A model of the euro-area yield curve with discrete policy rates*

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Abstract

This paper presents a no-arbitrage model of the yield curve where the central-bank policy rate is explicitly incorporated and that is consistent with the zero-lower-bound restriction. Notably, the framework makes it possible to account for the effects on the yield curve of the excess-liquidity situation that followed the implementation by the ECB of non-standard monetary policy measures in late 2008. After having estimated the model using daily euro-area data, I explore the behaviour of risk premia at the short end of the yield curve. These risk premia are neglected by the widely-used practice that consists in backing out market forecasts of future policy-rate moves from money-market forward rates. My results suggest that this practice is valid in terms of sign of the expected target moves, but that it tends to overestimate their size. As an additional contribution, the model is exploited to simulate forward-guidance measures. A credible commitment of the central bank to keep its policy rate unchanged for a deterministic period of time can result in substantial declines in yields. For instance, a central-bank commitment to keep the policy rate at 1% over the next 2 years would imply a decline in the 5-year rate of about 25 basis points.

JEL codes: E43, E44, E47, E52, G12.

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1 Introduction

The standard view of the monetary policy transmission mechanism suggests that central banks' actions are transmitted to the economy through their effect on market interest rates. According to this standard view, a restrictive monetary policy pushes up both short-term and long-term interest rates, leading to less spending by interest-sensitive sectors of the economy, and vice versa.

While there is a strong empirical support for the assertion that monetary policy is a major driver of the yield-curve fluctuations, the quantitative aspects regarding the transmission mechanism along the yield curve –from the overnight interbank market to longer-term interest rates– are less clear. Among the vast number of interest-rate term-structure models, only a very few explicitly deal with policy-rate decisions. This partly reflects the technical difficulties associated with accommodating the specificities of the policy-rate process.[1] Piazzesi (2005) and Fontaine (2009) propose term-structure models in which changes in the target for the policy rate have discrete supports. They estimate their models on U.S. data covering respectively the periods 1994-1998 (weekly) and 1994-2007 (daily). However, their models technically imply non-zero probabilities of negative interest rates for all maturities on the term structure. While this caveat may be tenable when the short-term interest rate is far enough from zero –the conditional probabilities of having negative interest in the subsequent periods being negligible–, it is more problematic in the current context of very low interest rates.

More generally, many of the tractable yield-curve models are not consistent with the zero-lower-bound (ZLB) restriction.[2] Furthermore, the crisis has highlighted several other limitations of various standard yield-curve models (e.g. homoskedasticity and linearity). Obviously, these limitations are particularly pronounced at a time

[1] See e.g. Rudebusch (1995), Hamilton and Jorda (2002), Balduzzi, Bertola and Foresi (1997) and Balduzzi et al. (1998) for models of the U.S. Federal Funds rate target.

[2] See Dai and Singleton (2003) or Piazzesi (2010). Hamilton and Wu (2012) propose a way to adapt the standard Gaussian framework to account for an extended period of constant short-term rate. However, they implicitly assume that when this phase ends, (a) such a phenomenon cannot happen again and (b), the short-term rate can turn negative again. Andreasen and Meldrum (2011) or Kim and Singleton (2011) show that the quadratic Gaussian framework can be used to preclude negative interest rates. However, these latter models can not accommodate long periods of unchanged interest rates.

when policymakers, and notably central bankers, have to consider all possible options to deal with the crisis.

In this paper, I propose a novel and tractable no-arbitrage term-structure model that addresses the above-mentioned limitations. The model is consistent with the zero-lower bound, can handle non-linarities and heteroskedasticity in interest rates. The tractability of the model is illustrated by estimating it on euro-area daily data covering the last 13 years. The estimation results shed light on the influence of the ECB monetary policy on the term-structure of interest rates. Notably, the results show the key effect of the monetary-policy phases –tightening, easing or status quo– on the shape of the yield curve. Besides, the analysis provides evidence of the existence of substantial risk premia at the short- to medium-end of the term structure of interest rates.[3] This implies notably that the common market practice that consists in backing out market forecasts of next policy-rate moves from money-market forward rates is biased.[4] More precisely, my results suggest that while this practice is valid in terms of sign of the expected target moves, it tends to overestimate their size. Besides, these risk premia turn out to be the most important when the monetary policy is in a tightening phase, the deviation between the 12-month-ahead risk-neutral forecast of the policy rate (this forecast is approximately a forward rate) and its physical counterpart being of about 50 basis points.

As an additional contribution, this model is exploited to assess the potential effects of so-called *forward policy guidance* measures. These measures, that consist of commitments of the central bank regarding the future path of its policy rate, are expected to provide more accommodation at the ZLB.[5] Indeed, the objective of these measures is to provide a stimulus to the economy by making market participants revising down their expectations of future short-term interest rates, thereby pushing down medium- to long-term interest rates. The effectiveness of such measures is the subject of substantial debate (Williams, 2011). Using new-Keynesian general

[3] The existence of such risk premia in the short end of the euro-area yield curve has notably been evidenced by Durré, Evjen and Pilegaard (2003).

[4] This common market practice implicitly assumes that the expectation hypothesis holds at the short-end of the yield curve.

[5] See Bernanke and Reinhart (2004) for a list and discussion of the potential policy options available to monetary-policy authorities when the zero bound is binding.

equilibrium models, Eggertsson and Woodford (2003), Campbell et al. (2012) or Levin et al. (2010), among others, investigate the impacts of forward policy guidance. While the former two studies find that forward guidance can be effective in terms of macroeconomic stabilisation, the latter shows that such measures may be insufficient to deal a "Great Recession"-style shock. As in Gagnon et al. (2011), Kool and Thornton (2012), Rudebusch and Bauer (2011) or Jardet, Monfort and Pegoraro (2009), I focus on the effects of unconventional monetary policies on the term structure of interest rates. Specifically, as in the latter paper, I use my model to simulate the effects of commitments of the central bank to keep its policy rate at its current level for (at least) a deterministic period of time. In the present framework, where the policy rate is explicit, such a simulation is carried out in a straightforward and consistent manner. According to the results, forward-guidance measures could lead to a substantial downward shift in the yield curve. The lower the policy rate, the larger the effect: for instance, in a context characterised by a policy rate of 1% (respectively 3.5%), the model predicts that the announcement of a commitment to keep the target rate unchanged for at least 2 years would be followed by a 25 bp (resp. a 5 bp) decline in the 5-year yield.

In my framework, changes in the monetary-policy rate are explicit and central. This implies that this model is particularly adapted to depict the dynamics of the short-end of the yield curve, where the influence of monetary policy decisions is the most evident (see Cochrane and Piazzesi, 2002). In my model, the shortest-term rate is the interbank overnight interest rate, which most central banks aim at stabilising to a level close to a policy (or target) rate. Target changes take place on pre-determined monetary-policy meeting dates and are positive multiples of 25 basis points (or 0.0025), implying a discrete support of the policy rate. Further, the model is consistent with the fact that target-rate changes occur infrequently, on a daily time scale, and with policy inertia (i.e. that target changes are often followed by additional changes in the same direction). These appealing features stem from an original use of regime-switching techniques, each regime being characterised by a given tick of the policy rate and a given monetary-policy phase: tightening, easing or status quo. The definition of these phases is consistent with observed central banks' target-setting behaviour and communication (see Smaghi, 2009). The probabilities of occurrences of target moves depend on the monetary-policy phase and on the level of the target rate.

The model is completed by specifying the dynamics of the so-called EONIA spread, that is the yield differential between the Euro Over-Night Index Average and the main policy rate. While this spread was mostly transitory before 2007, persistent deviations appeared in October 2008, following changes in the monetary-policy implementation in the euro area in response to the financial crisis. To capture that change in the behaviour of the EONIA spread, an additional two-state Markov-switching process is introduced, one of these two states corresponding to a situation in which banks' excess liquidity translates into a drop of the interbank rate with respect to the target (see Soares and Rodrigues, 2011).

Consistently with the choice of the EONIA as the shortest-term rate, the empirical exercise uses Overnight Index Swap (OIS) rates as longer-term yields.[6] An OIS is a fixed-for-floating interest rate swap with a floating rate leg tied to the index of daily interbank rates, that is the EONIA in the euro-area case. OIS have become especially popular hedging and positioning vehicles in euro financial markets and grew significantly in importance during the financial turmoil of the last few years.[7] The OIS curve is closely watched by practitioners to gauge what policy-rate changes the market has already priced in.

The model involves a lot of Markovian regimes (more than 200), which distinguishes it from earlier term-structure models involving regime switching.[8] In spite of this unusual feature, the approach remains tractable both in terms of bond pricing and estimation. The yields of different maturities turn out to be equal to linear combinations of the factors (including the regime variable), the factors loadings be-

[6] This is done only for the second part of my sample, i.e. 2005-2011. Indeed, long-term OIS are not avaiable before then. In the first part of the sample, I use EURIBOR swaps (see Subsection 2.2).

[7] While the United States has a liquid Fed Funds future contract (Gurkaynak, 2005 or Gurkaynak, Sack and Swanson, 2007), markets in most other countries rely exclusively on their local-currency-denominated OIS market for hedging central bank policy (Lang, 2010).

[8] See, e.g., Bansal and Zhou (2002), Dai, Singleton and Yang (2007), Ang, Bekaert and Wei (2008) or Lemke and Archontachis (2008).

ing given by simple formulas involving a limited number of matrix products.[9] The model can generate the usual shapes of the yield curve (steep, flat, inverse, humped, inverse-humped) and accommodates heteroskedasticity in the yield dynamics. As regards the estimation, a key point is that regimes are only partially hidden: a characteristic of the regimes, namely the central-bank policy rate, is observed by the econometrician.[10] Therefore, the econometric model can be seen as a six-state (three monetary-policy regimes and two liquidity regimes) Markov-switching model with heterogenous probabilities of transition, the latter depending on the observed target rate.

The model is estimated by maximum likelihood techniques. The computation of the log-likelihood is based on an innovative joint use of the Kitagawa-Hamilton's filter and so-called inversion techniques introduced by Chen and Scott (1993). The fit of the model is satisfying, the standard deviations of the pricing errors being of 8 basis points (from 1 month to 4 years). An important output of the approach are the probabilities of being in the different hidden Markovian states. To that respect, this approach is an illustration of the results of Bikbov and Chernov (2008) who underline the importance of using yield-curve information to identify monetary-policy regimes.

The remainder of the paper proceeds as follows. Section 2 presents the data and emphasizes stylized facts. Section 3 develops the model. Section 4 presents the estimation strategy and results. Section 5 documents the behavior of policy-rate-related risk premia. Section 6 derives some implications of the model regarding the commitment of the central bank to keep the target rate fixed for a given period of time. Section 7 concludes.

[9] In particular, the derivation of the term-structure of yields does not rely on the recursive algorithms usually used to solve discrete-time term structure models (as in Ang and Piazzesi, 2003). This point is crucial to make the model easily amenable to estimation using high-frequency data.

[10] I assume that market participants observe latent regimes and factors, as in most yield-curve studies involving latent factors.

2 Data and stylized facts

2.1 The EONIA and the Eurosystem's framework

Contrary to the Fed or the Bank of England, the ECB does not have an explicit interest-rate target. However, its aim is explicitly to *"influence money market conditions and steer short-term interest rates"* (ECB, 2011). This is done by using primarily the official interest rates: *"The (long) chain of causes and effect linking monetary policy decisions with the price level starts with a change in the official interest rates by the central bank on its own operations."*

In order to influence short-term money-market rates, a shortage of liquidity is created by imposing mandatory reserves on banks within the euro area. Specifically, credit institutions are required to hold compulsory cash deposits on accounts with the Eurosystem. The reserve requirements are based on the amount and profile of liabilities on a bank's balance sheet as of every month end. The banks can refinance themselves through the ECB's weekly Main Refinancing Operations (MROs). In these weekly refinancing operations, the ECB returns liquidity to the market by allowing banks to tender for cash (against collateral). In the remaining of this paper, by abuse of language, I use the term policy rate (or target rate) for the rate at which liquidity is supplied in the regular weekly monetary policy operations. However, there are two additional policy rates in the Eurosystem framework. Indeed, the latter is completed by a symmetric corridor bracketing the main policy rate.[11] The lower bound of the corridor, called the deposit-facility rate, is the rate at which counterparties can deposit cash overnight with the Eurosystem. The upper bound is the lending-facility rate, at which counterparties can borrow funds overnight from the Eurosystem. The target rate and the corridor is displayed in Panel A of Figure 1.

After having been fixed till June 2000, the MROs' rate then became variable.[12] In October 2008, in a context of worldwide financial stress, the Eurosystem adopted

[11] See Kahn (2010) for a comprehensive description and an international comparison of "corridor" systems.
[12] At that time, the target, or refi rate, acted as the minimum bid rate at the MRO.

a fixed-rate full allotment (FRFA) tender procedure: since then, the ECB accommodates any demand for liquidity its bank counterparties might express at the policy rate –against eligible collateral– in unlimited amounts.

While the policy rate defines the rate at which banks can refinance themselves through the ECB against collateral, EONIA (Euro OverNight Index Average) fixings reflect rates at which banks refinance themselves on the interbank market on an unsecured basis.[13] In "normal" circumstances, EONIA rates trade in close relation to ECB marginal rates but can also include a premium related to the unsecured nature of the lending.

Panel A of Figure 1 compares the fluctuations of the target with these of the EONIA. Changes in the policy rate are decided during the first of the bimonthly meetings of the ECB's Governing Council. On a daily scale, this implies a step-like behavior for the target rate. Over the estimation sample (January 1999 – February 2012), there were 18 rises in the target rate (16 of 25 bp and 2 of 50 bp) and 16 cuts in target rates (7 of 25 bp, 8 of 50 bp and one of 75 bp). Panel A of Figure 1 also suggests that the EONIA is closely linked to the target rate. However, by displaying the EONIA spread –i.e. the yield differential between the EONIA and the policy rate–, Panel B highlights the break in the relationships between these two rates that occurred in 2008. This break can be related to non-standard monetary-policy measures that were taken in response to the financial crisis. A particularly important decision was the one to move from variable rate tender procedures in liquidity providing operations to FRFA. Together with the expansion of the spectrum of maturities at which liquidity was being offered to the market, this measure generated a steady excess of liquidity balances in the overnight market, as banks began supplying in the interbank market the precautionary cash buffers that they were securing at the ECB.[14] An excess supply of liquidity in overnight trades put downward pressure

[13] The EONIA is computed as a weighted average of all overnight unsecured lending transactions undertaken in the interbank market, initiated within the euro area by the contributing banks. It is computed by the ECB at the end of every TARGET day (since January, 4 1999). The banks contributing to the EONIA are the same first class market standing banks as the panel banks quoting for Euribor. See www.euribor-ebf.eu for more details.

