Quantitative Easing, the Repo Market, and the Term Structure of Interest Rates

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Introduction

- Quantitative Easing in the E.U. and U.S. clearly shows that demand impacts

  i) the yield curve
  ii) repo rates

- Previous studies have considered these facts in isolation. Does the term structure interact with money markets where bonds collateralize loans?
- Standard models assume infinitely available bonds and exogenous short rate

- This paper: quantity-driven model where bonds serve as
  i) investment opportunities
  ii) collateral for overnight loans

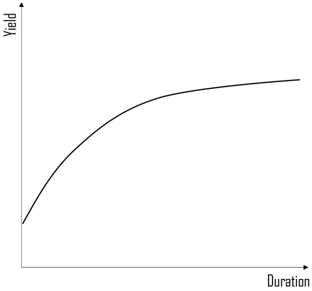
The Literature

Demand for bonds affects bond prices persistently
D'Amico and King 2013, Bernanke 2020, Vayanos and Vila 2021

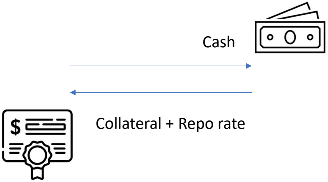
Demand for bonds also affects bond repo rates
Duffie 1996, Corradin and Maddaloni 2020, He, Nagel, and Song 2022

Problem: in TSMs, the short rate is *exogenous* to demand pressure
This paper: interactions of bond and repo markets along the yield curve

Bond Market



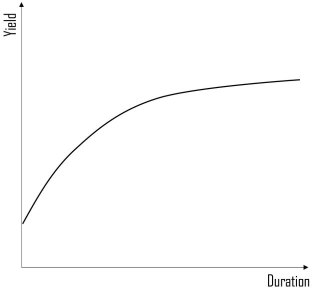
Repo Market



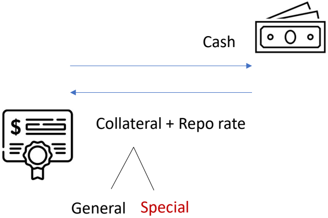
securities with identical CFs have same price
bonds reflect duration risk

bonds have a common exogenous repo rate

Bond Market



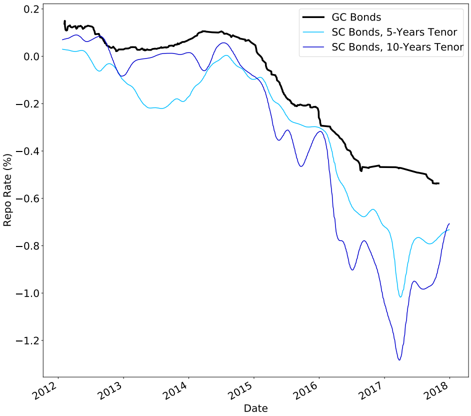
Repo Market



investors bid up the price of their preferred bonds
securities with identical CFs differ in price
bonds reflect duration risk and demand risk

arbitrageurs sell short overpriced bonds
cross section of repo rates interacts with demand

Motivation



ON repo rates on trades collateralized by German sovereign bonds, moving average. Source: MTS.

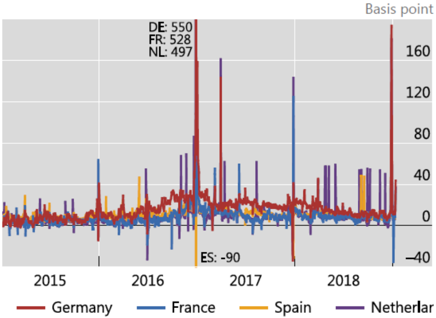
Repo specialness = GC rate – SC rate

US

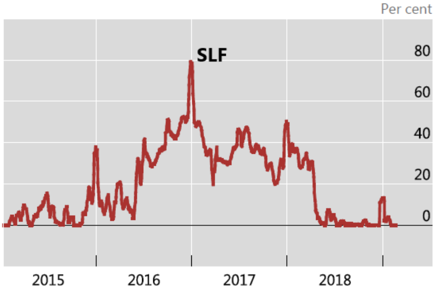
Persistence

Indicators of European bond specialness

European specialness spreads[1]



Share of special volume for Germany[2]



[1] The specialness is defined as the difference between the general collateral rate and the special rate. [2] 10-day moving average. Source: BIS.

