residuals $\widehat{u}_t$, calculate the structural shocks $\widehat{\varepsilon}_t = A^{-1}\widehat{u}_t$. Starting with $\widehat{z}_0 = 0$ the historical contribution of shock m to variable j is

calculated recursively

$$\widehat{z}_t(j) = \Phi \widehat{z}_{t-1} + A\widehat{\varepsilon}_t(m), \quad \Phi = J\widetilde{B}J',$$

where $\widehat{z}_t(j)$ is the ith row of $\widehat{z}_t$, and where $\widehat{\varepsilon}_t(m)$ is the mth column of $\widehat{\varepsilon}_t$.

B.2 Log-linearized Equations

Labor supply:

$$\widehat{mrs}_t = \varphi \widehat{h}_t + \widehat{c}_t$$

Consumption Euler equation:

$$\widehat{c}_t = \mathbb{E}_t[\widehat{c}_{t+1}] - (r_t - \mathbb{E}_t[\widehat{\pi}_{t+1}] - \nu_t)$$

No arbitrage:

$$\widehat{r}_t - \mathbb{E}_t[\widehat{\pi}_{t+1}] = (1 - \beta(1-\delta))\mathbb{E}_t[\widehat{r}^K_{t+1}] + \beta(1-\delta)\mathbb{E}_t[\widehat{q}_{t+1}] - \widehat{q}_t$$

Investment:

$$\widehat{x}_t = \frac{1}{1+\beta}\widehat{x}_{t-1} + \frac{1}{1+\beta}\mathbb{E}_t[\widehat{x}_{t+1}] + \frac{1}{(1+\beta)\eta}\widehat{q}_t$$

Capital utilization:

$$\widehat{r}^K_t = \omega \widehat{u}_t$$

Capital accumulation:

$$\widehat{k}_{t+1} + \nu_t = (1-\delta)\widehat{k}_t + \delta \widehat{x}_t$$

Real wage:

$$\widehat{w}_t = \widehat{mc}_t + \alpha(\widehat{k}_t + \widehat{u}_t) - \alpha \widehat{h}_t$$

Rental price:

$$\widehat{r}^K_t = \widehat{mc}_t + (\alpha-1)(\widehat{k}_t + \widehat{u}_t) + (1-\alpha)\widehat{h}_t$$

Price inflation:

$$\widehat{\pi}_t = \frac{(1-\lambda_p)(1-\beta\lambda_p)}{\lambda_p}\widehat{mc}_t + \beta\mathbb{E}_t[\widehat{\pi}_{t+1}]$$

Wage inflation:

$$\widehat{w}_t = \frac{1}{1+\beta}\frac{(1-\lambda_w)(1-\beta\lambda_w)}{\lambda_w(1+\epsilon_w\varphi)}(\widehat{mrs}_t - \widehat{w}_t) + \frac{1}{1+\beta}\widehat{w}_{t-1}$$
$$+ \frac{\beta}{1+\beta}\mathbb{E}_t[\widehat{w}_{t+1} + \widehat{\pi}_{t+1}] - \frac{1}{1+\beta}\widehat{\pi}_t$$

Production function:
$$\widehat{y}_t = \alpha(\widehat{k}_t + \widehat{u}_t) + (1-\alpha)\widehat{h}_t$$

Aggregate resource constraint:

$$\widehat{y}_t = s_c\widehat{c}_t + s_x\widehat{x}_t + s_u\widehat{u}_t$$
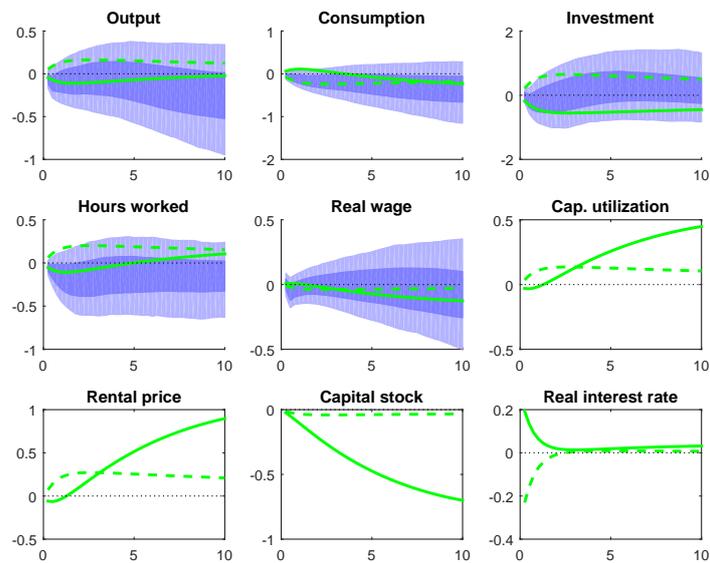
Output gap:

$$\widetilde{y}_t = \widehat{y}_t - \widehat{y}_t^n$$
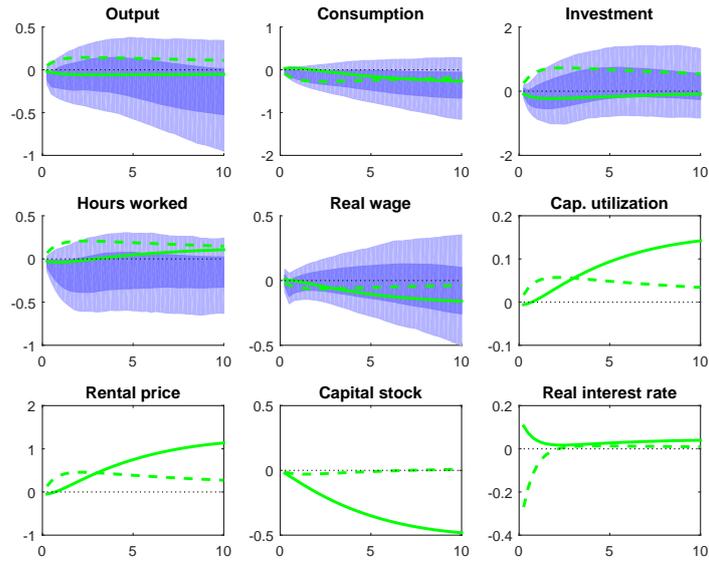
Monetary policy:
$$\widehat{r}_t = \rho_r\widehat{r}_{t-1} + (1-\rho_r)(\phi_\pi\widehat{\pi}_t + \phi_y\widehat{y}_t)$$

B.3 Variable Capital Utilization

Solid lines: model-based IRFs (Millian preferences). Dashed lines: model-based IRFs (Benthamite preferences). Shaded areas: 68% and 95% probability bands of VAR-based IRFs. Years on x-axis.

Figure 17: Model-based IRF to a Fertility Shock - High Elasticity of Capital Utilization ($\omega = 0.5$)

Solid lines: model-based IRFs (Millian preferences). Dashed lines: model-based IRFs (Benthamite preferences). Shaded areas: 68% and 95% probability bands of VAR-based IRFs. Years on x-axis.

Figure 18: Model-based IRF to a Fertility Shock - Low Elasticity of Capital Utilization ($\omega = 2$)