[14] A large share of the cash buffers is held with the ECB, the banks using massively the marginal deposit facility.

on the overnight interest rate, which drifted toward the lower limit of the monetary policy corridor (see Beirne, 2012 and Fahr et al., 2010).

[Insert Figure 1 about here]

2.2 The Overnight Index Swaps

An overnight index swap (OIS) is an interest rate swap whose floating leg is tied to an overnight rate (the EONIA in the euro-area case), compounded over a specified term. OIS contracts involve the exchange of only the interest payments, the principal amount being notional. That is, the two parties agree to exchange, on the agreed notional amount, the difference between interest accrued at the fixed rate and interest accrued through daily compounding (or geometric averaging) of the floating overnight index rate. While the tenor of these swaps was usually below 2 years before 2005, the OIS maturities were extended afterwards to more than 10 years (see Barclays, 2008). The OIS curve is more and more seen by market participants as a proxy of the risk-free yield curve (see e.g. Joyce et al., 2011).[15] In spite of that, OIS have failed to attract significant attention from academics for the time being.

As an interest-rate swap, an OIS can be used to manage interest-rate risks. In particular, the OIS are structured in such a way that if a bank (a) has some money available for investment, (b) has access to the overnight interbank market and (c) can enter OIS contracts, then this bank can synthetically create a fixed-income instrument that is equivalent to a maturity-h bond paying a coupon equal to the maturity-h OIS rate.

An important point that is going to be investigated below relates to the use of OIS curves to back out market expectations of future policy rate's moves. Heuristically, under the expectation hypothesis, the forward rates based on the OIS term structure should reflect the market expectations of the interbank rate, that is supposed to be

[15] While OIS rates reflect the credit risk of an overnight rate, this may be regarded as negligible in most situations. Besides, even during financial-markets turmois, the counterparty risk is limited in the case of a swap contract, due to netting and credit enhancement, including call margins (see Bomfin, 2003). To that respect, one can note that German sovereign bonds, usually perceived as being the European "safest haven" both in terms of credit quality and liquidity, trade at levels that have remained close to the OIS yield curve over the last years.

close to the target rate. This principle is widely used by market analysts, investors or central banks themselves.[16]

2.3 Data sources and treatments

The sample period is January 15, 1999 to February 17, 2012 (3416 dates). While the target rate and the EONIA series come from the ECB, the OIS yields are taken from Bloomberg. All yields are translated on a continuously compounded basis, and market holidays are filled with observations from the previous trading days' rates.[17] In the estimation, we consider six maturities (in addition to the overnight one): 1 month, 3 months, 6 months, 12 months, 2 years and 4 years.

As said above, OIS yields are not available for longer-than-one-year maturities before 2005. Before that date, we use EURIBOR swaps data in place of the 2-year and 4-year OIS yields. This appear to be a reasonable assumption given that the short-term EONIA swaps and maturity-matching EURIBORs had extremely close variations before 2007.[18] Swap yields are homogenous to coupon-bond yields. Since the pricing formula presented below (Subsection 3.2) are consistent with zero-coupon yields, zero-coupon yields are computed using classic bootstrapping methods.[19]

The estimation procedure involves survey-based forecasts of short-term yields (as in Kim and Orphanides, 2012; this is discussed in Section 4). Specifically, I use 12-month-ahead forecasts provided by the *Consensus Forecasts*: forecasts of the ECB's policy rate are available since July 2009; before that, I use 3-month EURIBOR forecasts.[20] Since EURIBOR and OIS were closely linked until summer 2007, using

[16] See e.g. Barclays, 2008, Joyce, Relleen and Sorensen, 2008, Joyce and Meldrum 2008, Bank of England, 2005 or Lang (2010).

[17] Let r denote a market-quoted interest rate (the OIS, say). Using the fact that money-market rate are based on the ACT/360 day-count basis, the corresponding continuously compounded rate is computed as $\ln(1 + d \times r/360) \times 365/d$, where d is the residual maturity of the instrument.

[18] During summer 2007, credit and liquidity risks affected unsecured interbank lending rates (IBOR), leading to a sudden widening of the IBOR-OIS spreads. Before that, this spread was small and steady. For each maturity (2-year and 4-year), I substract the 2005-2006 IBOR-OIS average spread from the EURIBOR swap series used in the estimation before 2005, which is about 10 basis points (standard deviation below 3 basis points).

[19] OIS rates with a maturities lower than one year are already homogenous to zero-coupon instruments. The bootstrapping methods are applied only for longer-than-one-year maturities. See Barclays (2008) for more information about EONIA swaps.

[20] Naturally, the fact that the nature of the forecasted rate changes in mid-2009 is taken into account in the estimation procedure.

EURIBOR forecasts instead of OIS forecasts is appropriate till then. In mid-2007 however, the widening in the EURIBOR-OIS spread is likely to induce a bias in the forecasts. This is addressed by substracting from the EURIBOR forecasts –from August 2007 to June 2009– the 1-year-ahead forward spread between the 3-month EURIBOR and OIS rates (averaged over the same period). All these survey-based expectations are available at the monthly frequency only and are released about mid-month. Using a cubic spline, this series is converted into a daily one. The discrepancies that arise from these approximations are expected to be captured by measurement errors of the state-space model that will be presented below.[21]

2.4 Preliminary analysis of the yields

Table 1 reports descriptive statistics for the different yields used in the analysis. These statistics suggest that yields are highly persistent. While the daily autocorrelation is nearly one, the correlations between the yields and their 1-year lags is still substantial (higher than 50%). The correlation across maturities is also extremely high, with near-unit correlations for adjacent maturities. Mean and median statistics show that the term structure is positively sloped on average.

The lowest Panel in Table 1 shows the results of a principal component analysis carried out on the set of seven spreads between OIS yields –with maturities of 1 day to 4 years– versus the policy rate. The three principal components are sufficient to explain most of the fluctuations of these spreads. Notably, the first principal component explains more than 90% of the variances of the spreads associated with yields of maturities comprised between 3 months and 1 year. This is graphically illustrated in Panel D of Figure 1, that highlights the common fluctuations in some of these spreads. Half of the variance of the EONIA spread and of the spread between the 4-year rate and the target rate is accounted for by this first factor, indicating that there are important correlations between the EONIA spread and longer-term spreads. However, further investigations mitigate this finding. Specifically, the same kind of

[21] Anticipating on the estimation results presented in Section 4, the standard deviation of the measurement errors associated to the forecasts is slightly larger than 20 bps (σ_{fcst} in Table 2), which is of the same order of magnitude as the errors expected from the previous points.

analysis has been carried out on a shorter sample, excluding the crisis period: 1999-2008 (bottom of Table 1). On that period, the EONIA spread turns out to be almost orthogonal to the first principal component. Therefore, the apparent comovement between the EONIA spread and the other spreads on the whole sample seems to be related to the fall in the EONIA spread that took place in mid-2008 (see Subsection 2.1 for a description of this phenomenon).

[Insert Table 1 about here]

3 The model

This section formulates a model of the daily dynamics of the overnight interbank interest rate.[22] Two dynamics are considered: the historical (or physical, or real-world) one and the risk-neutral (or pricing) one. The knowledge of the risk-neutral dynamics of the interbank rate makes it possible to price financial instruments –such as the OIS contracts– whose cash flows depend on the overnight interbank rate. The simultaneous knowledge of the two dynamics allows to study term premiums' behavior, as will be done in Section 5. The historical ($\mathbb{P}$) and the risk-neutral ($\mathbb{Q}$) dynamics of the different processes are of the same kind, but their respective parameterizations differ. These differences and the implied stochastic discount factor (s.d.f.) are detailed in Subsection 3.2, that also deals with the derivation of the term-structure of OIS rates. Before that, the next subsection presents the different components of the overnight interest rate.

3.1 The components of the overnight interest rate

The target rate prevailing at date t is denoted by $\bar{r}_t$. As is the case in most currency areas, the target rate is assumed to be a multiple of 0.25%. I proceed under the assumption that the target rate is lower than a maximal rate denoted by r_{max} and equal to $0.25\% \times N$, say. Therefore:

$$\bar{r}_t = \Delta' z_{r,t}$$

[22] The extension to a lower frequency is straightforward.

where $z_{r,t}$ is a selection vector, i.e. one of the column of I_{N+1}, the identity matrix of dimension $(N+1)\times(N+1)$ and where the entries of the vector Δ are the continuously-compounded possible policy rates. Specifically, using the money-market day-count convention, the ith entry of Δ is given by $\log(1 + (i-1)0.25\%/360)$. Note that at the daily frequency, many of the successive $\bar{r}_t$'s are equal. In particular, $\bar{r}_{t-1} = \bar{r}_t$ as soon as there are no policy meeting at date t. This results in a step-like process for the policy rate (as seen in Panel A of Figure 1).

The interbank overnight interest rate is denoted by r_t. Its deviations from the target rate are accounted for by three components: w_t, ξ_t and s_t:

$$r_t = \bar{r}_t + s_t + w_t + \xi_t \tag{1}$$

I assume that $\bar{r}_t$, s_t and (w_t, ξ_t) are independent of each others.[23] The variables s_t, w_t and ξ_t are unobservable but can be inferred from yields through the bond-pricing model. The historical dynamics of these factors are presented in the following. The risk-neutral dynamics are of the same kind, but their parameterizations is different from their physical counterparts. These differences are made explicit in Subsection 3.2.

3.1.1 The dynamics of the target rate $\bar{r}_t$

Central bankers can decide to change the target rate at their regular meetings. On these dates, the target can be raised or cut if the the tightening regime or the easing regime respectively prevail, but the target remains necessarily unchanged under the status quo regime. Formally, the monetary regime is represented by a 3-dimensional selection vector $z_{m,t}$ that is valued in the set of the three columns of the identity matrix I_3, corresponding respectively to the tightening, the status quo and the easing regimes. I assume that market participants observe the regime, this knowledge being based on a variety of detailed policy-relevant information that is not modeled here.

The Kronecker product of the selection vectors $z_{r,t}$ and $z_{m,t}$, denoted by $\bar{z}_t$, is

[23] Such independence assumptions are common in that literature (see Balduzzi et al., 1997 and 1998, or Piazzesi (2005)).

also a selection vector that is valued in the set of the columns of $I_{3(N+1)}$ (recall that $N+1$ is the number of possible values of the target rate, between 0% and $r_{max} = N \times 0.25\%$). The dynamics of $\bar{z}_t$ is described by a Markov chain. The matrix of transition probabilities of $\bar{z}_t$ is denoted by $\bar{\Pi}_t$. These matrices are time-inhomogenous, but in a deterministic way. Indeed, the matrices $\bar{\Pi}_t$ can take two values, one of them being specific to those days at which a monetary-policy meeting are scheduled.[24] The number of entries of these $\bar{\Pi}$ matrices is considerable: for $r_{max} = 10\%$, there are 15.129 of them. However, most of them are set to zero. Specifically, I assume that:

1. Conditionally on being in an easing, a status quo or a tightening regime, the target moves are respectively valued in $\{-0.50\%, -0.25\%, 0\}$, $\{0\}$ and $\{0, +0.25\%, +0.50\%\}$.

2. Easing or tightening phases are necessarily followed by status quo phases.

Even with these restrictions, many of $\bar{\Pi}_t$'s entries still require to be parameterized. Eight sets of probabilities needs to be defined: two of them contain the probabilities of switching to the status quo regime (the probability of exiting the easing and the tightening regimes are respectively denoted by p_{ES} and p_{TS}), two others are the probabilities of exiting the status quo regime (p_{SE} and p_{ST}), two of them contain the probabilities of 25-bp changes in the target rate (rise: p_{r25}; cut: p_{c25}) and the last two are the probabilities of 50-bp moves (rise: p_{r50}; cut: p_{c50}). These probabilities may vary with the policy rate. For instance, the probability of switching from the tightening to the status quo regime could be larger for higher target rates, say. In order to keep the model parsimonious, the probabilities are based on logit-based parametric functions of the target rate $\bar{r}$. Formally, let me define the function f by:

$$f(\bar{r}, [a_1, a_2]') = [1 + \exp(a_1 + a_2 \bar{r})]^{-1}. \quad (2)$$

For $i \in \{TS, ES, SE\}$, the probabilities p_i are characterized by some 2×1 vectors

[24] Contrary to the policy rate ($z_{r,t}$), that can change only following a monetary-policy meeting, the monetary-policy regime ($z_{m,t}$) can switch at any date. For instance, such changes could be triggered by ECB officials' speeches or the release of macroeconomic news or figures.

of parameters α_i and are given by $f(\bar{r}, \alpha_i)$. Further, so as to have $p_{ST} + p_{SE} < 1$, the probabilites p_{ST} are defined by $(1 - p_{SE}(\bar{r}))f(\bar{r}, \alpha_{ST})$. In order to keep the model parsimonious, I do not define α_i vectors for each of the four kinds of target move, but only two: one for the rises in the policy rate (α_r) and one for the cuts (α_c). Two additional parameters, k_r and k_c, are then introduced to share the rise and cut probabilities into those of 25-bp and 50-bp moves. Formally, the conditional probabilities of target-rate changes (i.e. p_{c25}, p_{r25}, p_{c50} and p_{r50}) are defined through:

$$\begin{cases} p_{r25}(\bar{r}) = k_r f(\bar{r}, \alpha_r) \quad \text{and} \quad p_{r50}(\bar{r}) = (1 - k_r) f(\bar{r}, \alpha_r) \\ p_{c25}(\bar{r}) = k_c f(\bar{r}, \alpha_c) \quad \text{and} \quad p_{c50}(\bar{r}) = (1 - k_c) f(\bar{r}, \alpha_c) \end{cases}$$

where k_c and k_r are valued in $[0, 1]$.[25] Eventually, the 15.129 entries of matrix $\bar{\Pi}$ are defined by 16 parameters only.