The Results

First quantity-driven term structure model with endogenous money market

Why do we care? We find that repo specialness

i) strengthens the local supply channel of QE

ii) dampens the duration extraction channel of QE

Theory

The Model

Cash market

- Continuous-time market for riskless ZCB with tenor τ and status $i = \{\text{g, s}\}$
- General (g) and special (s) bonds have equal cash flows, different demand

$$\text{Yield to maturity is } y_{i,t}^{\tau} = -\frac{1}{\tau} \log P_{i,t}^{\tau}.$$

Repo market

- Short rate is the GC repo rate (SOFR)

$$dr_t = \kappa_r(\bar{r} - r_t)dt + \sigma_r dv_t^r.$$

- What about the SC rate r_t^{τ}? Solve endogenously

Preferred-habitat investors

Bonds "*on special*" are issues subject to considerable demand pressure $Z_t^\tau(s)$.

$$Z_t^\tau(i) = \begin{cases} -\alpha_\tau \log P_{i,t}^\tau - \theta_\tau & i = s \quad \leftarrow \text{QE-eligible} \\ 0 & i = g \quad \leftarrow \text{QE-ineligible} \end{cases}$$

Market Clearing

Arbitrageurs connect prices over habitat segments (Modigliani and Sutch 1966)

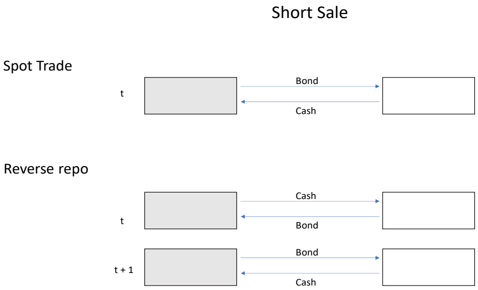
$$\underset{\text{habitat investors}}{\underset{\uparrow}{Z_{i,t}^{\tau}}} = -\underset{\text{arbitrageurs}}{\underset{\uparrow}{X_{i,t}^{\tau}}}$$

Arbitrageurs' problem

$$\max_{\{X_{i,t}^{\tau}\}} \quad \frac{\mathbb{E}_t\left[dW_t\right]}{dt} - \frac{\gamma}{2}\frac{\mathbb{V}_t\left[dW_t\right]}{dt}$$

Arbitrageurs

$$dW_t = r_t W_t dt + \underbrace{\int_0^\infty X_{g,t}^{\tau} \left(\frac{dP_{g,t}^{\tau}}{P_{g,t}^{\tau}} - r_t \right) d\tau}_{\text{General bonds}} + \underbrace{\int_0^\infty X_{s,t}^{\tau} \left(\frac{dP_{s,t}^{\tau}}{P_{s,t}^{\tau}} - r_t^{\tau} \right) d\tau}_{\text{Special bonds}}$$



General bonds, Special bonds

Segmentation: price is affine in short rate and, conditionally on status, demand

$$-\log P_{i,t}^{\tau} = \begin{cases} A_\tau r_t + B_\tau X_{s,t}^{\tau} + C_\tau & i = s, \\ A_\tau r_t + C_\tau & i = g. \end{cases}$$

Demand pressure $\Rightarrow$ bonds with identical cash flows trade at different prices

General bonds, Special bonds

Segmentation: price is affine in short rate and, conditionally on status, demand

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Demand pressure $\Rightarrow$ bonds with identical cash flows trade at different prices

$$-\log P^{\tau}_{i,t} = a_{i,\tau} r_t + b_{i,\tau} \theta^{\tau}_t + c_{i,\tau}$$

$$a_{i,\tau} = \frac{A_\tau}{1 + \alpha_\tau B_{i,\tau}},\; b_{i,\tau} = \frac{B_\tau}{1 + \alpha_\tau B_{i,\tau}},\; c_{i,\tau} = \frac{C_\tau}{1 + \alpha_\tau B_{i,\tau}},\; B_{i,\tau} = B_\tau \mathbb{1}_{[i=s]}$$

Equilibrium in the Bond Market

Repo rate

$$\mu_{i,t}^{\tau} - \underset{\downarrow}{r_t^{\tau}} = -a_{i,\tau}\lambda_{r,t}$$

- Repo specialness priced on the bond market
- Describes two term structures, general and special

Duration extraction

QE reduces arbitrageurs' portfolio duration flattening the yield curve

$$\text{Price of risk } \lambda_{r,t} = -\gamma \underbrace{\sigma_r^2 \int_0^\infty \left(a_{g,\tau} \overset{=0}{\overset{\downarrow}{X_{g,t}^{\tau}}} + a_{s,\tau} X_{s,t}^{\tau} \right) d\tau}_{\text{arbitrageurs' duration}}$$

