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The Varying Coefficient Bayesian Panel VAR Model

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Abstract

Interacted Panel VAR (IPVAR) models allow coefficients to vary as a deterministic function of observable country characteristics. The varying coefficient Bayesian panel VAR generalises this to the stochastic case. As an application of this framework, I examine if the impact of commodity price shocks on consumption and the CPI varies with the degree of exchange rate, financial, product and labour market liberalisation on data from 1976Q1-2006Q4 for 18 OECD countries. The confidence bands are smaller in the deterministic case and as a result most of the characteristics affect the transmission mechanism in a statistically significant way. But only financial liberalisation is an important determinant of commodity price shocks in the stochastic case. This suggests that results from IPVAR models should be interpreted with caution.

Keywords: Bayesian Panel VAR; Commodity Price shocks.

JEL **classification**: F32, E52, C11, C23.

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I. Introduction

A popular way of modelling time variation in VAR models is to allow the coefficients to stochastically vary as a random walk. But this approach is silent on the origins of the structural change. An alternative body of research attempts to answer that question by allowing the model coefficients to vary as a deterministic function of observable economic characteristics, such as the exchange rate regime, typically by pooling the data across countries and time in a panel VAR setup for that purpose. In this paper, I introduce the varying coefficient Bayesian panel VAR model, which also allows the coefficients to vary as a stochastic function of observable characteristics instead. As an application, I examine how the transmission of commodity prices shocks to real consumption and CPI inflation is affected by either exchange rate, financial, labour or product market liberalisation with data on 18 OECD countries over the period 1976Q1 – 2006Q4. I compare the results from the stochastic and deterministic approach to examine to which extent this assumption leads to econometric bias in the results.

There has been substantial interest in estimating VAR models with time-varying coefficients to document stylised facts about the transmission mechanism of monetary policy (See for example Cogely and Sargent (2005); Primiceri (2005)), fiscal policy (Perreira and Lopes, 2010) and commodity price (Baumeister and Peersman, 2010) shocks to output and inflation. Most papers in this literature assume that coefficients evolve stochastically according to a slowly moving random walk. While this means that changes in the coefficients will reflect permanent structural changes, it is not possible to infer why the structural change has happened.

A separate body of work relates changes in the transmission of shocks to observable economic characteristics, such as financial or labour market liberalisation with VARs estimated for individual countries. For example, Mertens (2008) and Olivei and Teynero (2007; 2008) allow the coefficients of their VARs to depend on regulation Q in the US and wage rigidity in the US, Japan, UK, France and Germany, respectively, to examine the impact of changes in these economic characteristics on the monetary policy transmission mechanism. Similarly, Iacoviello and Minetti (2003) estimate the impact of financial liberalisation on monetary transmission to house prices by estimating single-country VARs for several countries before and after financial liberalization. But there is of course always a question whether the degree of time-series variation in a single country is sufficient to estimate such effects satisfactorily. It is for that reason that some researchers choose to use the panel VAR approach, to exploit the cross-sectional variation in economic structures across countries, instead. For example Assenmacher-Wesche and Gerlach (2010) and Calza et al (2013) both estimate panel VARs on a set of countries with more and less developed financial (mortgage) markets to infer the impact of mortgage market development on the monetary policy transmission mechanism. Similarly, Mendoza, Ileztzki and Vegh (2012) estimate fiscal policy VARs over several groups of countries to examine to which extent trade openness, capital account openness, financial fragility and the exchange rate regime affect government spending multipliers.

If the economic characteristic in question can be observed both over time and in the cross-section, it might, of course, be more desirable to estimate a model that exploits all of the variation across both of these dimensions. In particular, Broda (2001; 2004) estimates a panel VAR to examine if the impact of commodity prices shocks in developing countries varies with the degree of the exchange rate regime. He interacts all of the model coefficients with an indicator of exchange rate regime flexibility, that varies by country and time, for that purpose. This is what Towbin and Weber (2013) refer to as the 'interacted panel VAR' approach (IPVAR) in their exploration of the role of changes in developing countries' financial structure and exchange rate regimes in the transmission of commodity price shocks.

The underlying assumption in the IPVAR approach is that coefficients are a deterministic function of the country characteristics of interest. This means that all of the heterogeneity in VAR coefficients is explained by these economic characteristics and allows estimation of the interacted model by pooling the data across time and countries. If this assumption is violated, the estimates will be subject to dynamic heterogeneity bias (Pesaran and Smith, 1995). Sa, Towbin and Wieladek (2014) use the mean group estimator to address this potential problem, but since that approach requires estimation country-by-country, degrees of freedom considerations typically constrain the number of economic characteristics that can be analysed to two. The main contribution of this paper is to develop a Bayesian shrinkage estimator for panel VAR models that allows modelling the coefficients as a stochastic function of multiple structural characteristics. In similar spirit to previous applications of this approach, I examine whether the transmission of commodity price shocks to consumption and the CPI varies with exchange rate regime flexibility, financial, labour and product market liberalisation. The model is estimated on data for 18 OECD countries from 1976Q1- 2006Q4 for that purpose. I also estimate a model where the coefficients are a deterministic function of economic characteristics, to compare the results from the proposed, to the previously used, approach.

Economic theory makes clear predictions about how these economic characteristics should affect the transmission of commodity price shocks to CPI and consumption. Greater exchange rate regime flexibility allows the exchange rate to react to commodity price shocks and hence should weak their impact on domestic consumption and CPI. Similarly, it is frequently argued that labour and product market liberalisation should weaken the domestic propagation of, and hence the dynamics associated with, such shocks. Finally, to the extent that financial liberalisation allows for greater risk-sharing across countries, consumption should react less in more financially liberalised countries.

The results suggest that only financial liberalisation has a significant impact on the transmission of commodity price shocks that is consistent with theory. However, when the model is estimated subject to the assumption that the VAR coefficients are a deterministic function of the economic characteristics, exchange rate regime flexibility and product market liberalisation spuriously emerge as important and statistically significant determinants of the commodity price transmission mechanism. This suggests that allowing for coefficients to vary as a stochastic function, as in the new estimator proposed in this paper, is important in studying whether the impact of a shock is affected by structural characteristics or not.

The remainder of the paper is structured as follows: Section two describes the varying coefficient Bayesian panel VAR model. Section three discusses the application, the data and the results. Section four concludes.