3.1.2 The dynamics of ξ_t and w_t

The variable ξ_t is a volatile and short-lived process. Basically, it aims at capturing white noise in the EONIA spread. Looking at Panel B of Figure 1, it clearly appears that the distribution of these shocks has changed over the last years. This relates to the implementation of non-standard monetary-policy measures, giving rise to a *banks' excess liquidity regime* (see Subsection 2.1). This is handled by introducing an additional Markovian regime process $z_{exc,t}$ that can take two values $[1, 0]'$ (no excess-liquidity conditions) or $[0, 1]'$ (excess liquidity conditions). The matrix of transition probabilities associated with this process is time-homogenous and is denoted by Π_{exc}.[26] Formally, ξ_t is given by:

$$\xi_t = \left[\begin{array}{cc} \xi_{norm,t} & \xi_{exc,t} \end{array} \right] z_{exc,t}$$

[25] Before November 2001, possible changes in the policy rate were discussed in each of the bi-weekly meetings of the ECB Governing Council. Since then, they are considered during the first of these two bi-weekly meetings only. Accordingly, for the first part of the sample (up to November 2001), the target-moves probabilities are divided by two so as to result in (approximately) the same probabilities of target moves over a month.

[26] The columns of Π_{exc} sum to one.

where, for $i \in \{norm, exc\}$, the $\xi_{i,t}$ follow specific distributions that I denote by $\Xi(p_i, \alpha_{P,i}, \beta_{P,i}, \alpha_{N,i}, \beta_{N,i})$, with $i \in \{norm, exc\}$. The Ξ distribution is designed in wuch a way that it accommodates non-zero skewness and fat tails. Let me make this distribution more precise. A random variable follows the distribution $\Xi(p, \alpha_P, \beta_P, \alpha_N, \beta_N)$ if it is equal to $\mathbb{I}_{\{u=0\}} v_P - \mathbb{I}_{\{u=1\}} v_N$, where u is Bernoulli distributed with success probability p, and where v_P and v_N follow beta distributions with respective parameters $(\alpha_{P,i}, \beta_{P,i})$ and $(\alpha_{N,i}, \beta_{N,i})$ ["P" and "N" respectively stand for "positive" and "negative"]. In addition, the support of this distribution is the compact $[-1, 1]$ (in annualized terms), which is consistent with the fact that the EONIA is bounded by the corridor set by the ECB's standing facilities.[27] The Laplace transform of such a distribution –which is required to derive bond prices (see Subection 3.2)– is provided in Appendix C.

The variable w_t –which also depends on the excess-liquidity regime variable $z_{exc,t}$– captures highly persistent and steady deviations from the target. It can take two values: w_{norm} and w_{exc}. Formally, w_t is given by:

$$w_t = \begin{bmatrix} w_{norm} & w_{exc} \end{bmatrix} z_{exc,t}.$$

3.1.3 The dynamics of s_t

The variable s_t is aimed at contributing to persistent fluctuations of yields that can not be accounted for by the regime variables ($z_{r,t}$, $z_{m,t}$ and $z_{exc,t}$). Combined with ξ_t, the latter are expected to account for most of EONIA's fluctuations. Therefore, the variable s_t is expected to have a far lower impact on the overnight rate than on longer-term yields. To obtain this, I define $s_{1,t}$ and $s_{2,t}$ that are such that $s_t = s_{1,t} + s_{2,t}$,

[27] Note that the width of the corridor has changed over time (between 150 and 200 bp). However, taking into account such a variability would induce severe complexity in the framework.

with:

$$\begin{bmatrix} s_{1,t} \\ s_{2,t} \end{bmatrix} = \Phi \begin{bmatrix} s_{1,t-1} \\ s_{2,t-1} \end{bmatrix} + \Sigma\varepsilon_t, \quad \varepsilon_t \sim \text{i.i.d. } \mathcal{N}(0, I) \tag{3}$$

$$\text{where} \qquad \Phi = \begin{bmatrix} \rho_1 & \beta \\ 0 & \rho_2 \end{bmatrix} \quad \text{and} \quad \Sigma = \begin{bmatrix} 0 & 0 \\ 0 & \sigma \end{bmatrix}$$

The smaller β, the less variable $s_{1,t}$ is. In the limit, if β is equal to zero and if $s_{1,t}$ was zero at some point in the past, then $s_t = s_{2,t}$. I assume this is the case under the physical measure, but not under the risk-neutral one. Under the latter measure, if the ρ_i^*'s –the risk-neutral counterpart of the ρ_i's– are close to one, a shock on $s_{2,t}$ can have a very persistent impact on s_t. In addition, if β^* is large enough, these effects are multiplied by feeding through $s_{1,t}$. Therefore, s_t's innovations may have a far more long-lasting impact under the risk-neutral measure than under the physical measure. This implies that s_t may account for a far larger variance of long-term yields than of short-term yields.[28]

3.1.4 Definition of the single regime variable z_t

Defining a single regime variable will prove convenient for notational reasons in the remaining of this paper. Accordingly, I introduce the selection vector z_t, defined as the Kronecker product of $\bar{z}_t$ and $z_{exc,t}$. Since $\bar{z}_t$ is itself the Kronecker product of $z_{r,t}$ and $z_{m,t}$, I have:

$$z_t = z_{r,t} \otimes z_{m,t} \otimes z_{exc,t}.$$

Hence, z_t is valued in the set of the columns of $I_{6(N+1)}$, each of the $6(N+1)$ different regimes being characterized by the policy rate (there are $N+1$ of them), a monetary-policy stance (there are three of them) and the situation of Eurosystem's liquidity (the situation being "normal" or "in surplus"). While z_t is observed by market participants, $z_{m,t}$ and $z_{exc,t}$ are not observed by the econometrician.

Given the assumption of independence between $\bar{r}_t$ and (w_t, ξ_t), the matrix of transition probabilities of z_t, denoted by Π_t, is equal to the Kronecker product of $\bar{\Pi}_t$

[28] The choice of this dynamics builds on Dubecq and Gourieroux (2011).

and Π_{exc}.

3.1.5 About the seasonality of the EONIA spread

This framework do not account for potential seasonality in the EONIA spread. While this could bias the pricing of short-term yields (with maturities of one week, say), this simplification has a limited impact for longer maturities. As noted by Balduzzi et al. (1998), only little seasonal variability of the overnight interest rate should be tansmitted to longer-term rates, since seasonal variability is "averaged out" in the expectation process (especially if one considers maturities that are multiple of the reserve maintenance period, which is the case in that study).

3.2 Pricing

3.2.1 The stochastic discount factor (s.d.f.)

I assume that the risk-neutral dynamics of z_t and s_t are of the same kinds as their historical couterparts except that the Π_t's and Φ are respectively replaced by Π_t^*'s and Φ^* matrices, that depend on the same number of free parameters.[29] In this context, it can be shown that the stochastic discount factor (s.d.f.), or pricing kernel, is explicit.[30] Specifically, the s.d.f. $M_{t-1,t}$ between $t-1$ and t is given by:

$$M_{t-1,t} = \exp\left(-\Delta_m^{'} z_{t-1} - s_{t-1} - w_{t-1} - \xi_{t-1} - \frac{1}{2}\nu_{t-1}^{'}\nu_{t-1} + \nu_{t-1}^{'}\varepsilon_t + z_{t-1}^{\prime}\delta_t z_t\right)$$

where Δ_m is the concatenation of six vectors Δ, that is $\Delta_m = 1_{6\times 1} \otimes \Delta$, which reflects the fact that there are three monetary regimes ($z_{m,t}$) and two Eurosystem-liquidity situations ($z_{exc,t}$), and where the risk sensitivities δ_t and ν_t –that price respectively the risks associated to the regime shifts and to the Gaussian shocks ε_t– are defined

[29] In particular, the p_{ES}^*, p_{SE}^*, p_{ST}^*, p_{TS}^*, p_{r25}^*, p_{c25}^*, p_{r50}^* and p_{c50}^*, that define the Π_t^*'s matrices, are based on functions $f(\bar{r}, \bullet)$. Still using the superscript $*$ to denote risk-neutral parameters, these probabilities depend on vector α_i^* (see end of Subsection 3.1.1).

[30] See Monfort and Renne (2011).

by:

$$\begin{cases} \delta_{ij,t} & = \log\left(\Pi^*_{t,ij}/\Pi_{t,ij}\right) \\ \nu_t & = \Sigma^{-1}(\Phi^* - \Phi)\left[\begin{array}{cc} s_{1,t} & s_{2,t} \end{array}\right]' \end{cases} \quad \forall\ i,j,t. \tag{4}$$

3.2.2 Bond prices

It is well-known that the existence of a positive stochastic discount factor is equivalent to the absence of arbitrage opportunities (see Hansen and Richard, 1987) and that the price at t of a zero-coupon bond with residual maturity h, denoted by $P(t,h)$ is given by:

$$\begin{aligned} P(t,h) &= E_t\left(M_{t,t+1} \times \ldots \times M_{t-h-1,t-h}\right) \\ &= E_t^{\mathbb{Q}}\left(\exp\left[-r_t - \ldots - r_{t+h-1}\right]\right). \end{aligned} \tag{5}$$

Substituting equation (1) into equation (5) leads to:

$$P(t,h) = E_t^{\mathbb{Q}}\left(\exp\left[-\sum_{i=0}^{h-1}\left(\bar{r}_{t+i} + s_{t+i} + w_{t+i} + \xi_{t+i}\right)\right]\right) \tag{6}$$

Under the assumption that $\bar{r}_t$, s_t and (w_t, ξ_t) are independent processes, I get:

$$P(t,h) = \left(E_t^{\mathbb{Q}} e^{-\sum_{i=0}^{h-1}\bar{r}_{t+i}}\right)\left(E_t^{\mathbb{Q}} e^{-\sum_{i=0}^{h-1}(w_{t+i}+\xi_{t+i})}\right)\left(E_t^{\mathbb{Q}} e^{-\sum_{i=0}^{h-1}s_{t+i}}\right)$$

Let me denote by $P_1(t,h)$, $P_2(t,h)$ and $P_3(t,h)$ the three terms that respectively appear on the right-hand side of equation (11), Appendix B details the computation of these terms. It is important to stress that explicit formulas are available to compute each of these three terms, each of them turning out to be exponential affine in $(z_t', s_t)'$. Accordingly, the yields associated with zero-coupon bonds of maturity h, denoted by $y(t,h)$, are of the form:

$$y(t,h) = -\frac{1}{h}\left[G(t,h)z_t + A_h + B_h s_t\right] \tag{7}$$

Note that $G(t,h)$ is deterministic (i.e., the only stochastic components of the yields are z_t and s_t).

4 Estimation

4.1 The state-space form of the model

Kim and Orphanides (2012) have shown that the estimation of dynamic no-arbitrage term structure models with a flexible specification of the market price of risk is beset by a severe small-sample problem arising from the highly persistent nature of interest rates. They show that using survey-based forecasts of a short-term interest rate as an additional input to the estimation can overcome this problem. I follow their approach and I enlarge the state-space model to make the estimated model consistent with 12-month-ahead forecasts of short-term rates provided by the *Consensus Forecasts*.[31]

Let me denote by R_t a vector of M observed yields of maturities $h_1, \ldots, h_M$, that is $R_t = [y(t,h_1), \ldots, y(t,h_M)]'$. Equation (7) shows that the these yields are affine in (z_t, s_t). It is straightforward to show that it is also the case for the 12-month-ahead forecasts included in the estimation. These forecasts are denoted by CF_t. Introducing some vectors of –supposedly i.i.d. normal– measurement errors denoted by ξ, I have:

$$\begin{cases} R_t & = \Lambda_{z,R} z_t + \Lambda_{s,R} s_t + \xi_t^R \\ CF_t & = \Lambda_{z,C} z_t + \Lambda_{s,C} s_t + \xi_t^C \end{cases} \tag{8}$$

where the Λ matrices are functions of the model parameters (see Subsection 3.2). The model admits a Markov-switching state-space representation whose measurement equations are given by (8). The dynamics of the state vectors s_t and z_t are respectively defined by equation (3) and by the matrices of transition probabilities Π_t.

[31] Other methodologies have been proposed to address this problem, see e.g. Jardet, Monfort and Pegoraro (2009).

4.2 Computation of the log-likelihood

Whereas the Markov chain $z_{r,t}$ is observed, the remaining state variables (s_t, $z_{m,t}$ and $z_{exc,t}$) are not. This latency is handled by using an estimation strategy building on Monfort and Renne (2011). The approach consists in applying inversion techniques *à la* Chen and Scott (1993) together with the Kitagawa-Hamilton filter to address the hidden nature of the switching regimes. The idea of the inversion technique is the following: assuming that a combination of the yields –gathered in the vector R_t– is observed without error, one can recover the latent variables. Further, one can compute the likelihood function based on the specified dynamics of the latent factor as well as on the distribution of the (remaining) pricing errors. Usually, one uses trivial perfectly-priced combinations of yields: specifically, if there are m latent factors with continuous support in the model, one assumes that m yields are priced without error. However, as noted for instance by Piazzesi (2010), the choice of this maturity is arbitrary. Therefore, I resort to an alternative approach and choose s_t in order to minimize the average squared pricing errors across the different maturities.[32] This is simply obtained by using the OLS formula:[33]

$$s_t = (\Lambda'_{s,R}\Lambda_{s,R})^{-1}\Lambda'_{s,R}(R_t - \Lambda_{z,R}z_t). \tag{9}$$

Since s_t is a given combination of the yields, there is the same information in R_t as in $\{s_t, \tilde{R}_t\}$, where $\tilde{R}_t$ is any subvector of R_t containing $M-1$ yields of distinct maturities. Without loss of generality, I assume that $\tilde{R}_t = [y(t,h_2),\ldots,y(t,h_M)]'$. As a consequence, from an econometric point of view, the model reads:

$$\begin{cases} \tilde{\Gamma}\xi_t^R & = \tilde{\Gamma}\left\{(I-\Lambda_R)R_t - (I-\Lambda_R)\Lambda_{z,R}z_t\right\} \\ \xi_t^C & = CF_t - (\Lambda_{z,C} - \Lambda_C\Lambda_{z,R})z_t - \Lambda_C R_t \\ \varepsilon_t & = \frac{1}{\sigma}\Lambda_s\left[(R_t - \rho_2 R_{t-1}) - \Lambda_{z,R}(z_t - \rho_2 z_{t-1})\right] \end{cases}$$

[32] I am grateful to Simon Dubecq for providing me with this procedure. To our knowledge, though particularly efficient compared to classic inversion techniques, it has not been used in the existing literature.