- Duration risk of arbitrageurs ↓ premium for *shorting* long bonds ↓ and TP ↓

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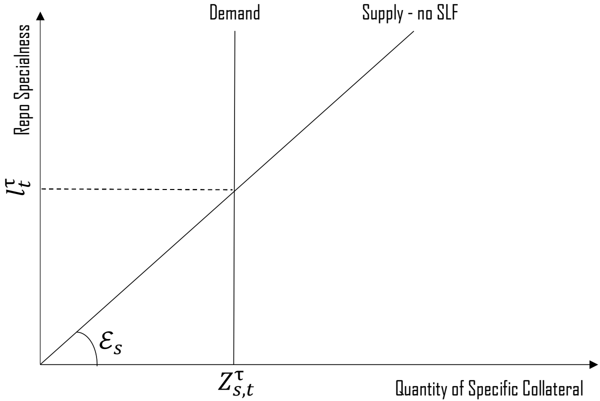
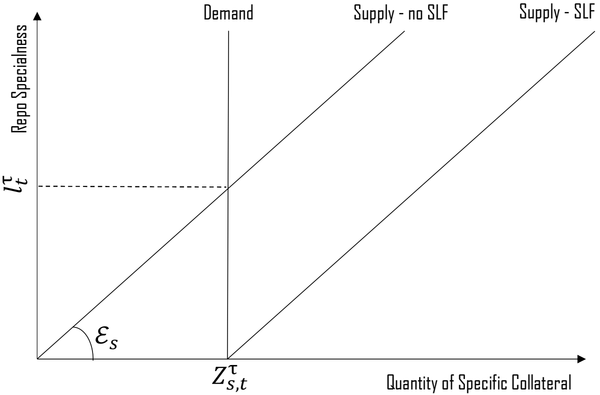
Equilibrium in the Repo Market



No SLF - Specialness

$$l_t^\tau = \mathcal{E}_i Z_{i,t}^\tau$$

Equilibrium in the Repo market



No SLF - Specialness

$$l_t^\tau = \mathcal{E}_i Z_{i,t}^\tau$$

SLF - No specialness

$$l_t^\tau = \mathcal{E}_i (X_{i,t}^\tau + Z_{i,t}^\tau) = 0$$

Bond Price and Repo Rates

Bond price is inversely related to PDV of its repo rates

$$P^{\tau}_{i,t} = \exp\left(- A_{\tau} r_t - B_{i,\tau} X^{\tau}_{i,t} - C_{\tau} \right) = \mathbb{E}^{\mathbb{Q}}_t \left[\exp\left(- \int_0^{\tau} r^{\tau-u}_{t+u} du \right) \right]$$

Special bonds have higher prices and lower repo rates

$$\underset{\text{Special bond premium}}{\exp\left(B_{i,\tau} X^{\tau}_{s,t} \right)} = \frac{\mathbb{E}^{\mathbb{Q}}_t \left[\exp\left(- \int_0^{\tau} r_{t+u} du \right) \right]}{\mathbb{E}^{\mathbb{Q}}_t \left[\exp\left(- \int_0^{\tau} r_{t+u} du \right) \exp\left(-\mathcal{E}_s \int_0^{\tau} \underset{\text{PDV(specialness)}}{X^{\tau-u}_{t+u}} du \right) \right]}$$

General Equilibrium (i) SLF

Bond pricing coefficients

$$a_{i,\tau} = \frac{1 - e^{-\kappa_r^* \tau}}{\kappa_r^*}$$

$$b_{i,\tau} = 0$$

$$c_{i,\tau} = \kappa_r^* \overline{r}^* \int_0^\infty a_{i,u} du - \frac{\sigma_r^2}{2} \int_0^\infty a_{i,u} du$$

Money market rates

The short rate r_t is unique

Vayanos and Vila 2021 model

General Equilibrium (ii) No SLF

Bond pricing coefficients

$$a_{i,\tau} = \frac{1 - e^{-\kappa_r^* \tau}}{\kappa_r^*}$$

$$b_{i,\tau} = \frac{\mathcal{E}_i (1 - g_\tau)(1 - e^{-\int \overline{\theta}_\tau d\tau})}{\overline{\theta}_\tau}$$

$$c_{i,\tau} = \kappa_r^* \overline{r}^* \int_0^\infty a_{i,u} du - \frac{\sigma_r^2}{2} \int_0^\infty a_{i,u} du$$