2. The varying coefficient Bayesian panel VAR model

Consider the following, time-varying coefficient panel VAR model:

$$\boldsymbol{Y_{c,t} = X_{c,t}B_{c,\tau} + E_{c,t}} \qquad \boldsymbol{E_{c,t} \sim N(0, A'_{c,\tau}\Sigma_c A_{c,\tau})} \tag{1}$$

Where $Y_{c,t}$ is and $1\ x\ N$ matrix of N endogenous variables for country c at time t. $X_{c,t}$ contains the lags of $Y_{c,t}$ and a constant term. The total number of lags is *L*, and *K=L+1*. The total number of countries (time series) is C (T). I assume that $A_{c,\tau}$ is lower triangular. Now let $y_{c,t} \equiv vec(Y_{c,t})$, $\beta_{c,\tau} \equiv vec(B_{c,\tau})$, $a_{c,\tau} \equiv vec(A_{c,\tau})$ and $e_{c,t} \equiv vec(E_{c,t})$. To estimate this model, we need to make a prior assumption about the time-varying coefficients, $\beta_{c,\tau}$ and $a_{c,\tau}$. Previous work has typically assumed that these coefficients evolve according to a stochastic random walk (Cogely and Sargent, 2005; Primiceri, 2005; Canova and Cicarelli, 2008) or a markov-switching (Sims and Zha, 2006) process. In this paper I make the prior assumptions that they vary as a function of observables:

$$\boldsymbol{\beta_{c,\tau}\,|y_{c,t}, X_{c,t}, a_{c,\tau}, \Sigma_c \sim N(D_{c,\tau}\delta_B, \Lambda_{Bc})} \tag{2}$$

$$\boldsymbol{a_{c,\tau}\,|y_{c,t}, X_{c,t}, \beta_{c,\tau}, \Sigma_c \sim N(D_{c,\tau}\delta_A, \Lambda_{Ac})} \tag{3}$$

where $\boldsymbol{\delta_B}$, $\boldsymbol{\delta_A}$ is a matrix of pooled coefficients across countries, which relate the weakly exogenous variables $\boldsymbol{D_{c,t}}$ to the individual country coefficients $\boldsymbol{\beta_{c,\tau}}$, $\boldsymbol{A_{c,\tau}}$, with the variances $\boldsymbol{\Lambda_{Bc}}$, $\boldsymbol{\Lambda_{Ac}}$ determining the tightness of these priors. I parameterize $\boldsymbol{\Lambda_{Bc} = \lambda_B L_{Bc}}$ and $\boldsymbol{\Lambda_{Ac} = \lambda_A L_{Ac}}$, $\boldsymbol{\lambda_A}$ and $\boldsymbol{\lambda_B}$ are shrinkage parameters, which are estimated from the data. For these parameters, I follow the approach in Jarocinski (2010) and assume an inverted Gamma density:

$$\boldsymbol{\lambda_B|y_{c,t}, X_{c,t}, \beta_{c,\tau}, \Sigma_c \sim IG_2 \propto \lambda_B^{-\frac{v+2}{2}} exp\left(-\frac{1}{2}\frac{s}{\lambda_B}\right)} \tag{4}$$

$$\boldsymbol{\lambda_A|y_{c,t}, X_{c,t}, a_{c,\tau}, \Sigma_c \sim IG_2 \propto \lambda_A^{-\frac{v+2}{2}} exp\left(-\frac{1}{2}\frac{s}{\lambda_A}\right)} \tag{5}$$

The greater $\boldsymbol{\lambda_B}$ and $\boldsymbol{\lambda_A}$ the larger the degree to which the country-specific coefficients are allowed to differ from the common mean. If $\boldsymbol{\lambda_B} \rightarrow \infty$ and $\boldsymbol{\lambda_A} \rightarrow \infty$, this approach will lead to country-by-country estimates, while $\boldsymbol{\lambda_B = 0}$ and $\boldsymbol{\lambda_A = 0}$ implies pooling across all countries of the dynamic and contemporaneous coefficients, respectively. The parameterisation of $\boldsymbol{\Lambda_{Bc}}$ and $\boldsymbol{\Lambda_{Ac}}$in this manner has the econometrically convenient property that it is necessary only to estimate two hyper-parameters $\boldsymbol{\lambda_B}$ and $\boldsymbol{\lambda_A}$ to determine the degree of heterogeneity in the lagged dependent variable and contemporaneous coefficients, respectively. But there is of course one drawback: the coefficients in $\boldsymbol{\beta_{c,\tau}}$ and $\boldsymbol{a_{c,\tau}}$ may have different magnitudes. In specifying a single parameter that determines the degree of heterogeneity, there is therefore the risk that some coefficients are allowed to differ from the common mean by a small fraction of their own size, while others can differ by orders of magnitude.

Following the approach proposed in Jarocinski (2010) and a procedure analogous to the Litterman (1986) prior, $\boldsymbol{L_{Bc}}$ is a matrix of scaling factors used to address this problem. In particular, $\boldsymbol{L_{Bc}(k,n)} = \frac{\sigma_{cn}^2}{\sigma_{ck}^2}$, where $\boldsymbol{c}$ is the country, $\boldsymbol{n}$ the equation and $\boldsymbol{k}$ the number of the variable regardless of lag. $\boldsymbol{\sigma_{cn}^2}$ is the estimated variance of the residuals of a univariate auto-regression of the endogenous variable in equation $\boldsymbol{n}$, of the same order as the VAR, and is obtained pre-estimation. $\boldsymbol{\sigma_{ck}^2}$ is the corresponding variance for variable $\boldsymbol{k}$ and obtained in an identical manner. $\boldsymbol{L_{Ac}}$ is obtained in a similar manner. To the extent that unexpected movements in variables will reflect the difference in the size of VAR coefficients, scaling by this ratio of variances allows us to address this issue. Finally, note that $A'_{c,\tau}$ is assumed to be lower triangular, with ones on the diagonal, following the approach in Primiceri (2005). As a result, $|A_{c,\tau}| = \Pi a_{c,\tau,ii} = 1$ and $|A'_{c,\tau}\Sigma_c A_{c,\tau}| = |\Sigma_c|$.

Previous work has adopted three different ways of estimating panel VAR models with the structure as set out in **(1) – (5)**. Abritti and Weber (2010) and Towbin and Weber (2013) assume that $\boldsymbol{\lambda_B = 0}$, which means that $\boldsymbol{B_{c,\tau}}$ is a deterministic function the vector of weakly exogenous variables, $\boldsymbol{D_{c,\tau}}$. In that case equations **(2)** and **(3)** can be substituted back into equation **(1)** and the model can be easily estimated by OLS, equation by equation. If this is assumption is violated, as is likely to be the case with macroeconomic data, estimating the model with country fixed effects will lead to dynamic heterogeneity bias (Pesaran and Smith, 1995). Sa, Towbin and Wieladek (2014) use the mean group estimator to address this problem. But to the extent that this approach requires estimation country-by-country, modelling variation in coefficients as a set of more than two exogenous variables is typically not feasible, even in moderately sized VARs, due to degrees of freedom considerations.

Finally, it is important to note that the coefficients, $\boldsymbol{\beta}_{c,\tau}$ and $\boldsymbol{a}$, vary with $\boldsymbol{\tau}$, as oppose to, $\boldsymbol{t}$. This mixed frequency structure is an advantage of our framework, since the country-specific economic characteristics in $\boldsymbol{D}_{c,\tau}$ are available only at an annual, as opposed to quarterly, frequency. This means that $\boldsymbol{\beta}_{c,\tau}$ and $\boldsymbol{a}_{c,\tau}$ will vary by year as oppose to quarter.