[33] Note that this procedure results in one s_t conditionally to each of the different hidden regimes.

where $\Lambda_s = (\Lambda'_{s,R}\Lambda_{s,R})^{-1}\Lambda'_{s,R}$, $\Lambda_R = (\Lambda'_{s,R}\Lambda_{s,R})^{-1}\Lambda_{s,R}\Lambda'_{s,R}$, $\Lambda_C = (\Lambda'_{s,R}\Lambda_{s,R})^{-1}\Lambda_{s,C}\Lambda'_{s,R}$ and where $\tilde{\Gamma}$ is the $(M-1)\times M$ matrix that picks up the last $M-1$ entries of an $M\times 1$ vector.

Assuming that the ε_t's, the ξ_t^{R}'s and the ξ_t^{cf}'s are i.i.d. normal, the computation of the log-likelihood associated with the previous model is easily obtained by applying the Kitagawa-Hamilton filter. However, this likelihood is the one associated with the vector $\{s_t, \tilde{R}_t, CF_t\}_{t=1,\dots,T}$, while we need to maximize the one associated with actually observed data $\{R_t, CF_t\}_{t=1,\dots,T}$. The latter is obtained by multiplying the former by the determinant of the Jocobian resulting from this change in variables, that is $\left|\partial\left[s_t, \tilde{R}'_t\right]/\partial R_t\right| = \frac{1}{\Lambda'_{s,R}\Lambda_{s,R}}\Lambda_{s,R,1}$ where $\Lambda_{s,R,1}$ is the first entry of $\Lambda_{s,R}$.

4.3 Estimation results

The parameter estimates are obtained by maximizing the log-likelihood.[34] In order to avoid that the factor s_t, thanks to its flexible Gaussian dynamics, explains too large a share of the yield fluctuations, I limit the size of its unconditional variance in the estimation. Specifically, I impose that the unconditional standard deviation of the s_t-related component of the one-year yield is lower than 10 basis points. Eventually, fifty one parameters have to be estimated.

Table 2 reports the parameter estimates. The computation of the estimates' standard errors are based on the outer product of the first derivative of the likelihood function. The standard deviation of the pricing error –i.e. the deviation between modeled and observed yields) is equal to eight basis points–, which is comparable to Piazzesi's (2005) fit of the U.S. yield curve.[35] Panels B, C and D of Figure 2 respectively show the fit of the 3-month, the 2-year and the 4-year yields. These plots also show the part of those yields that is explained by the regime variable z_t. It appears that most of the yields' fluctuations can be accounted for by z_t: more than 95% of the sample variances of yields with maturities lower than 2 years are

[34] Numerical optimizations, based on the Nelder-Mead methods, have been carried out under the Scilab software.

[35] Note however that the sample period used by Piazzesi (2005) is shorter (4 years against 13 here) and the frequency is higher here (daily vs. weekly).

captured by the term $G(t,h)z_t$ appearing in equation (7).[36]

[Insert Figures 2 to 6 about here]

Panel A of Figure 3 illustrates the ability of the model to reproduce survey-based forecasts of the target rate. Panel B and Panel C respectively present the estimated (smoothed) probabilities of being in the different monetary-policy regimes ($z_{m,t}$) and in the liquidity-surplus regime ($z_{exc,t}$) characterized by the disconnection of the EONIA from the main ECB policy rate.[37] According to the estimation, the first period of the liquidity-surplus regime is October 17, 2008, i.e. a few days after the announcement of the fixed-rate full-allotment procedure by the ECB. This regime was interrupted three times since then. The last interruption ended on August 2, 2011, two days before the ECB announced supplementary very long-term refinancing operations (VLTRO) in a context of renewed financial tensions.[38]

Searching for potential explanations of each change in regime is beyond the scope of this paper. Nevertheless, let me focus on an episode where monetary-policy-regime shifts can be directly related to central bankers' announcements.[39] During the press conference following the ECB Governing Council that took place on 5 June 2008, J.-C. Trichet said: "we could decide to move our rates [by] a small amount in our next meeting". As is shown in Figure 4, this triggered a change in the monetary-policy regime, from status quo to tightening. A rate hike was then decided by the Governing Council in the next meeting, on 3 July 2008. The latter meeting was however followed by a more dovish press conference by Trichet, which induced a return to the status-quo regime in the next few days.

Figure 5 illustrates the influence of the monetary-policy regimes on the term structure of interest rates. For three dates, the modeled yields are compared with the observed ones. For each date, three additional yield curves are displayed, each of them corresponding to one of the three monetary-policy regimes. The modeled yield curve corresponds to one of these three curves, the attribution being based on the

[36] 85% of the variance of the 4-year yield is accounted for by $G(t,h)z_t$.
[37] Smoothing is based on Kim's (1993) algorithm.
[38] See the press release at http://ecb.int/press/pr/date/2011/html/pr110804_1.en.html.
[39] Naturally, central bankers' speeches are key events that are subject to indicate changes in monetary-policy regimes (see e.g. Rosa and Verga, 2008 in the euro-area case).

smoothed probabilities associated with the Markov chain $z_{m,t}$.[40] The two remaining curves are the answers to the question: what if the monetary-policy stance were different on that date? These plots show that monetary-policy regimes are key to shape the yield curve. Furthermore, this figure illustrates the ability of the model to reproduce various shapes of the yield curve (steep, flat, humped, inverse-humped).

As discussed in Section 2.1, non-conventional monetary policy measures implemented during Fall 2008 were followed by a sharp decline in the EONIA rate with respect to the rate of the main refinancing operations (the latter being called policy rate hereinabove). The fact that this decrease of the short-term rate is transmitted along the yield curve is captured by the present model.[41] Figure 6 illustrates this point by comparing, for a day when the estimation indicates that the excess-liquidity regime prevails, the fitted yield curve with the one that one would observed absent the excess-liquidity situation. The simulation results point to a decline in medium-term yields of about 50 basis points; this effect slightly decays along the maturity axis.

[Insert Figures 7 and 8 and about here]

Figure 7 displays the 30-day-ahead probabilities of change in the monetary-policy regime and in the target for the policy rate. Both historical and risk-neutral probabilities are reported. Interestingly, all three monetary-policy regimes are more persistent under the risk-neutral measure than under the physical one, which can be seen from the fact that the risk-neutral probabilities of exiting a given monetary-policy phase are lower than their historical counterparts. The implications of the differences between the two dynamics (historical vs. risk-neutral) are explored in Section 5. Overall, the probabilities of monetary-policy changes substantially depend on the target rate: This appears on the plots of Figure 7 and is also reflected by the statistical significance of the parameters a_2 that relate the probabilities of changes in the

[40] In the present case, the smoothing algorithm results in a clear-cut identification of the hidden monetary-policy regime: Most of the time, the smoothed probabilities are either 1 or 0.

[41] This effect is captured by the term $w_t + \xi_t$. More precisely, the yield-curve effect stems from the fact that (risk-neutral) expectation of $w_t + \xi_t$ (for a given horizon, or maturity) depends on the regime $z_{exc,t}$ (excess-liquidity situation or not).

policy rate or in the monetary-policy regime to the level of the policy rate (see Table 2).

In this model, the volatility of the policy rate, and hence of the whole term structure of interest rates is not trivial. This is illustrated in Figure 8, that displays the standard deviation associated with the model-imlpied 3-month-ahead forecasts of the policy rate. The left-hand (right-hand) side plot regards the historical (risk-neutral) measure. The volatility of the policy rate turns out to strongly depend on the level of the target rate as well as with the monetary-policy phase. Notably, these results echo those of Fontaine (2009) who finds –using U.S. data– that the uncertainty is lowest (highest) in tightening (loosening) cycles.

[Insert Table 2 about here]

5 Term premia associated with target changes

The fact that the historical ($\mathbb{P}$) and the risk-neutral ($\mathbb{Q}$) dynamics of $\bar{r}_t$ differ gives rise to target-related risk premia.[42] The existence of such term premia is important in several respects. Let me highlight two of them. First, if these risk premia are sizeable, OIS forward rates should not be interpreted as the market perceptions of future target rates, though this is the basis of a widespread market practice (see Subsection 2.2). Second, the existence of risk premia at the short-end of the yield curve implies that excess returns associated with a long position in money-market instruments may be partially predictable or, alternatively said, that the expectation hypothesis does not hold at the short-end of the yield curve. While there is strong evidence against the expectation hypothesis for long-term yields, the evidence is weaker for short-term ones (see Longstaff, 2000).

[Insert Figure 9 about here]

In order to assess the size of target-related risk premia, I compare policy-rate forecasts computed under the two different measures. Conceptually, under the risk-neutral measure $\mathbb{Q}$, the forecasted paths of the policy rate are very close to the

[42] These target-related premia contribute to the total term premia, that also include risk premia associated with the s_t component of the EONIA.

term structure of forward annualized rates (up to small Jensen-inequality correction terms). Note that here, I focus on the risk premia associated with policy-rate changes, those associated with the s_t process having a straightforward and orthogonal influence.[43] Figure 9 displays the term structure of the policy-rate forecasts. Nine pairs of plots are reported. Each pair of plot corresponds to a given policy rate (1%, 2.5% or 4%) and a given monetary-policy phase (tightening, status quo or easing). For each pair of charts, an upper plot presents the forecasts of the policy rate (w.r.t. the horizon forecast, on the x axis) and a lower one displays the associated risk premia, i.e. the spread between the $\mathbb{Q}$ and $\mathbb{P}$ forecasts. 90% confidence intervals for the risk premia are reported in the lower charts.[44] These premia are discussed in the following.

First, it appears that the risk premia can be substantial, even at the short end of the yield curve. In particular, under the tightening regime (see the first column of charts in Figure 9), the risk premia are higher than 50 basis points for maturities higher than 12 months. Furthermore, for policy rates that are higher than the sample average (of about 2.5%), the risk premia associated with tightening and easing monetary-policy regimes turn out to have opposite signs at the short- to medium-end of the yield curve (see the second and third rows of pairs of charts, corresponding respectively to a 2.5% and a 4% policy rates). This stems from the fact that the probabilities of remaining in tightening and the easing regimes are higher under the risk-neutral measure than under the historical one (as shown in Figure 7), implying higher life expectancies for these regimes and, thereby, a higher probability –compared with the physical measure $\mathbb{P}$– of having several policy-rate moves in the next months or quarters. This translates into positive (negative) risk premia at the short end of the yield curve when the tightening (easing) regime prevails. Therefore, the estimation results suggest that under the risk-neutral measure, the central bank is more "aggressive", in the sense that the yield curve reflects the behavior of a central

[43] The mean reversion of s_t being far larger under the historical measure than under the risk-neutral measure, the risk premia associated with this factor are almost $-B_h s_t/h$ (see Subsection 3.2 and equation (7) for details regarding the latter expression).

[44] The confidence intervals are based on bootstrap techniques, the parameter estimates being drawn from their asymptotic distribution, see Figure 9's caption for more details.

bank that tends to rise (respectively cut) the policy rate in a more rapid way than under the real-world measure when in the tightening (resp. easing) regime.[45] This supports the findings of Balduzzi et al. (1997), who observe that the target-change predictions that may be obtained from the short-end of the yield curve –under the expectation hypothesis– are correct in terms of sign, but tend to overestimate the size of realized target moves.

6 Estimated impact of forward policy guidance

In my framework, the behavior of the central bank is modeled through a set of probabilities: some of them correspond to probabilities of switching from one regime to the other (tightening, easing and status quo), some of them correspond to probabilities of rises or cuts in the target rate (the latter being conditional to the monetary-policy regime). If a change in these probabilities is made public, it may have an impact on the whole yield curve because the pricing of financial assets depend in part on the entire expected future path of short-term interest rates. This expectation channel of monetary policy transmission is at the heart of the rationale for *forward policy guidance* measures. In the current context in which the zero bound is binding for the overnight nominal interest rate, these measures are aimed to provide additional stimulus to the economy by pushing down medium- to long-term interest rates and, thereby, to support other asset prices (see e.g. Bernanke and Reinhart, 2004).

My framework makes it possible to assess the impacts of such announcements in a straightforward and consistent manner. Here, I consider a basic form of forward guidance. Specifically, I consider a central bank's commitment to maintain its target rate constant for (at least) a deterministic period of time. The recent decision by the U.S. Federal Reserve to release federal funds rate forecasts and to extend its pledge to keep rates near zero at leat through late 2014 is of that kind.[46] In the past, other

[45] Regarding the rise in rate, this is not any more the case for high target rates, since the risk-neutral probability of a rise in the target is lower than its historical counterpart when the policy rate is above 4%. However, note that the unconditional probability of being in the targeting regime when the target rate is higher than 4% is low (see lowest Panel of Figure 7).

[46] See the Fed press release at http://www.federalreserve.gov/monetarypolicy/files/fomcprojtabl20120125.pdf

central banks have signalled future policy intentions through official communication. For instance, the Bank of Canada announced on April 21, 2009 its conditional commitment to *"hold current policy rate [close to zero] until the end of the second quarter of 2010."*[47]

As in nearly all of the existing literature, the following simulations abstract from issues that could arise under imperfect credibility of the central bank and focus on the case where the monetary authorities benefit from a perfect commitment technology.