Money market rates

Cross-section of repo rates $r_t^\tau = r_t - \mathcal{E}_i Z_{i,t}^\tau$

Two-markets GE exchange economy

Conventional Monetary Policy

Repo specialness impairs transmission of rate hikes to money and bond markets (Nguyen, Tomio, and Vari 2023, Eisenschmidt, Ma, and Zhang 2024)

- SC rates respond to changes in r_t only by $1 - \alpha_\tau a_{s,\tau} \mathcal{E}_s$

$$r_t^\tau = r_t + \mathcal{E}_s \underbrace{\left[\theta_\tau - \alpha_\tau \left(a_{s,\tau} r_t + b_{s,\tau}\theta_\tau + c_{s,\tau}\right)\right]}_{X_{s,t}^\tau}$$

- SC yields respond to rate hikes less than yields of GC

$$A_\tau (1 + \alpha_\tau B_{s,\tau})^{-1} < A_\tau$$

Quantitative Easing

Repo specialness dampens the effects of QE ($\Delta\theta_\tau < 0$) on the term premium TP

$$\underbrace{\frac{1}{\tau}\gamma A_\tau^2 \sigma_r^2 \int_0^\infty A_u du}_{\frac{\partial TP}{\partial \theta_\tau},\ \text{SLF}} \geq \underbrace{\frac{1}{\tau}\gamma A_\tau \sigma_r^2 a_{s,\tau}(1-\alpha_\tau b_{s,\tau}) \int_0^\infty A_u du}_{\frac{\partial TP}{\partial \theta_\tau},\ \text{No SLF}}$$

Intuitive, specialness shrinks arbitrageurs' holdings $X_{s,t}^\tau$

$$\downarrow X_{s,t}^\tau = \theta_\tau - \alpha_\tau \left(a_{s,\tau} r_t + \uparrow b_{s,\tau}\theta_\tau + c_{s,\tau}\right)$$

QE extracts more duration risk (TP↓) when money markets are functional

Repo specialness and the term premium

- QE $\uparrow \implies$ duration of private sector's portfolio $\downarrow \implies$ term premium $\downarrow$
- Private sector: sells long bonds short, and invests at the risk-free rate

- Short selling is easy if bonds are elastically supplied...
- ...but is costly if there is a specialness premium

The trade-off inherent in quantitative policies

FOC of arbitrageurs

$$\mu_{i,t}^{\tau} - r_t + l_t^{\tau} = -a_{i,\tau}\lambda_{r,t}$$

Repo specialness $l_t^{\tau} \uparrow$ for given r_t, either

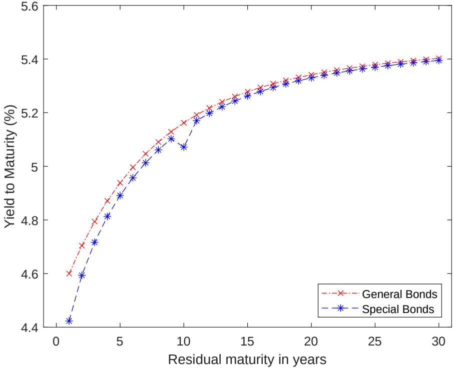
i) Stronger local supply $\mu_{i,t} \uparrow$ relative to duration risk $a_{i,\tau}$ (price anomaly)

ii) Weaker duration extraction $\lambda_{r,t} \downarrow$ (steeper YC)

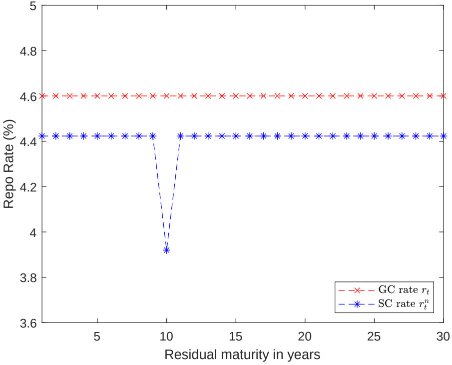
Trade-off applies to both QE and QT

Calibration

Repo dual to the cash market

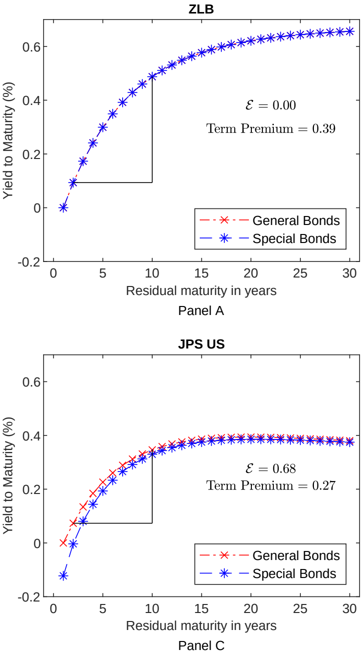


Long Rates



Short Rates

- Repeated interventions smooth out distortions in the repo market over time
- Repo specialness strengthens the local supply channel of QE