Subject to these assumptions, the likelihood function will be proportional to[1]:

$$\prod_c\prod_t |\boldsymbol{\Sigma}_c| \exp\left(-\frac{1}{2}\sum_t\sum_c (\boldsymbol{y}_{c,t}-\widetilde{\boldsymbol{X}}_{c,t}\boldsymbol{\beta}_{c,\tau})'\left(\boldsymbol{A}_{c,\tau}'\boldsymbol{\Sigma}_c\boldsymbol{A}_{c,\tau}\right)^{-1}(\boldsymbol{y}_{c,t}-\widetilde{\boldsymbol{X}}_{c,t}\boldsymbol{\beta}_{c,\tau})\right)$$

$$\boldsymbol{\lambda_B}^{-\frac{TCNK}{2}} \boldsymbol{exp}\left(-\frac{1}{2}\sum_c\sum_\tau(\boldsymbol{\beta}_{c,\tau}-\overline{\boldsymbol{\beta}}_{c,\tau})'\boldsymbol{L}_{Bc}^{-1}\boldsymbol{\lambda_B}^{-1}(\boldsymbol{\beta}_{c,\tau}-\overline{\boldsymbol{\beta}}_{c,\tau})\right)\prod_c\prod_t|\boldsymbol{\Sigma}_c|^{-\frac{N+1}{2}}\boldsymbol{\lambda_B}^{-\frac{v+2}{2}}\boldsymbol{exp}\left(-\frac{1}{2}\frac{\boldsymbol{s}}{\boldsymbol{\lambda_B}}\right)$$

$$\boldsymbol{\lambda_A}^{-\frac{TN(N-1)}{2}} \boldsymbol{exp}\left(-\frac{1}{2}\sum_c\sum_\tau(\boldsymbol{a}_{c,\tau}-\boldsymbol{a}_{c,\tau})'\boldsymbol{L}_{Ac}^{-1}\boldsymbol{\lambda_A}^{-1}(\boldsymbol{a}_{c,\tau}-\overline{\boldsymbol{a}}_{c,\tau})\right)\prod_c\prod_\tau|\boldsymbol{\Sigma}_c|^{-\frac{N+1}{2}}\boldsymbol{\lambda_A}^{-\frac{v+2}{2}}\boldsymbol{exp}\left(-\frac{1}{2}\frac{\boldsymbol{s}}{\boldsymbol{\lambda_A}}\right)$$

where $\widetilde{\boldsymbol{X}}_{c,t} \equiv \boldsymbol{I_N}\otimes\boldsymbol{X}_{c,t}$, $\overline{\boldsymbol{\beta}}_{c,\tau} \equiv \boldsymbol{vec}\left(\boldsymbol{D}_{c,\tau}\boldsymbol{\delta_B}\right)$, $\overline{\boldsymbol{a}}_{c,\tau} \equiv \boldsymbol{vec}(\boldsymbol{D}_{c,\tau}\boldsymbol{\delta_A})$. and $\boldsymbol{\Upsilon} = \frac{\boldsymbol{T}}{\boldsymbol{\Xi}}$. $\boldsymbol{T}$ is the total number of time series observations and $\boldsymbol{\Upsilon}$ is the total number of time periods that $\boldsymbol{\beta}_{c,\tau}$ and $\boldsymbol{a}_{c,\tau}$ are allowed to vary for. In our empirical application, one of the labour market indicator is only available every 5 years before 2000. We therefore set $\boldsymbol{\Xi} = \boldsymbol{20}$, which means that with a $\boldsymbol{T}$ of 120, $\boldsymbol{\Upsilon} = \boldsymbol{6}$, and that $\boldsymbol{\beta}_{c,\tau}$ and $\boldsymbol{a}_{c,\tau}$ vary for 6 periods each. The other advantage of this approach is that 5-year averages of economic characteristics are less likely to be endogenous at business cycle frequency. Below I list the conditional distributions for the Gibbs sampler of this model and the full derivation is listed in the appendix of this paper.

The country-specific VAR coefficients $\boldsymbol{\beta}_{c,\tau}$ are drawn from:

$$\boldsymbol{p}(\boldsymbol{\beta}_{c,\tau} \mid \overline{\boldsymbol{\beta}}_{c,\tau}, \boldsymbol{Y}_c, \boldsymbol{\Lambda}_{Bc}) = \boldsymbol{N}\left((\boldsymbol{G}_c)^{-1}\left(\left(\boldsymbol{A}_{c,\tau}'\boldsymbol{\Sigma}_c\boldsymbol{A}_{c,\tau}\right)^{-1}\otimes\boldsymbol{X}_{c,t}'\right)\boldsymbol{y}_{c,t}+\boldsymbol{L}_{Bc}^{-1}\boldsymbol{\lambda_B}^{-1}\overline{\boldsymbol{\beta}}_{c,\tau}, (\boldsymbol{G_c}^{-1})\right) \quad \textbf{(7)}$$

where $\boldsymbol{G_c} = \left(\boldsymbol{A}_{c,\tau}'\boldsymbol{\Sigma}_c\boldsymbol{A}_{c,\tau}\right)^{-1}\otimes\boldsymbol{X}_{c,t}'\boldsymbol{X}_{c,t}+\boldsymbol{L}_{Bc}^{-1}\boldsymbol{\lambda_B}^{-1}$. $\boldsymbol{\delta_B}$ is drawn from:

$$\boldsymbol{p}(\boldsymbol{\delta_B} \mid \boldsymbol{\beta}_{c,\tau}, \boldsymbol{\Lambda}_{Bc}) = \boldsymbol{N}\left(\left(\sum_c\sum_\tau\boldsymbol{D}_{c,\tau}'\boldsymbol{\Lambda}_{Bc}^{-1}\boldsymbol{D}_{c,\tau}\right)^{-1}\sum_c\sum_\tau\boldsymbol{D}_{c,\tau}'\boldsymbol{\Lambda}_{Bc}^{-1}\boldsymbol{\beta}_{c,\tau}, \left(\sum_c\sum_\tau\boldsymbol{D}_{c,\tau}'\boldsymbol{\Lambda}_{Bc}^{-1}\boldsymbol{D}_{c,\tau}\right)^{-1}\right) \quad \textbf{(8)}$$

[1] See appendix for detailed derivation.

$\boldsymbol{\lambda_B}$ is treated as a hyper parameter and drawn from the following inverse gamma 2 distribution:

$$p(\boldsymbol{\lambda_B} \mid \overline{\boldsymbol{\beta}}, \boldsymbol{\beta_c}, \boldsymbol{L_c^{-1}}) = \boldsymbol{IG_2}(\boldsymbol{s} + \sum_\tau \sum_c (\boldsymbol{\beta}_{c,\tau} - \overline{\boldsymbol{\beta}}_{c,\tau})' \boldsymbol{L}_{Bc}^{-1} \boldsymbol{\lambda_B}^{-1} (\boldsymbol{\beta}_{c,\tau} - \overline{\boldsymbol{\beta}}_{c,\tau}), \boldsymbol{YCNK} + \boldsymbol{v}) \qquad (9)$$

A completely non-informative prior with s and v set to 0 results in an improper posterior in this case. We therefore set both of the quantities to very small positive numbers, which is equivalent to assuming a weakly informative prior. But it is important to point out that $\boldsymbol{\lambda}$ is estimated from the total number of coefficients that this prior is applied to, namely the product of country (**C**), equations (**N**) and total number of coefficients in each equation (**K**). Given this large number of effective units, any weakly informative prior will be dominated by the data.