Let me assume that the central bank has announced at date t that it will keep its policy rate unchanged for the next p periods. Then, equation (7) can be used to compute the yields of different maturities, up to a few parameters' adjustments: the matrix $G(t,h)$ has simply to be replaced by $\tilde{G}(t,h)$, the latter being computed in the same way as the former (i.e. using the formulas presented in Appendix B) after having modified the matrices of Π^*_{t+i}, $i \leq p$ by setting the probabilities of policy-rate moves to zero.[48]

[Insert Figure 10 about here]

Figure 10 displays the results of four simulations. These simulations are based on two different target rates (1% and 3.5%) and two commitment durations (12 months and 24 months). Consistently with the fact that the policy rate is fixed for several months, the monetary-policy regime is set to the status-quo one (in the baseline as well as in the counterfactual case). The results suggest that such measures would have a statistically significant downward impact on the yield curve (90% confidence intervals of the downard effects are reported for each of the four cases presented in Figure 10). The impact appears to be far larger when the current target rate is low. For instance, a commitment to keep the target rate unchanged for the next 24 months leads to a decrease in the 5-year yield of about 25 bp when the target rate is of 1% and of about 5 bp when the target rate is of 3.5%.

[47] There exist older cases of forward policy guidance: the Reserve Bank of New Zealand announced a path for its 3-month bank bill rate in 1997, it was followed by the Norges Bank and the Riskbank in 2005 and 2007, respectively.

[48] The fact that the probabilities of having policy-rate moves over the next p periods implies that the same is true under the risk-neutral measure because $\mathbb{P}$ and $\mathbb{Q}$ are equivalent measures. If this was not the case, it would imply the existence of infinitely large Sharpe ratios associated with policy-rate changes.

7 Conclusion

While central banks' decisions are obvious drivers of the fluctuations of the term structure of interest rates, only few of the available yield-curve models feature a realistic modeling of the policy rate. In this paper, I propose a framework that captures simultaneously the dynamics of the policy rate and the yields of longer maturities. Importantly, this model is consistent with the existence of the zero-lower-bound restriction, making it appealing in the current context of extremely low interest rates.

A key ingredient of the model is an extensive and innovative use of switching-regime features. Each regime is characterized by (a) a target level, (b) a monetary-policy regime (easing, tightening or status quo) and (c) the Eurosystem aggregate liquidity situation (normal or "in surplus"). The latter is introduced so as to accommodate the recent situation in which banks resort massively to the ECB deposit facility, which has an impact on the overnight interbank rate –the shortest-maturity yield considered in the model.

In order to illustrate the flexibility of the model, I estimate it using daily data covering the last thirteen years. Consistently with the choice of the interbank rate (EONIA) as the shortest yield, the overnight index swap (OIS) curve is fitted. Being impressively tractable, the model is estimated by standard maximum likelihood techniques. In order to alleviate potential small-sample bias and, hence, to properly estimate the physical dynamics of the processes, the estimation data set includes survey-based forecasts of short-term rates.

Various by-products are available, including the estimation of the market-perceived monetary-policy regime (at the daily frequency). In addition, the model is used in order to explore the size and influence of risk premia at the short end of the yield curve, the approach making it possible to exhibit risk premia associated with target moves. My analysis suggests that market yields reflect the behavior of a central bank that would tend to rise (respectively cut) the target rate more rapidly than is physically observed when in a tightening (resp. easing) phase. This has implications regarding the common practice that consists in inverting the OIS yield curve to extract market-based short-term forecasts of the policy-rate path. Specifically, it means that such a practice –that assumes that the expectation hypothesis holds at the short-end of the yield curve– is valid in terms of sign of next target changes, but tend to overestimate their size.

Finally, the model is exploited to predict the potential effects of a forward policy guidance measure that consists of a commitment of the central bank to keep its rate unchanged for a deterministic period of time. The simulations show that, in the current context of low short-term rates and with a commitment duration of 2 years, such an (unanticipated) announcement would be followed by a decrease of about 25 basis points of the 5-year rate.

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A Multi-horizon Laplace transform of a (homogenous) Markov-switching process

In the following, I consider a n-state Markov process z_t, valued in $\{e_1, \dots, e_n\}$, the set of columns of I_n, the identity matrix of dimension $n \times n$. I assume that the matrix of transition probabilities is deterministic and denoted by P_t (the columns sum to one). We have: $\mathbb{P}(z_{t+1} = e_i | z_t) = e_i' P_{t+1} z_t$.

Computation of $E_t(\exp(\alpha' z_{t+1}))$

$$
\begin{aligned}
E_t(\exp(\alpha' z_{t+1})) &= \sum_{i=1}^{n} \exp(\alpha_i) e_i' P_{t+1} z_t \\
&= \left(\sum_{i=1}^{M} \exp(\alpha_i) e_i' \right) P_{t+1} z_t \\
&= \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} D(\exp \alpha) P_{t+1} z_t
\end{aligned}
$$

where $\exp \alpha$ is the vector whose entries are the $\exp(\alpha_i)$'s and where $D(x)$ is a diagonal matrix whose diagonal entries are the elements of the vector x.

Computation of $E_t(\exp[\alpha_1' z_{t+1} + \alpha_2' z_{t+2}])$

The law of iterated expectations leads to:

$$
\begin{aligned}
E_t(\exp[\alpha_1' z_{t+1} + \alpha_2' z_{t+2}]) &= E_t(E_t[\exp[\alpha_1' z_{t+1} + \alpha_2' z_{t+2}] | z_{t+1}]) \\
&= E_t(\exp[\alpha_1' z_{t+1}] E_t[\exp[\alpha_2' z_{t+2}] | z_{t+1}])
\end{aligned}
$$

Then, using the previous case:

$$
\begin{aligned}
& E_t(\exp[\alpha_1' z_{t+1} + \alpha_2' z_{t+2}]) \\
= \; & E_t\left(\exp[\alpha_1' z_{t+1}] \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} D(\exp \alpha_2) P_{t+2} z_{t+1}\right) \\
= \; & E_t\left(\begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} D(\exp \alpha_2) P_{t+2} z_{t+1} \exp[\alpha_1' z_{t+1}]\right) \\
= \; & E_t\left(\begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} D(\exp \alpha_2) P_{t+2} z_{t+1} z_{t+1}' D(\exp \alpha_1) \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}'\right).
\end{aligned}
$$

Using the facts that $z_{t+1} z_{t+1}'$ commutes with any matrix and that $z_{t+1} z_{t+1}' \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}' = z_{t+1}$, we get:

$$
\begin{aligned}
E_t(\exp[\alpha_1' z_{t+1} + \alpha_2' z_{t+2}]) &= E_t\left(\begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} D(\exp \alpha_2) P_{t+2} D(\exp \alpha_1) z_{t+1}\right) \\
&= \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} [D(\exp \alpha_2) P_{t+2}] [D(\exp \alpha_1) P_{t+1}] z_t.
\end{aligned}
$$

Generalization

It is straightforward to generalize and to show that:

$$E_t(\exp[\alpha_1' z_{t+1} + \ldots + \alpha_h' z_{t+h}]) = \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} [D(\exp\alpha_h)P_{t+h}] \times \ldots$$
$$\ldots \times [D(\exp\alpha_1)P_{t+1}]\, z_t.$$

B Pricing formulas

In this appendix, I detail the computation of the three multiplicative components of $P(t,h)$ (the price at date t of a bond with residual maturity h), namely $P_1(t,h)$, $P_2(t,h)$ and $P_3(t,h)$. More precisely, this appendix propose a way to compute $G_1(t,h)$, $G_2(t,h)$, A_h and B_h that are such that:

$$\begin{cases} P_1(t,h) & = G_1(t,h)z_t \\ P_2(t,h) & = G_2(t,h)z_t \\ P_3(t,h) & = \exp(A_h + B_h s_t) \end{cases}$$

These formulas leads to equation (7).[49]

B.1 Computation of $P_1(t,h)$

The targets $\bar{r}_t$ are the only stochastic variables involved in the computation of $P_{1,}(t,h)$. The previous Appendix shows that the expectation of an exponential-affine combination of a variable that follows a Markov-switching process is available in closed form. This leads to the following formula:

$$P_1(t,h) = E_t^{\mathbb{Q}}\left(\exp\left[-\sum_{i=0}^{h-1}\bar{r}_{t+i}\right]\right) = G_1(t,h)z_t \tag{10}$$
$$\text{with } G_1(t,h) = \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} \left[\prod_{i=h-1}^{1} D(\exp[-\Delta_m])\Pi^*_{t+i}\right] D(\exp[-\Delta_m])$$

and where

- $D(x)$ is a diagonal matrix whose diagonal entries are those of the vector x.
- The matrices Π^*_t, which are of dimension $6(N+1)\times 6(N+1)$, contain the risk-neutral probabilities of switching from one regime –defined by a policy rate, a monetary-policy regime and a bank's liquidity situation– to another. As their physical-measure counterparts, these matrices can take two values, depending on whether a monetary-policy meeting is scheduled at date t or not.
- The product operator $\prod$ works in a backward direction: if X_1 and X_2 are some square matrices, $\prod_{i=2}^{1} X_i = X_2 X_1$

[49] In equation (7), the i^{th} entry of $G(t,h)$ is the logarithm of the i^{th} entry of $G_1(t,h) + G_2(t,h)$.

It is important to stress that this formula does not require the use of time-demanding recursive algorithms used by most alternative discrete-time affine term-structure models. Since policy meetings do not take place at a fully regular frequency, the matrices G_t should be computed for every date. As in Piazzesi (2005), I resort however to an intermediate approach where I consider only the exact number of days until the next decision meeting whereas subsequent meetings are assumed to be equally spread (every 30 days). The latter approximation, that leads to the computation of (only) 31 matrices G_i (instead of one per day), results in negligible pricing errors.

B.2 Computation of $P_2(t,h)$

The computation of $E^{\mathbb{Q}}\left(\exp\left[-\sum_{i=0}^{h-1}(w_{t+i}+\xi_{t+i})\right]\right)$ is very close to this of $P_1(t,h)$. Indeed, using the law of iterated expectations, it comes:

$$P_2(t,h) = E_t^{\mathbb{Q}}\left(E^{\mathbb{Q}}\left[\exp\left(-\sum_{i=0}^{h-1}(w_{t+i}+\xi_{t+i})\right)\right] \middle| z_{exc,t+1},\dots,z_{exc,t+h-1}\right). \quad (11)$$

Then remark that $w_{t+i}+\xi_{t+i}=\left[\begin{pmatrix} w_{norm}+\xi_{norm,t+i}\end{pmatrix} \quad \begin{pmatrix} w_{exc}+\xi_{exc,t+i}\end{pmatrix}\right] z_{exc,t+i}$ and recall that the ξ's follow Ξ distributions based on beta distributions. Appendix C gives the Laplace transform of a variable drawn from a Ξ distribution, which provides us with $E(\exp(-\xi_{j,t}))$ for $j \in \{norm, exc\}$. This leads to:

$$E_t^{\mathbb{Q}}\left(\exp\left[-\sum_{i=0}^{h-1}(w_{t+i}+\xi_{t+i})\right]\right) = E_t^{\mathbb{Q}}\left(\exp\left[\sum_{i=0}^{h-1}\left[\begin{array}{cc}\vartheta_{norm} & \vartheta_{exc}\end{array}\right] z_{exc,t}\right]\right)$$

where $\exp\vartheta_j = E(\exp(-w_j-\xi_{j,t}))$. Then, using Appendix A again, one obtains:

$$\begin{aligned} P_2(t,h) &= G_2(t,h) z_t \quad (12)\\ \text{with } G_2(t,h) &= \left[\begin{array}{cc} 1 & 1\end{array}\right]\left\{D\left(\exp\left[\begin{array}{c}\vartheta_{norm}\\ \vartheta_{exc}\end{array}\right]\right)\Pi_{exc}\right\}^{h-1} D\left(\exp\left[\begin{array}{c}\vartheta_{norm}\\ \vartheta_{exc}\end{array}\right]\right) H_{exc}\end{aligned}$$

where H_{exc} is the selection matrix (whose entries are 0 or 1) that is such that $z_{exc,t} = H_{exc} z_t$.

B.3 Computation of $P_3(t,h)$

We have

$$\begin{aligned} P_3(t,h) &= E_t^{\mathbb{Q}} e^{-\sum_{i=0}^{h-1} s_{t+i}} = E_t^{\mathbb{Q}} e^{-\sum_{i=0}^{h-1} s_{1,t+i}+s_{2,t+i}} \\ \text{where } \left[\begin{array}{c} s_{1,t}\\ s_{2,t}\end{array}\right] &= \Phi^* \left[\begin{array}{c} s_{1,t-1}\\ s_{2,t-1}\end{array}\right] + \Sigma\varepsilon_t^*, \quad \varepsilon_t^* \sim \text{i.i.d. } \mathcal{N}^{\mathbb{Q}}(0,I).\end{aligned}$$

In that Appendix, I describe an algorithm originally presented by Borgy et al. (2011). This algorithm results in the same matrices than the recursive formula given in the seminal paper by Ang and Piazzesi (2003). However, this latter approach turns out to be time-demanding for high-frequency (weekly or daily) processes. As shown by Borgy et al. (2011), the algorithm described below is substantially quicker when h is large.