ZLB
VV
JPS US
JPS EU
Yield to Maturity (%)
Residual maturity in years
$\mathcal{E}$ = 0.00
Term Premium = 0.39
$\mathcal{E}$ = 0.00
Term Premium = 0.24
$\mathcal{E}$ = 0.68
Term Premium = 0.27
$\mathcal{E}$ = 0.78
Term Premium = 0.29
General Bonds
Special Bonds
Panel A
Panel B
Panel C
Panel D

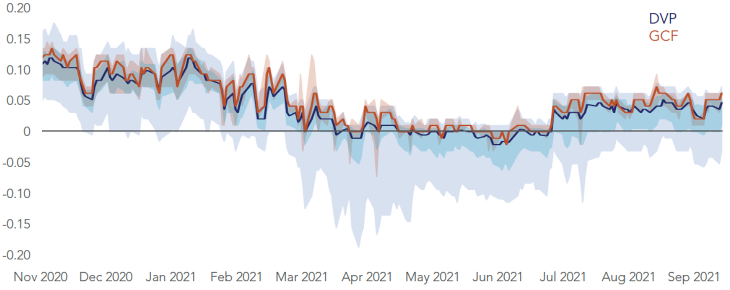
Conclusion

New framework to think about term structure and money market

intuitive: bond scarcity, local supply $\uparrow$ duration extraction $\downarrow$
realistic: specialness endogenous in arbitrageurs' short-selling
tractable: admits solutions in closed form

Policy implications

MP should be paired with securities lending facilities
Policy may induce long-term investors to lend securities



- US Treasury data, source OFR

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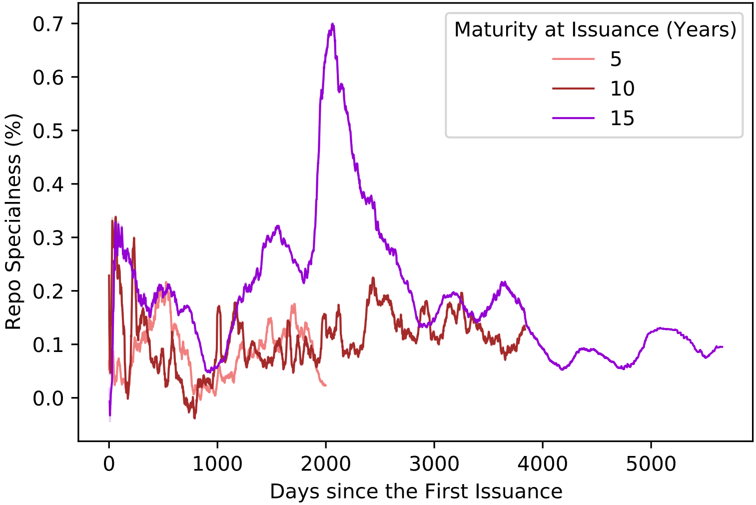

- MTS German data show that repo specialness is persistent

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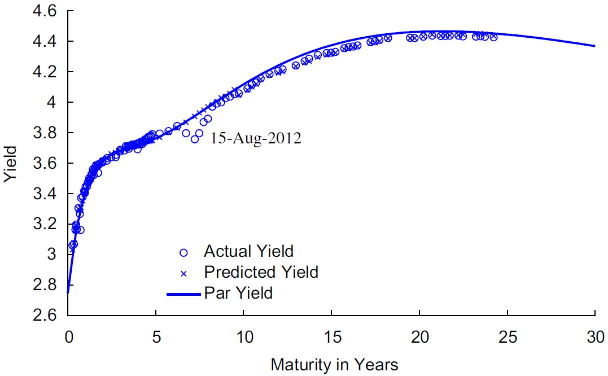

Fig. 4. Premium for the cheapest-to-deliver issue on May 24, 2005.

- Figure from Gürkaynak, Sack, and Wright 2007

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