Similarly, given that $\boldsymbol{A_{c,\tau}}$ is lower-triangular with ones on the diagonal,the appendix shows that $\boldsymbol{a_{c,\tau}^j}$, where j refers to the equation, can be drawn equation by equation from:

$$p(\boldsymbol{a_{c,\tau}^j} \mid \overline{\boldsymbol{a}}_{c,\tau}^j, \boldsymbol{E_c}, \boldsymbol{\Lambda_{Ac}}) = \boldsymbol{N}(\boldsymbol{F_c^{-1}}(\boldsymbol{\Sigma_c^{-1}} \otimes \boldsymbol{EJ'_{c,t}})\boldsymbol{e_{c,t}} + \boldsymbol{L_{Ac}^{-1}} \boldsymbol{\lambda_A}^{-1} \overline{\boldsymbol{a}}_{c,t}^j, \boldsymbol{F_c^{-1}}) \qquad (10)$$

where $\boldsymbol{F_c} = \boldsymbol{\Sigma_c^{-1}} \otimes \boldsymbol{EJ'_{c,t}} \boldsymbol{EJ_{c,t}} + \boldsymbol{L_{Ac}^{-1}} \boldsymbol{\lambda_A}^{-1}$, $\boldsymbol{e_{c,t}}$ is the error term of equation j and $\boldsymbol{EJ'_{c,t}}$ contains all of the other relevant $\boldsymbol{e_{c,t}}$'s as explanatory variables for that equation. Given that $\boldsymbol{A_{c,\tau}}$ is lower-triangular, this means that in the case of the second equation, $\boldsymbol{EJ'_{c,t}}$ will consist of one other error term, in the case of the third equation of two ,etc. $\boldsymbol{\delta_A}$ is drawn from:

$$p(\boldsymbol{\delta_A} \mid \boldsymbol{a_{c,\tau}}, \boldsymbol{\Lambda_{Ac}}) = \boldsymbol{N}((\sum_c \sum_\tau \boldsymbol{D'_{c,\tau}} \boldsymbol{\Lambda_{Ac}^{-1}} \boldsymbol{D_{c,\tau}})^{-1} \sum_c \sum_\tau \boldsymbol{D'_{c,\tau}} \boldsymbol{\Lambda_{Ac}^{-1}} \boldsymbol{a_{c,\tau}}, (\sum_c \sum_\tau \boldsymbol{D'_{c,\tau}} \boldsymbol{\Lambda_{Bc}^{-1}} \boldsymbol{D_{c,\tau}})^{-1}) \quad (11)$$

$\boldsymbol{\lambda_A}$ is treated as a hyper parameter and drawn from the following inverse gamma 2 distribution:

$$p(\boldsymbol{\lambda_A} \mid \overline{\boldsymbol{a}}_{c,\tau}, \boldsymbol{a_{c,\tau}}) = \boldsymbol{IG_2}(\boldsymbol{s} + \sum_\tau \sum_c (\boldsymbol{a_{c,\tau}} - \overline{\boldsymbol{a}}_{c,\tau})' \boldsymbol{L_{Ac}^{-1}} (\boldsymbol{a_{c,\tau}} - \overline{\boldsymbol{a}}_{c,\tau}), \boldsymbol{YN}(\boldsymbol{N} - \boldsymbol{1})/\boldsymbol{2} + \boldsymbol{v}) \qquad (12)$$

Finally, the country-specific variance matrix of the residuals, $\boldsymbol{\Sigma_c}$, is drawn from an inverse-Wishart distribution:

$$p(\boldsymbol{\Sigma_c} \mid \boldsymbol{A_{c,\tau}^{-1}}, \boldsymbol{\beta_{c,\tau}}) = \boldsymbol{IW}(\boldsymbol{U'_c U_c}, \boldsymbol{T_c}) \qquad (13)$$

where $\boldsymbol{U_c} = [\boldsymbol{U_{c,1}} \dots \boldsymbol{U_{c,T}}]'$, $\boldsymbol{U_{c,t}} = \boldsymbol{A_{c,\tau}^{-1}} \boldsymbol{E_{c,t}}$ and $\boldsymbol{T_c}$ is the number of observations for each country. For the application below, I estimate this model by repeatedly drawing from the posteriors of the Gibbs sampling chain in **(7) – (13)** 150,000 times, discarding the first 50,000 draws as burn-in and retaining every 100th of the remaining draws for inference.

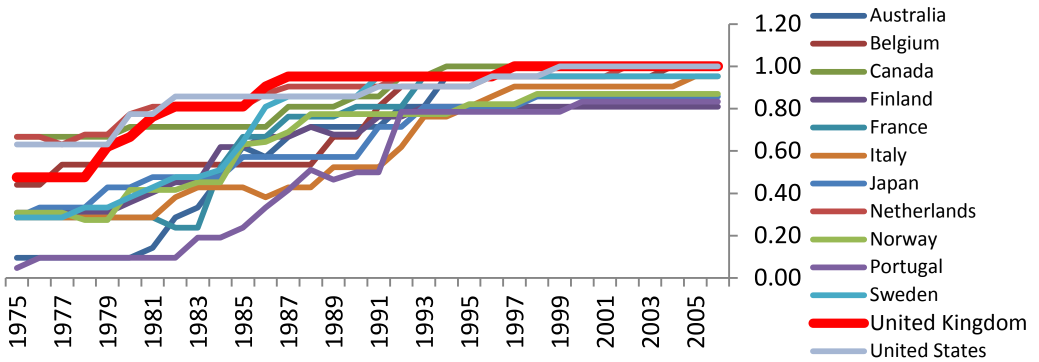
3. An application: Examining the transmission of Commodity Price Shocks

As an application of this model I examine how exchange rate regime, financial, labour and product market liberalisation has affected the transmission of real commodity price shocks to real consumption and CPI in OECD countries with impulse response analysis. This follows the initial applications of the IPVAR methodology presented in Broda (2001), Radatz (2007) and Towbin and Weber (2013). I first describe the data and then the results.

3.1 Data

I explore whether VAR coefficients vary with the degree of exchange rate flexibility, financial, labour and product market deregulation. I describe each index in turn.

Figure 1: Index of Financial Liberalisation

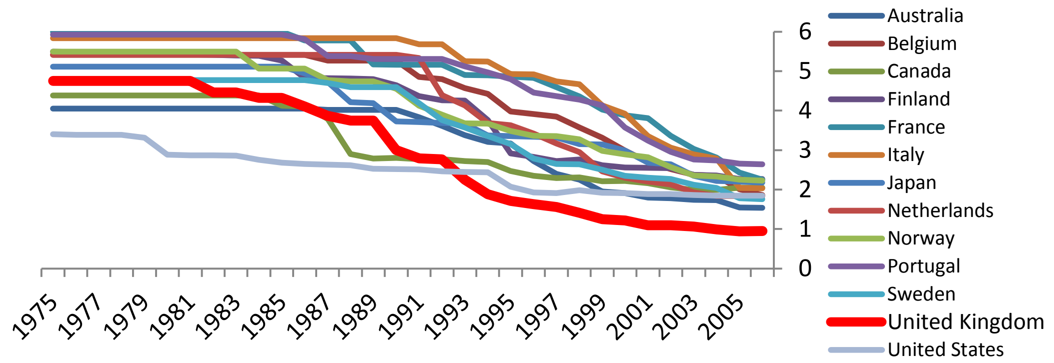


Sources & Notes: Abiad et al (2010). Higher values indicate greater liberalisation.