Let me denote by X_t the vector $[s_{1,t}, s_{2,t}, s_{1,t-1}, s_{2,t-1}]'$. X_t follows:

$$X_t = \tilde{\mu}^* + \tilde{\Phi}^* X_{t-1} + \tilde{\Sigma}\varepsilon_t^*, \quad \varepsilon_t^* \sim \mathcal{N}^{\mathbb{Q}}(0, I),$$

where $\tilde{\mu}^*$, $\tilde{\Phi}^*$ and Σ^* are easily deduced from μ^*, Φ^* and Σ. In the following, I show how to compute the vectors A_h and C_h that are such that

$$P_3(t,h) = E_t^{\mathbb{Q}}(\exp(\delta' X_{t+1} + \ldots + \delta' X_{t+h})) = \exp\left(A_h + C_h X_t\right)$$

where $\delta = [0,0,1,1]'$. Denoting by $F_{t,t+h}$ the random variable $X_{t+1} + \ldots X_{t+h}$, we get:

$$P_3(t,h) = E_t^{\mathbb{Q}}(\exp(\delta' F_{t,t+h})$$

Note that$F_{t+h,h}$ is a Gaussian random variable. We have

$$\begin{aligned} F_{t+h,h} &= \left(hI + (h-1)\Phi^* + \ldots + \Phi^{*h-1}\right)\mu^* + \\ &\quad \left(\Phi^* + \Phi^{*2} \ldots + \Phi^{*h}\right) X_t + \\ &\quad \left(I + \ldots + \Phi^{*h-1}\right)\varepsilon_{t+1}^* + \left(I + \ldots + \Phi^{*h-2}\right)\varepsilon_{t+2}^* + \ldots + \varepsilon_{t+h}^*. \end{aligned}$$

Therefore $F_{t+h,h} \sim \mathcal{N}^{\mathbb{Q}}(\Lambda_{0,h} + \Lambda_h X_t, \Omega_h)$ with

$$\begin{cases} \Lambda_h &= \Phi^* \left(\Phi^{*h} - I\right)\left(\Phi^* - I\right)^{-1} \\ \Lambda_{0,h} &= \left[\chi_{1,h} - hI\right]\left(\Phi^* - I\right)^{-1}\mu^* \end{cases}$$

and with

$$\begin{aligned} \Omega_h &= \operatorname{Var}\left(\left(I + \ldots + \Phi^{*h-1}\right)\varepsilon_{t+1}^* + \left(I + \ldots + \Phi^{*h-2}\right)\varepsilon_{t+2}^* + \ldots + \varepsilon_{t+h}^*\right) \\ &= \left(\Phi^* - I\right)^{-1}\Big[\left(\Phi^{*h} - I\right)\Sigma\Sigma'\left(\Phi^{*h} - I\right)' + \ldots \\ &\quad + \left(\Phi^* - I\right)\Sigma\Sigma'\left(\Phi^* - I\right)'\Big]\left(\Phi^* - I\right)'^{-1} \\ &= \left(\Phi^* - I\right)^{-1}\left[(h-1)\Sigma\Sigma' - \Lambda_h\Sigma\Sigma' - \Sigma\Sigma'\Lambda_h' + \Pi(h, \Phi^*, \Sigma)\right]\left(\Phi^* - I\right)'^{-1} \end{aligned}$$

where $\Pi : (h, \Phi^*, \Sigma) \rightarrow \left(\Phi^{*h}\right)\Sigma\Sigma'\left(\Phi^{*h}\right)' + \ldots + \left(\Phi^*\right)\Sigma\Sigma'\left(\Phi^*\right)' + \Sigma\Sigma'$. Instead of using a brute-force approach (based on h loops) to compute $\Pi(h, \Phi^*, \Sigma)$, we exploit the fact that $\Pi(kp, \Phi^*, \Sigma) = \Pi(k, \Phi^{*p}, \Pi(p, \Phi^*, \Sigma) - \Sigma\Sigma') + \Sigma\Sigma'$. This can substantially reduce the computation time to compute. It suffices to apply the latter formula a few times, based on an integer factorization of h. Finally

$$\begin{cases} A_h &= \delta'\Lambda_{0,h} + \frac{1}{2}\delta'\Omega_h\delta \\ C_h &= \delta'\Lambda_h. \end{cases}$$

Finally, since $s_{1,t} \equiv 0$ under $\mathbb{P}$, denoting by B_h the second column of C_h, we get:

$$P_3(t,h) = \exp\left(A_h + B_h s_t\right). \tag{13}$$

C The distribution of ξ_t

Computation of $E(\exp[\xi])$ if $\xi \sim \Xi(p, \alpha_P, \beta_P, \alpha_N, \beta_N)$:

$$\begin{aligned} E(\exp[\xi]) &= pE(\exp \xi_P) + (1-p)E(\exp \xi_N) \\ &= p.f(\alpha_P, \beta_P) + (1-p).f(\alpha_N, \beta_N) \end{aligned}$$

where $f(\alpha, \beta)$ is given by:

$$f(\alpha, \beta) = 1 + \sum_{k=0}^{\infty} \frac{1}{k!} \left(\prod_{i=0}^{k-1} \frac{\alpha + i}{\alpha + \beta + i} \right).$$

D Tables and Figures

Tab. 1: **Descriptive statistics of yields**

Notes: The table reports summary statistics for selected yields. The data are monthly and cover the period from January 1999 to February 2012. Two auto-correlations are shown (the 1-day and the 1-year auto-correlations). The yields are continuously compounded and are in percentage annual terms. Panel B presents the covariances and the correlations of the yields. The EONIA spread is the yield differeintial between the (annualized) EONIA and the target rate. Panel C reports some results of a principal-component analysis carried out on the spreads between the yields and the target rate. More precisely, it shows the share of the variances of the different spreads that are explained by the first three principal components. Two samples are cnosidered: January 1999 to February 2012 and January 1999 to August 2008.

Panel A - Descriptive statistics 1999-2012								
	Target	EONIA	1-mth	3-mth	6-mth	12-mth	2-yr	4-yr
Mean	2,64	2,56	2,58	2,60	2,64	2,74	2,94	3,35
Median	2,50	2,57	2,60	2,64	2,67	2,77	2,91	3,37
Standard dev.	1,15	1,34	1,33	1,34	1,34	1,34	1,28	1,16
Skewness	0,14	0,14	0,14	0,14	0,13	0,13	0,17	0,22
Kurtosis	1,91	2,04	2,01	1,95	1,93	1,92	2,02	2,23
Auto-cor. (1 day)	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00
Auto-cor. (1 year)	0,52	0,56	0,57	0,57	0,57	0,58	0,60	0,62
Panel B – *Correlations* \ **Covariances**								
Target	1,31	1,51	1,50	1,49	1,48	1,44	1,34	1,16
EONIA	*0,98*	1,80	1,77	1,77	1,75	1,71	1,60	1,39
1-mth OIS	*0,98*	*0,99*	1,77	1,77	1,76	1,73	1,62	1,41
3-mth OIS	*0,98*	*0,99*	*1,00*	1,78	1,79	1,76	1,65	1,43
6-mth OIS	*0,96*	*0,98*	*0,99*	*1,00*	1,80	1,78	1,68	1,46
12-mth OIS	*0,94*	*0,96*	*0,97*	*0,99*	*1,00*	1,79	1,70	1,48
2-yr OIS	*0,91*	*0,93*	*0,95*	*0,96*	*0,98*	*0,99*	1,65	1,46
4-yr OIS	*0,87*	*0,90*	*0,91*	*0,93*	*0,94*	*0,96*	*0,99*	1,34
Panel C – Principal component analysis of spreads vs. target								
1999-2012								
	Eonia	1-mth	3-mth	6-mth	12-mth	2-yr	4-yr	Total
1st PC	0,51	0,80	0,93	0,96	0,93	0,80	0,48	0,77
2d PC	0,86	0,96	0,97	0,96	0,96	0,99	0,91	0,94
3rd PC	0,99	0,96	0,99	1,00	0,98	0,99	0,99	0,98
1999-2008								
1st PC	0,03	0,58	0,89	0,93	0,95	0,89	0,63	0,70
2d PC	0,84	0,74	0,90	0,93	0,96	0,93	0,71	0,86
3rd PC	0,99	0,85	0,97	0,96	0,96	0,99	0,95	0,95

Tab. 2: **Parameter estimates**

Notes: The table reports the estimates of the parameters defining the dynamics of the factor under historical and risk-neutral measures. The estimation data are daily and span the period from January 1999 to February 2012. Standard errors are reported in parenthesis. The sign "*" (after a number) denotes significance at the 5% level. The parameters a_i relate the probabilities of changes in the policy rate or in the monetary-policy regime to the level of the policy rate (see Subsection 3.1.1 and notably equation 2). The parameters that define the risk-neutral dynamics are indicated by *. The dynamics of the Markov chain $z_{exc,t}$ is defined by $p_{exc,exc}$ and $p_{norm,norm}$ which are, respectively, the probabilities of remaining in the excess-liquidty regime and the non-excess-liquidty regime. σ_{fcst} and σ_{pric} are, respectively, the standard deviations of the measurement errors ξ_t^C and of the pricing errors ξ_t^R (see equation 8).

	a_1^*	a_2^*	k^*		a_1	a_2	k
rise in	-0	0.11*	0.61		1.2*	-0.17*	0.82*
the target	(0.81)	(0.036)	(0.58)		(0.23)	(0.014)	(0.105)
cut in	0.41	0.098*	0.38		-0.00011	0.11*	0.49*
the target	(0.46)	(0.02)	(0.44)		(0.31)	(0.0104)	(0.14)
ES	3.8*	0.4*			3.9*	0.065*	
	(0.065)	(0.022)			(0.22)	(0.032)	
SE	9*	-0.87*			9*	-1.3*	
	(0.49)	(0.091)			(0.32)	(0.072)	
TS	4.7*	0.25*			4.1*	0.19*	
	(0.068)	(0.018)			(0.37)	(0.058)	
ST	5.3*	0.109			5.4*	-0.33*	
	(0.081)	(0.058)			(0.16)	(0.09)	

$\alpha_{P,norm}$	$\alpha_{N,norm}$	$\beta_{P,norm}$	$\beta_{N,norm}$	μ_{norm}	p_{norm}
0.76*	0.22*	8.3*	0.86*	0.04*	0.48*
(0.035)	(0.0056)	(0.13)	(0.064)	(0.00012)	(0.0105)
$\alpha_{P,exc}$	$\alpha_{N,exc}$	$\beta_{P,exc}$	$\beta_{N,exc}$	μ_{exc}	p_{exc}
0.75*	0.5*	4.6*	9*	-0.65*	0.55*
(0.05)	(0.038)	(0.25)	(1.6)	(0.0012)	(0.021)

ρ_1^*	ρ_2^*	β^*		ρ_2		σ	
0.9999*	0.9999*	16*		0.94*		0.00007*	
(0)	(0)	(6.1)		(0.0012)		(0.00003)	

$p^*_{norm,norm}$	$p^*_{exc,exc}$		$p_{norm,norm}$	$p_{exc,exc}$		σ_{fcst}	σ_{pric}
0.9999*	0.9999*		0.99*	0.9999*		0.23*	0.085*
(0.0001)	(0.00014)		(0.00093)	(0)		(0.0029)	(0.00021)

Fig. 1: **Target rate, EONIA and OIS**

Notes: Panel A shows the target rate together with the overnight interbank interest rate (EONIA). Panel B displays the EONIA spread, i.e. the spread between the EONIA and the target. The four vertical bars in Panel B indicate the four following dates, respectively: 8 October 2008 (introduction of Fixed-Rate Full Allotment procedures), 3 December 2009 (announcement of the phasing out of the very long-term refinancing operations), 4 August 2011 (given the renewed financial-market tensions, announcement of supplementary VLTRO), 8 December 2011 (3-year VLTRO). Panel C presents the target rate and two OIS yields, the spreads between the latter and the target being reported in Panel D.

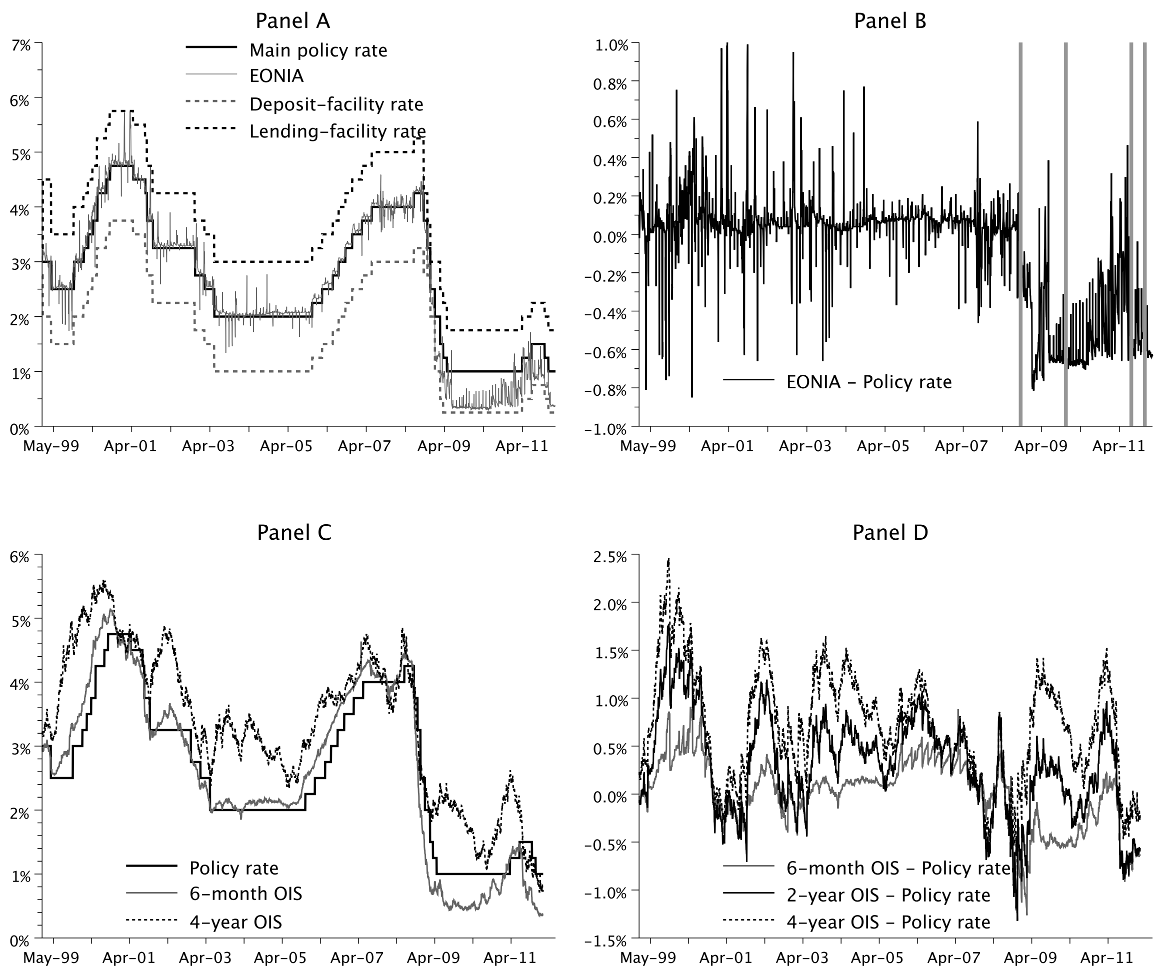

Fig. 2: **Estimated s_t process and model fit**

Notes: Panel A displays the estimated s_t process (see equation (9)). Panels B, C and D compare model-implied yields with their data (actual) counterparts. The latter panels also display (grey dashed line) the part of the model-implied yields that is accounted for by the regime variable z_t (that is $-\frac{1}{h}\left[G(t,h)z_t + A_h\right]$ in equation 7).