Figure 1 shows the financial liberalisation index for each of countries in our study. This is taken from Abiad et al (2010) and has seven different components of the dataset. These are: credit controls, interest rate controls, entry barriers, state ownership in the banking sector, prudential regulation, securities market policy and capital account restrictions. Each component can take the values {0,1,2,3} with higher values meaning fewer restrictions. I sum all components to come up with the aggregate financial liberalization index we use in our empirical exercise. This index is normalised to 1.

As a proxy for product market regulation I use the ETCR index constructed by Conway and Nicoletti (2006), which is shown in figure 2. This captures the level of regulation in seven non-manufacturing sectors: airlines, telecommunication, electricity, gas, post, rail and road freight. These sectors represent a substantial proportion of economic activity and the area in which domestic economic regulation is most concentrated and has a major impact due to limited import competition. The index takes into account characteristics of the markets, such as the presence of barriers to entry, public ownership, vertical integration, monopolies and the presence of legally imposed price controls, which can distort competition in these sectors.

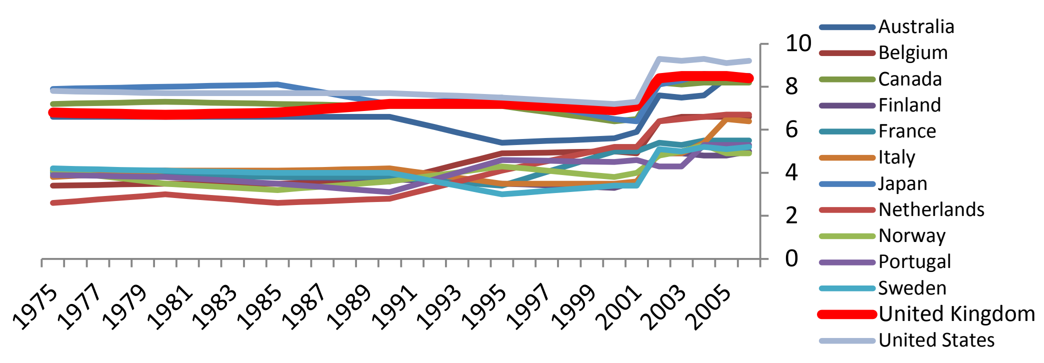
Figure 2: Index of Product Market Liberalisation



Sources & Notes: Conway and Nicoletti (2006). Lower values indicate greater liberalisation.

Figure 3 shows the index of labour market liberalisation that I use. This broadly reflects minimum wage regulation, hiring and firing practices, the share of the labour force whose wages are set by centralized collective bargaining, unemployment benefits and use of conscription to obtain military personnel.

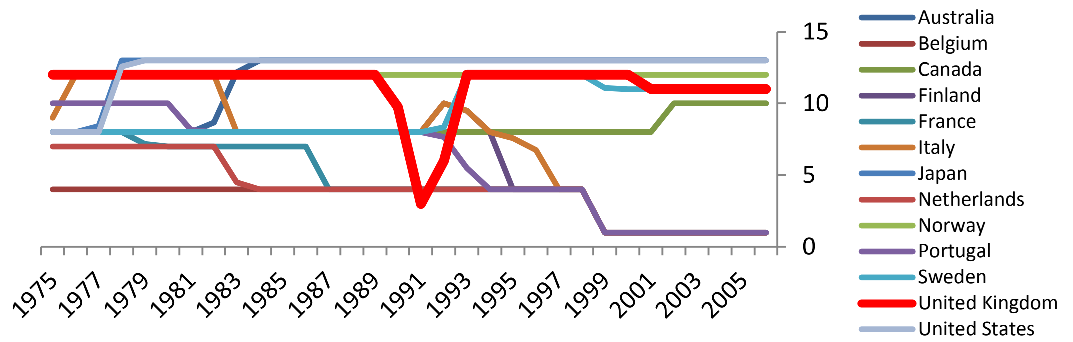
Figure 3: Index of Labour Market Liberalisation.



Sources & Notes: Fraser Institute. Higher values mean greater liberalisation. Up until 2000, these are only available every 5 years, and the chart shows linearly interpolated values.

Figure 4 shows the indicator of exchange rate regime flexibility that I use, which is taken from Ilzetzki, Reinhart and Rogoff (2012). Finally, our VAR model consists of three endogenous variables: Real imported commodity price growth, Quarterly real consumption growth and CPI inflation. CPI data were taken from the OECD Main Economic Indicators. The remaining variables were taken from the OECD Economic Outlook database.

Figure 4: Indicator of the Exchange Rate Regime



Sources & Notes: Ilzetzki, Reinhart and Rogoff (2012). Higher values indicate greater flexibility.

3.2 Impulse Response Analysis

In this section I present the results from the application of the varying coefficient Bayesian Panel VAR model. In particular, I want to examine how exchange rate regime, financial, labour and product market liberalisation affect the transmission of commodity price shocks to real consumption and the CPI?

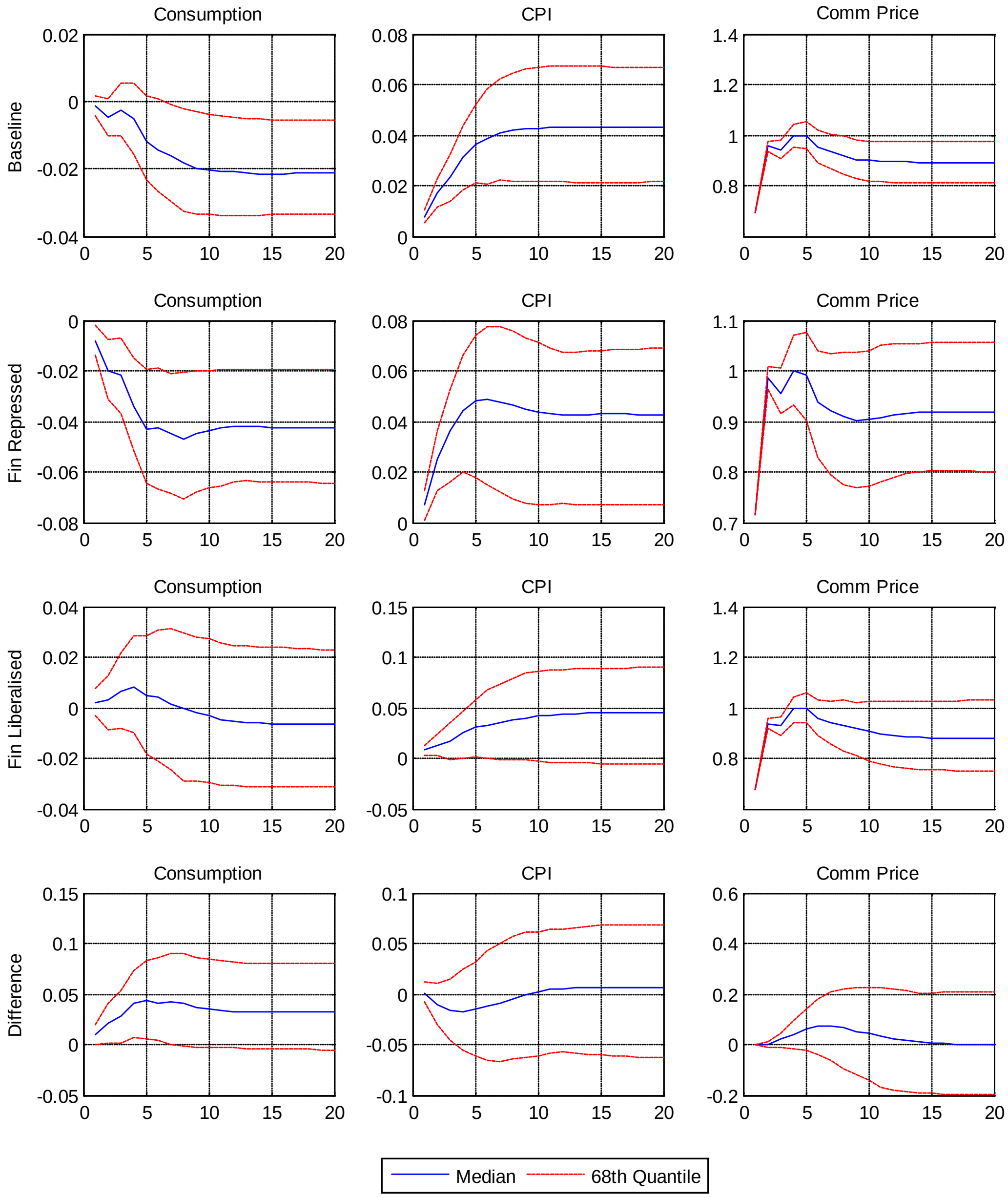
For this purpose, I estimate the model on a VAR with three endogenous variables for each country: Real Commodity Price Inflation, Real Consumption Growth and CPI inflation. To identify commodity price shocks, I follow previous work (cite a paper of …. Here) and use a lower triangular identification scheme with commodity price inflation ordered first.