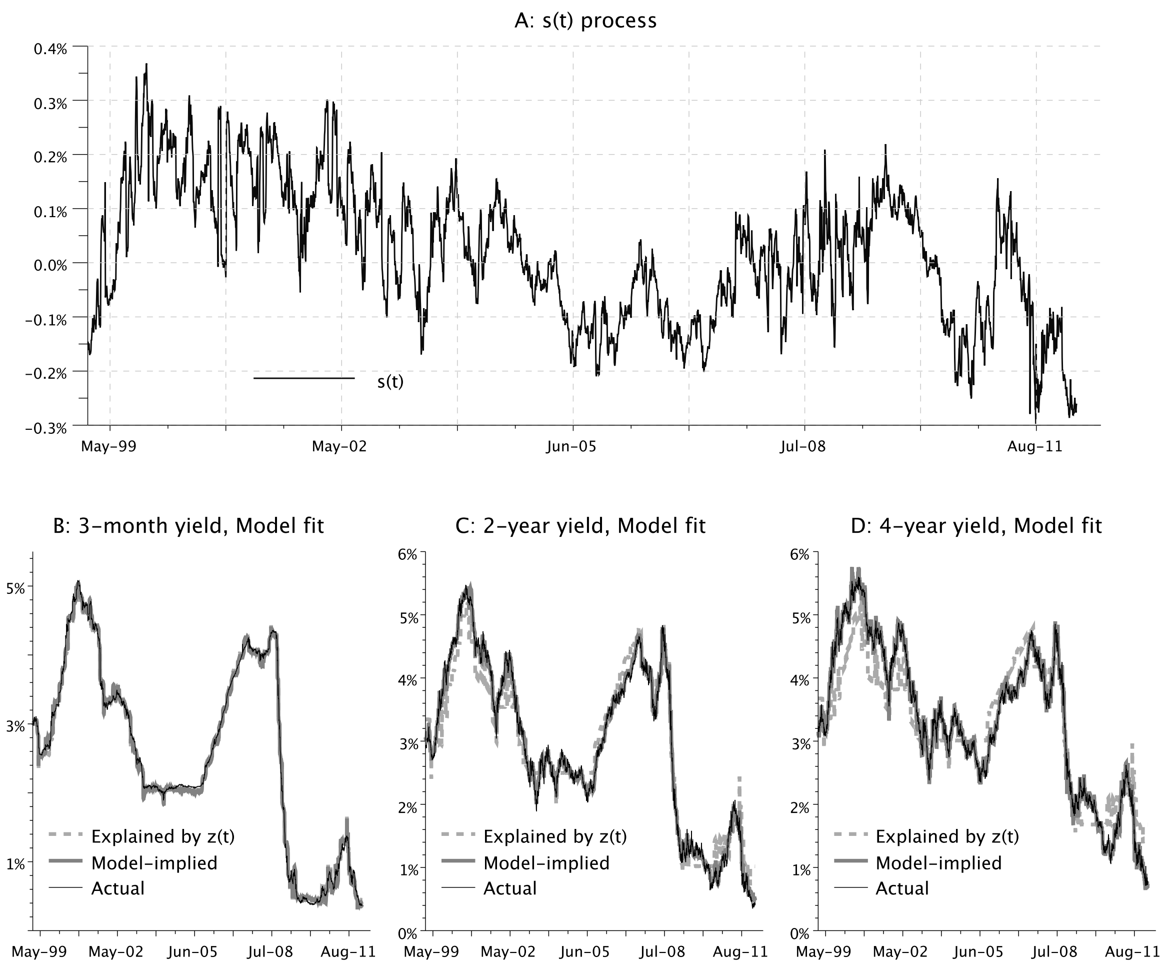

Fig. 3: **Regimes' estimates**

Notes: Panel A compares the model-implied forecasts with the survey-based ones (*Consensus Forecasts*). Panel B displays the (smoothed) probabilities of being in the different monetary-policy regimes. Panel C shows the smoothed probabilities of being in the excess-liquidity regime. The four vertical lines reported in Panel C indicate the following dates: 8 October 2008 (introduction of Fixed-Rate Full Allotment procedures), 3 December 2009 (announcement of the phasing out of the very long-term refinancing operations), 4 August 2011 (given the renewed financial-market tensions, announcement of supplementary VLTRO), 8 December 2011 (3-year VLTRO).

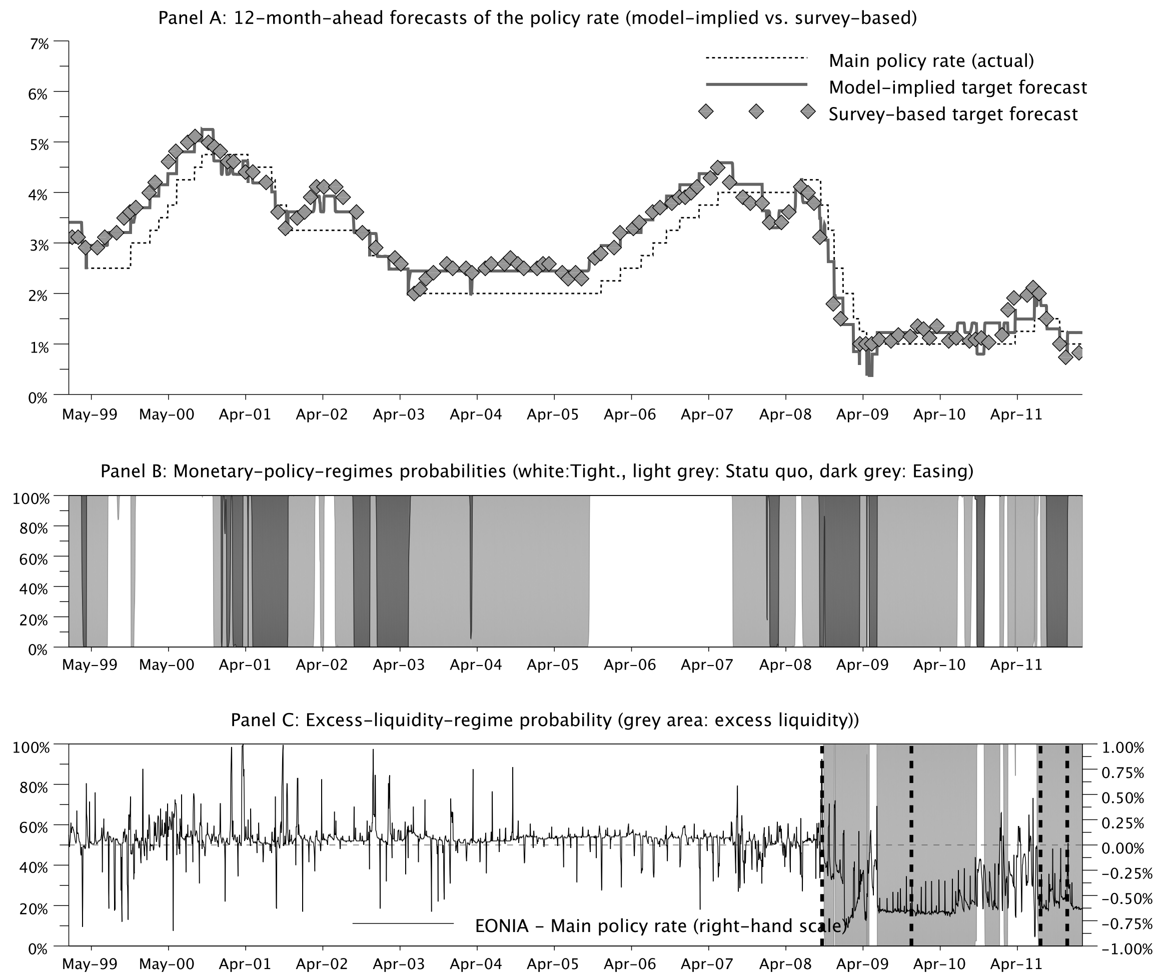

Fig. 4: **Changes in monetary-policy regimes and central-bankers' announcements**

Notes: This Figure relates some specific (estimated) changes in the monetary-policy regimes to specific central bankers speeches (summer 2008). The grey shaded area corresponds to the (smoothed) probability of being in the tightening monetary-policy regime. The two vertical bars indicate the dates of two subsequent ECB governing councils (5 June 2008 and 3 July 2008). The quotation from J.-C. Trichet comes from the "Questions & Answers" part of the Press Conference held at the ECB on 5 June 2008. The extract from the *Financial Times* comes from www.ft.com (article entitled "ECB raises interest rates to 4.25%", by Ralph Atkins).

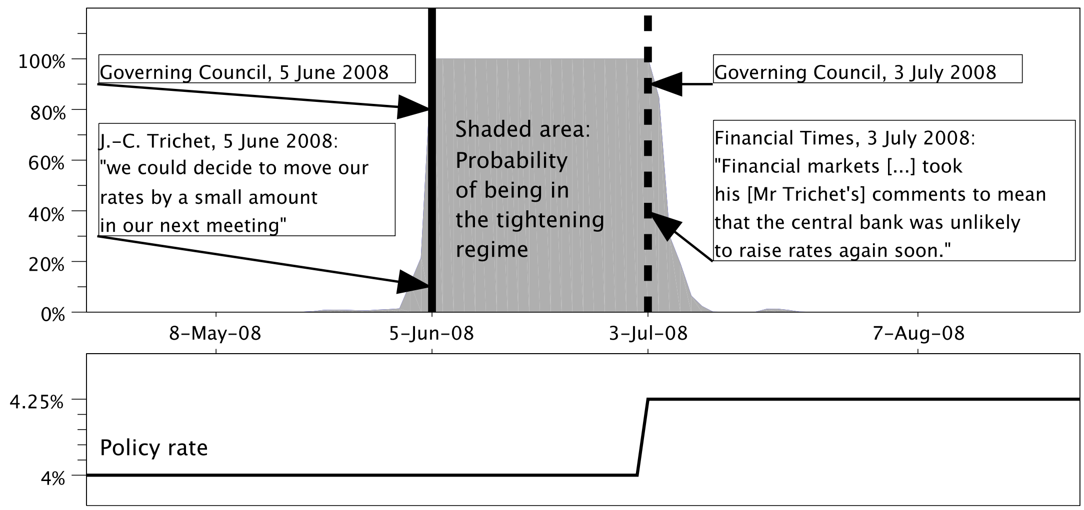

Fig. 5: **Fitted yield curves and influence of monetary-policy regimes**

Notes: These plots compare model-implied (diamonds) with observed (black circles) yield curves at different dates. In addition, each plot reports the (model-implied) yield curves that would have been obtained if other monetary-policy regimes had prevailed. The seven circles (and diamonds) correspond respectively to the following maturities: 1 day, 1, 3, 6 months, 1, 2 and 4 years.

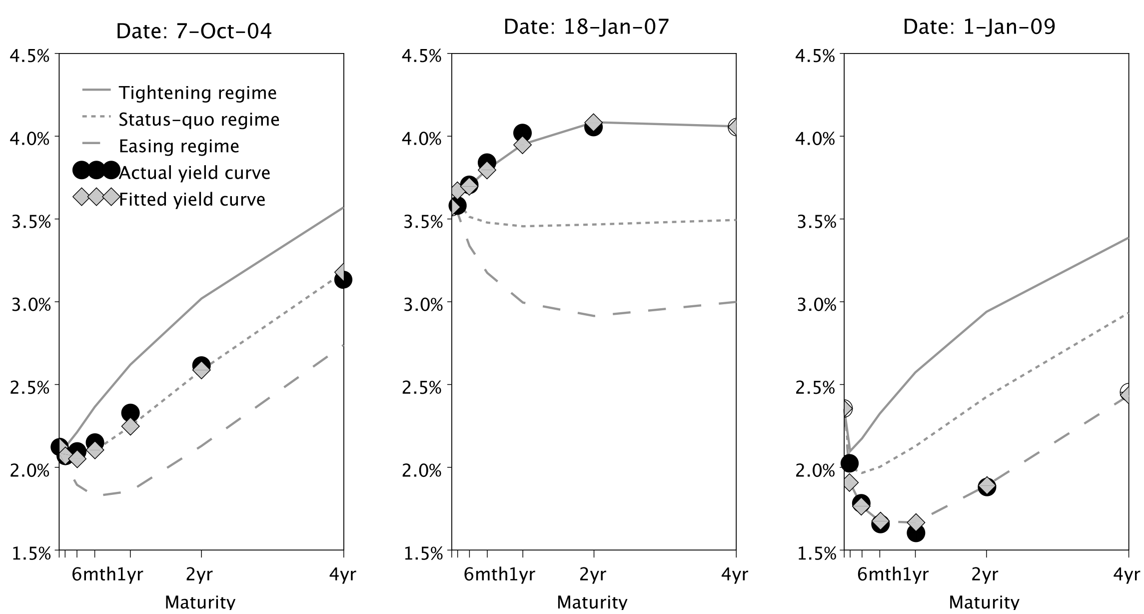

Fig. 6: **The effect of the excess-liquidity situation on the yield curve**

Notes: This plot illustrates the effect on the yield curve of the excess-liquidity situation. For a selected date (12 March 2009) for which the estimates of the Markov chain $z_{exc,t}$ indicate that the excess-liquidity regime prevailed, it compares the fitted yield curve with the (fitted) one that would have been observed absent the excess-liquidity situation.