From equations **(2)** and **(3)**, it is easy to see that these VAR coefficients are a function of

$$\boldsymbol{D_{c,t} = [1\ FIN_{c,t}\ \ FX_{c,t}\ \ LABOUR_{c,t}\ \ PROD_{c,t}]}\,,$$

where $\boldsymbol{FIN_{c,t}}$, $\boldsymbol{FX_{c,t}}$, $\boldsymbol{LABOUR_{c,t}}$ and $\boldsymbol{PROD_{c,t}}$ are indices of financial liberalisation, exchange rate regime flexibility, labour market and product market liberalisation, respectively. Prior to structural analysis, the individual elements of $\boldsymbol{D_{c,\tau}}$ need to be fixed at certain values. For example, to obtain average VAR coefficients across time and country, it is necessary to evaluate all of the elements of $\boldsymbol{D_{c,\tau}}$ at their median values. From (2) and (3), this would yield draws of $\boldsymbol{\beta_{c,\tau}^{MED}}$ and $\boldsymbol{a_{c,\tau}^{MED}}$, which can then be used for identification. Similarly, it is possible to examine how these coefficients, and the implied impulse responses, are affected by financial, product and labour market liberalisation in the following manner. First, evaluate the structural characteristic of interest, for instance financial liberalisation, at a high value (defined as the 90th percentile of values realised in the sample) with all the other characteristics evaluated at their medians to obtain draws of $\boldsymbol{\beta_{c,\tau}^{FINHIGH}}$ and $\boldsymbol{a_{c,\tau}^{FINHIGH}}$ and the associated distribution of impulse responses. Repeat the previous step, but this time with a low value of financial liberalisation (defined as the 10th percentile) to obtain draws of $\boldsymbol{\beta_{c,\tau}^{FINLOW}}$ and $\boldsymbol{a_{c,\tau}^{FINLOW}}$. A comparison of these two distributions, subject to the same size shock, allows us to infer the effect of financial liberalisation on the transmission of real commodity price shocks. This exercise can be repeated for each structural characteristic in turn to learn about their individual amplification/ propagation properties.

Figure 5: The effect of financial liberalisation on the commodity price transmission mechanism



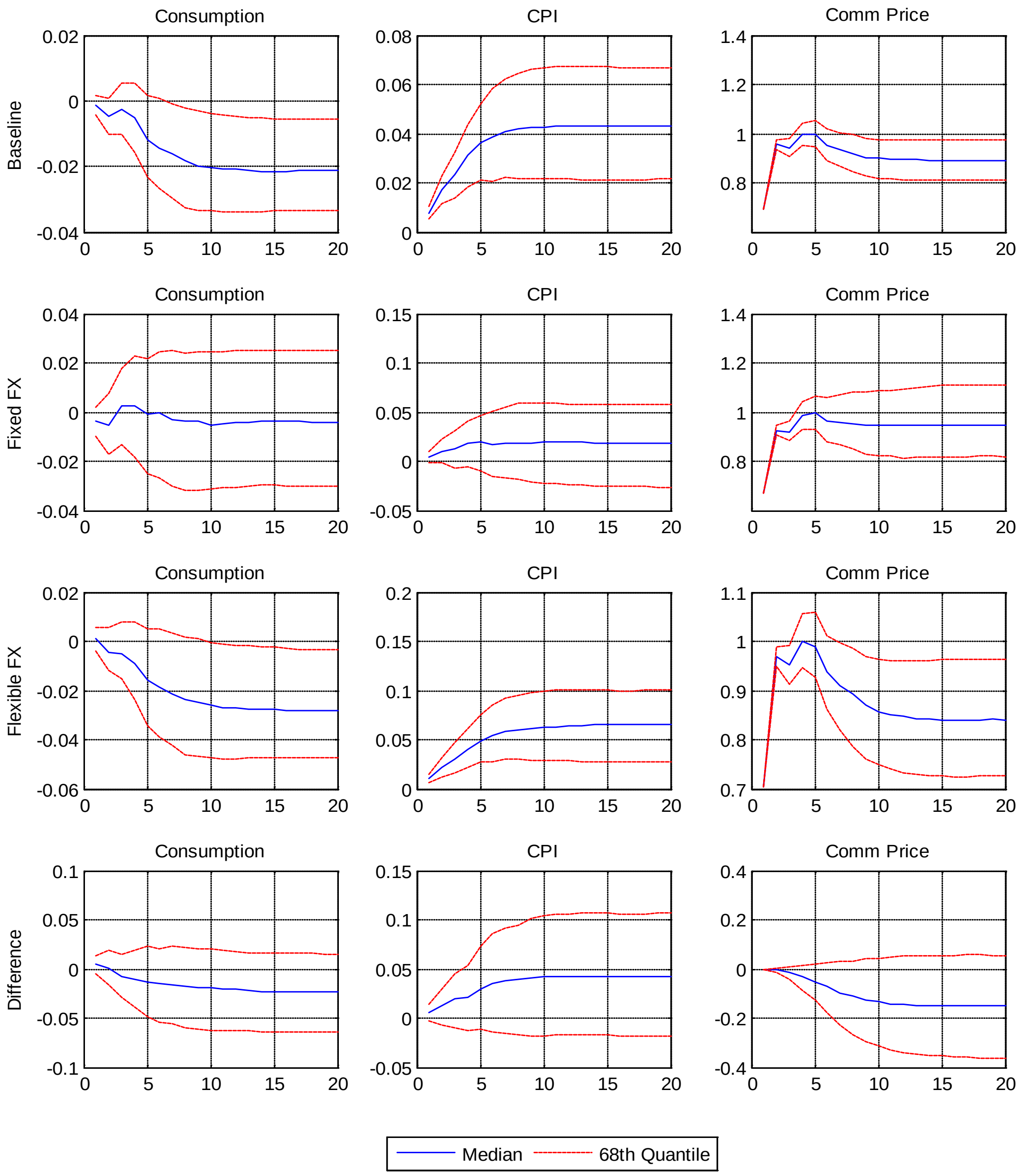
Note: Figure 5 shows the effect of financial liberalisation on the transmission of an unexpected 1% rise in commodity prices. Column one, two and three show impulse responses to a 1% rise in commodity prices of real consumption, the CPI and the real commodity price index. Cumulated impulse responses are shown, as all of the the variables enter the model in log differences. Row one shows the responses when all of the the exchange rate, financial, labour and product market indices have been evaluated at the sample medians. Row two shows the responses when the financial liberalisation index has been evaluated at the 10th percentile of values realised at the sample, with all the other indices evaluated at their medians. Row three repeats this exercise, with the financial liberalisation index now evaluated at the 90th percentile of the values in the sample. Row four reports the median and 68% quantiles based on the difference in impulse responses that were used to obtain the corresponding statistics for rows two and three, respectively.