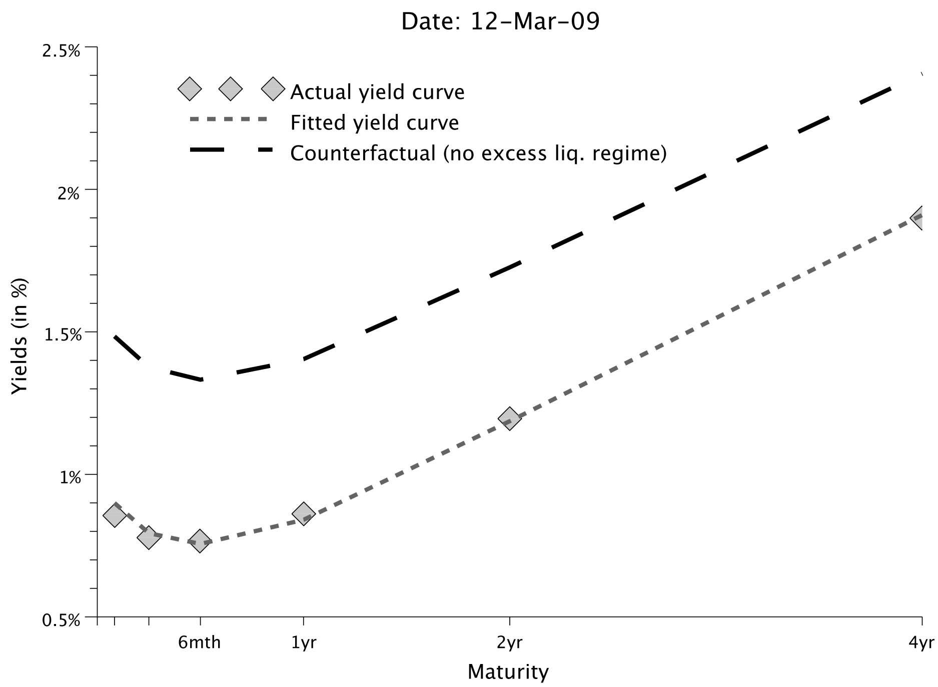

Fig. 7: **Estimated probabilities of regime changes**

Notes: These plots show the estimated probabilities of regime change over the next 30 days (period which includes only one monetary-policy meeting). As detailed in Subsection 3.1, these curves are based on some parametric forms of the target rate $\bar{r}_t$. Each plot displays the historical, or physical, probabilities as well as the risk-neutral ones. The upper four panels define the probabilities of monetary-policy-regime changes, the lower two show the target-change probabilities. Alltogether, these probabilities define the matrices Π_t and Π_t^* describing respectively the dynamics of the Markov chain z_t (indicating the current target rate and the monetary-policy regime) under the physical and the risk-neutral measures.

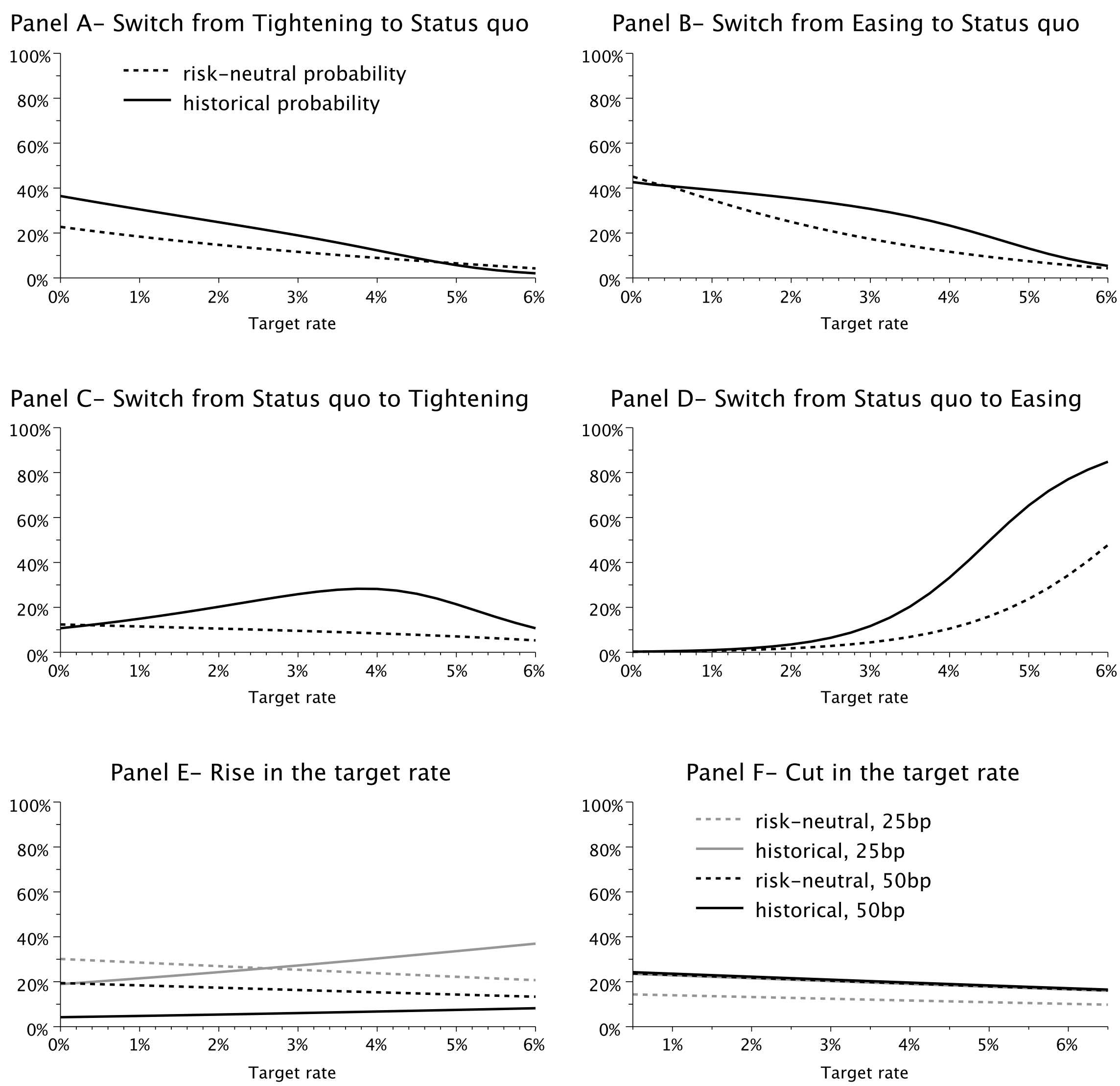

Fig. 8: **Standard deviations associated with the 3-month-ahead forecasts of the policy rate**

Notes: These plots present the standard deviations (reported in basis points) associated with the 3-moth-ahead forecasts of the policy rate. These standard deviations depend on the target rate and on monetary-policy phase, which illustrates the heteroskedasticity of the policy rate in the model.

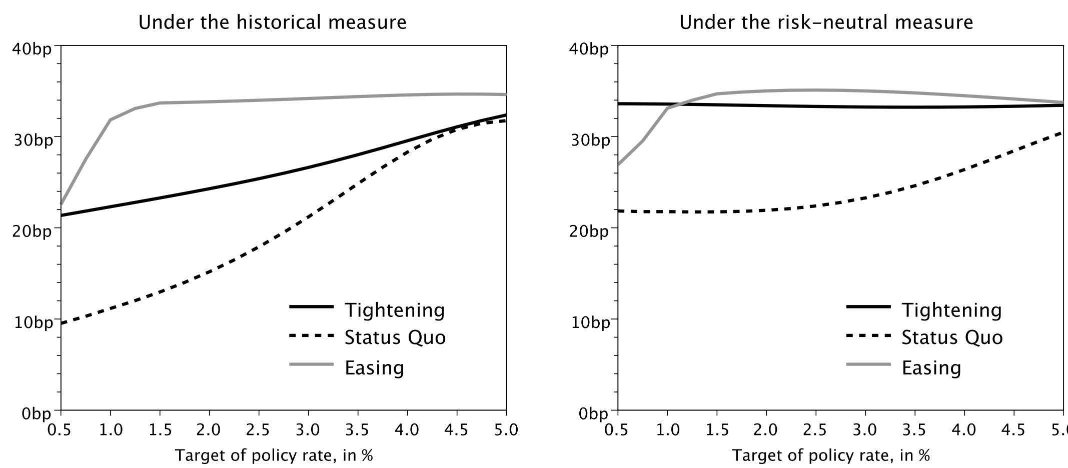

Fig. 9: Risk-neutral vs. physical policy-rate forecasts, and associated risk premia

Notes: These plots show the term structures of the forecasts of the policy rate under the physical (grey circles) and the risk-neutral (black circles) measures. Up to the Jensen inequality, these curves can be considered as forward rates of the policy rate (as regards the risk-neutral measure). The three columns of plots correspond to the current (period 0) monetary-policy regime (either tightening, status quo or easing). The three rows of plots correspond to different (current) policy rates (1%, 2.5% and 3.5%). Each of the 9 plots presenting the policy-rate forecasts is completed by a plot (placed below the first plot) of the corresponding risk premia, i.e. the spread between the two forecast curves (in basis points). 90% confidence intervals are reported. These confidence intervals are based on bootstrap techniques: the asymptotic distribution of the parameter estimates is used to draw 1000 alternative sets of parameter estimates that, in turn, are used to compute 1000 sets of alternative risk premia; the dashed lines correspond to the 5 and 95 percentiles of the obtained risk-premia distributions.

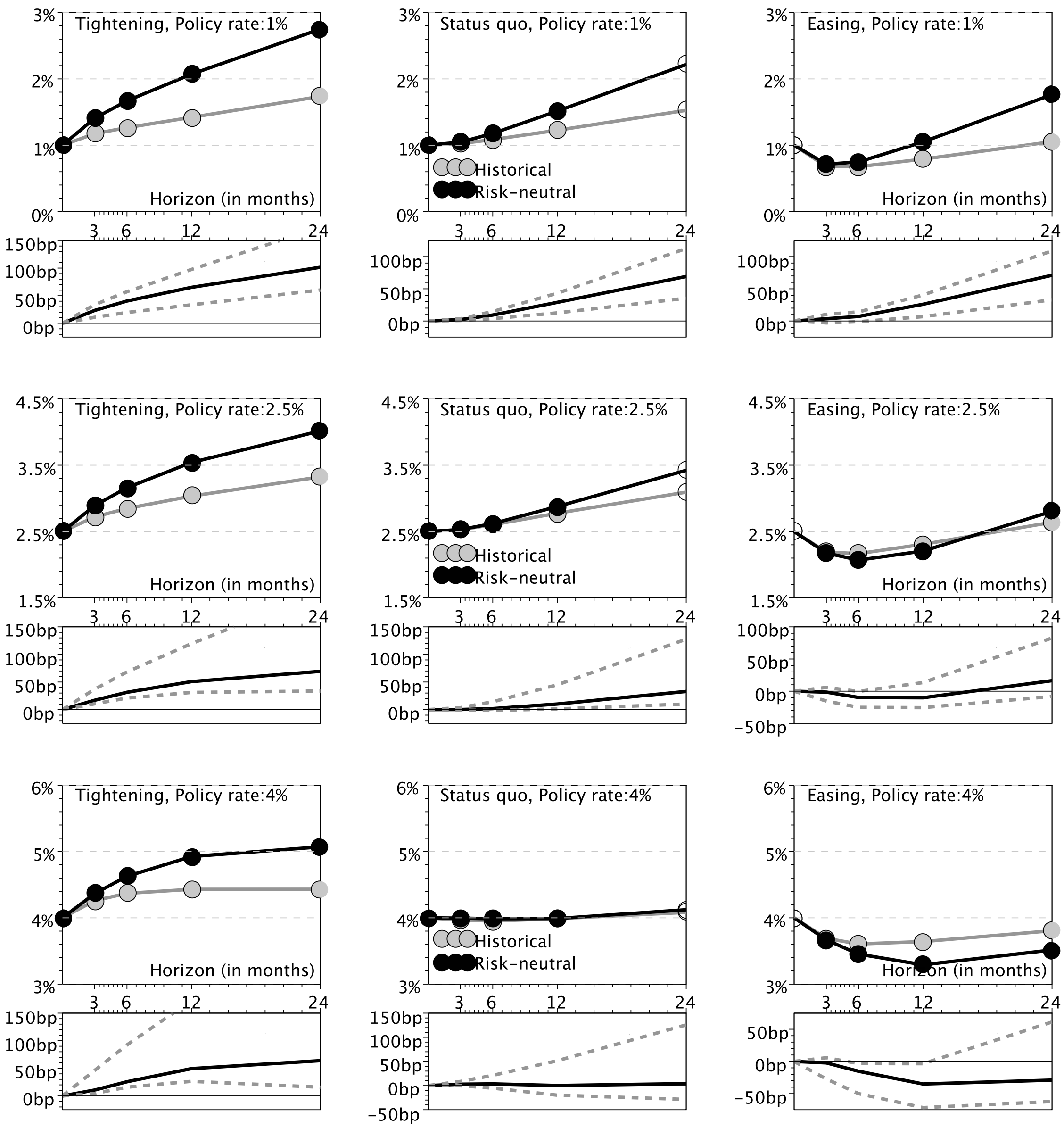

Fig. 10: **Simulation of forward-guidance measures**

Notes: These plots show the term-structure impact of a central bank's commitment to keep the target rate at its current level for 12 or 24 months. Two different policy rates are considered (1% and 3.5%). For each policy rate (1% or 3.5%) and each commitment durations (12 or 24 month), two plots are reported: the upper one displays yield curves with/without commitment of the central bank, the lower plot present the associated downward effect of the forward-guidance measure (that is the spread between the two curves plotted in the upper plot, in basis points). Note that here, I abstract from the effects of the excess-liquidity regime (w_t) and s_t is set at 0, its unconditional level (the rationale behind this is that in my framework, these two latter factors are independent from the policy rate, which is the only factor affected by the measure). In the baseline as well as in the counterfactual case, the monetary-policy regime is set to the status-quo regime. Regarding the downward effect of the measure (lower plots of each pair of charts), 90% confidence intervals are reported. These confidence intervals are based on bootstrap techniques: the asymptotic distribution of the parameter estimates is used to draw 1000 alternative sets of parameter estimates that, in turn, are used to compute 1000 sets of alternative effects of the measure; the dashed lines correspond to the 5 and 95 percentiles of the obtained downward-effects distributions.