Figure 5 shows impulse responses for the level of real consumption, the CPI and the commodity price index. The first row, labelled as baseline, shows impulse responses obtained with all structural characteristics evaluated at their medians. The second row shows impulse responses from coefficients that have been evaluated at the 10th percentile of the financial liberalisation index, with all of the remaining coefficients evaluated at their medians, which yields impulse responses for a financially repressed economy. The third row repeats this exercise, but with the financial liberalisation index evaluated at the 90th percentile. In all of these cases, the size of the commodity price shock is always standardised to 1 percent at the peak. The fourth row shows the median, the 16th and the 84th quantile of the difference between the distributions of impulse responses in rows two and three to test for statistical significance. It should be noted that the median of the differences is not the difference of the medians, but rather a median of the difference in impulse responses.

The results in the first row show that consumption falls and CPI rises following a rise in commodity prices, which is a result consistent with many previous studies of oil-price shocks on the macroeconomy (Killian (XXXX)). Interestingly, there is no statistically significant impact on real consumption in financially liberalised economies. In financially repressed economies, on the other hand, there is a negative and statistically significant effect on real consumption. As the ultimate row shows, this difference is statistically significant. This finding is consistent with economic theory in the sense that one would expect greater risk-sharing in more financially liberalised economies. But the CPI reaction seems to be similar regardless of the degree of financial liberalisation.

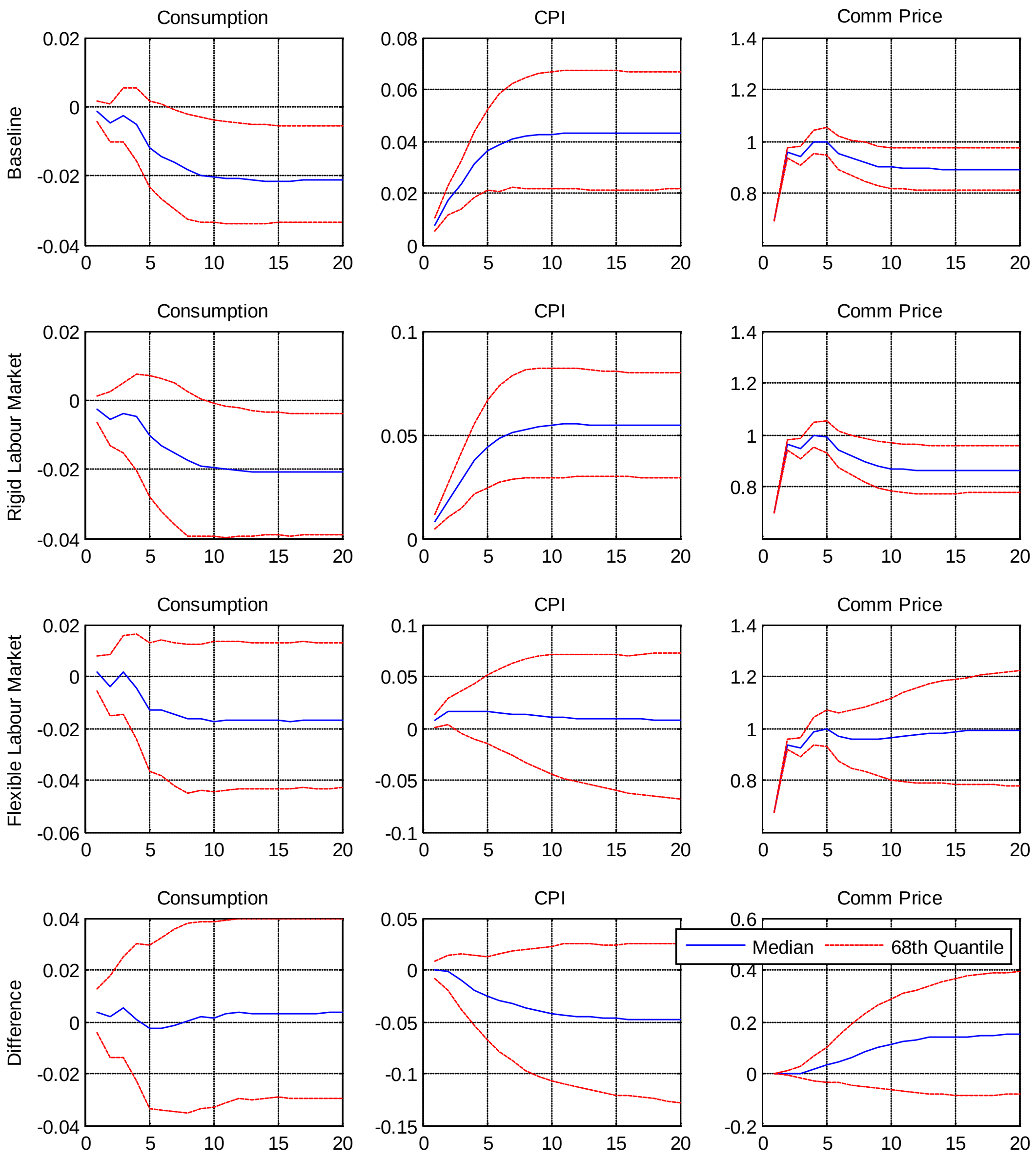
Figure 6 repeats the same exercise for the exchange rate regime. Interestingly, the reaction of real consumption and CPI seem stronger in the flexible exchange rate regime case, but this difference is not statistically significant. In other words, the type of exchange rate regime does not seem to affect the transmission of commodity price shocks in a statistically significant way.

Figure 6: The effect of the FX Rate Regime on the commodity price transmission mechanism



Note: Figure 6 shows the effect of exchange rate regime liberaliation on the transmission of an unexpected 1% rise in the real price of imported commodities. Column one, two and three show impulse responses to a 100 basis point monetary policy expansion of real consumption, the CPI and the real imported commodity price. Cumulated impulse responses are shown, as these variables enter the model in log differences. Row one shows the responses when all of the the exchange rate, financial, labour and product market indices have been evaluated at the sample medians. Row two shows the responses when the FX regime index has been evaluated at the 10th percentile of values realised at the sample, with all the other indices evaluated at their medians. Row three repeats this exercise, with the FX regime index now evaluated at the 90th percentile of the values in the sample. Row four reports the median and 68% quantiles based on the difference in impulse responses that were used to obtain the corresponding statistics for rows two and three, respectively.

Figure 7: The effect of labour market liberalisation on the commodity price transmission mechanism



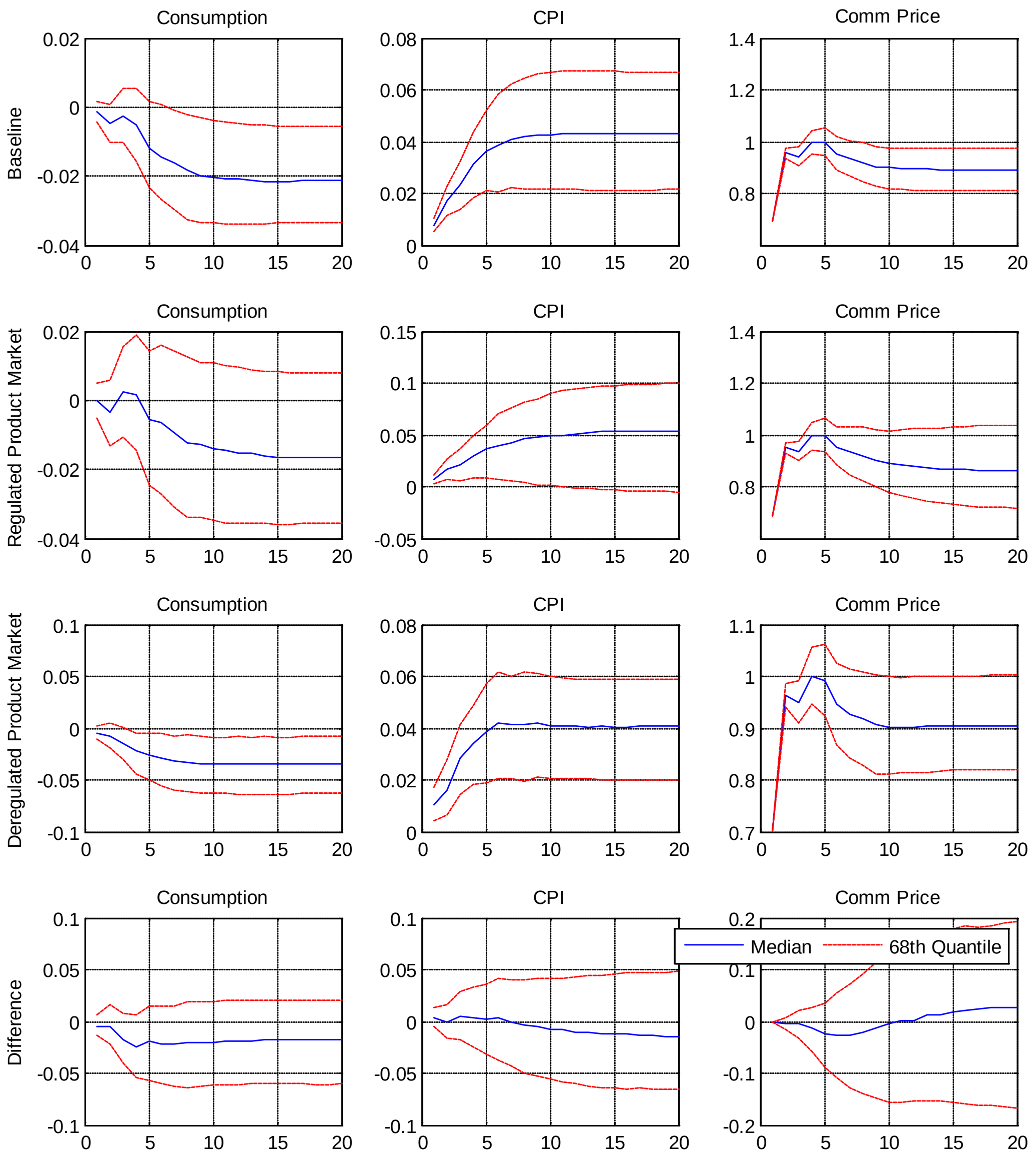
Note: Figure 7 shows the effect of labour market liberalisation on the transmission of an unexpected 1% rise in real imported commodity prices. Column one, two and three show impulse responses to a 1% rise in real commodity prices of real consumption, the CPI and real imported commodity prices. Cumulated impulse responses are shown, as all variables enter the model in log differences. Row one shows the responses when all of the the exchange rate, financial, labour and product market indices have been evaluated at the sample medians. Row two shows the responses when the labour market liberalisation index has been evaluated at the 10th percentile of values realised at the sample, with all the other indices evaluated at their medians. Row three repeats this exercise, with the labour market liberalisation index now evaluated at the 90th percentile of the values in the sample. Row four reports the median and 68% quantiles based on the difference in impulse responses that were used to obtain the corresponding statistics for rows two and three, respectively.

Figure 7 shows results for changes to labour reform. The real consumption response is very similar for either the case of an economy with a rigid or flexible labour market, meaning that labour market reform does not seem to affect the response of this variable to a commodity price shock. But the CPI response is much stronger in an economy with rigid labour markets, as predicted by economic theory. Nevertheless, as a result of the wide confidence band, the difference in the CPI responses is not statistically significant.

Figure 8 presents the results from the deregulation of the product market. With regulated product markets, the response of real consumption and the CPI to a 1% rise in real imported commodity prices is weaker than with deregulated product markets. But as before, the difference is not statistically significant.

Overall the evidence therefore suggests that only financial liberalisation affects the transmission of commodity price shocks to real consumption and CPI. Consistent with the idea that financial liberalisation promotes risk-sharing, the results suggest that real consumption does not react to commodity price shocks in financially liberalised economies and this difference is statistically significant. However, as a result of the wide confidence bands, none of the other types of liberalisation seem to affect this transmission mechanism in a statistically significant way.

Figure 8: The effect of product market liberalisation on the commodity price transmission mechanism



Note: Figure 8 shows the effect of product market liberalisation on the transmission of an unexpected 1% rise in commodity prices. Column one, two and three show impulse responses to a 1% rise in commodity prices of real consumption, the CPI and commodity prices. Cumulated impulse responses are shown, as all of these variables enter the model in log differences. Row one shows the responses when all of the the exchange rate, financial, labour and product market indices have been evaluated at the sample medians. Row two shows the responses when the product liberalisation index has been evaluated at the 10th percentile of values realised at the sample, with all the other indices evaluated at their medians. Row three repeats this exercise, with the product market liberalisation index now evaluated at the 90th percentile of the values in the sample. Row four reports the median and 68% quantiles based on the difference in impulse responses that were used to obtain the corresponding statistics for rows two and three, respectively.

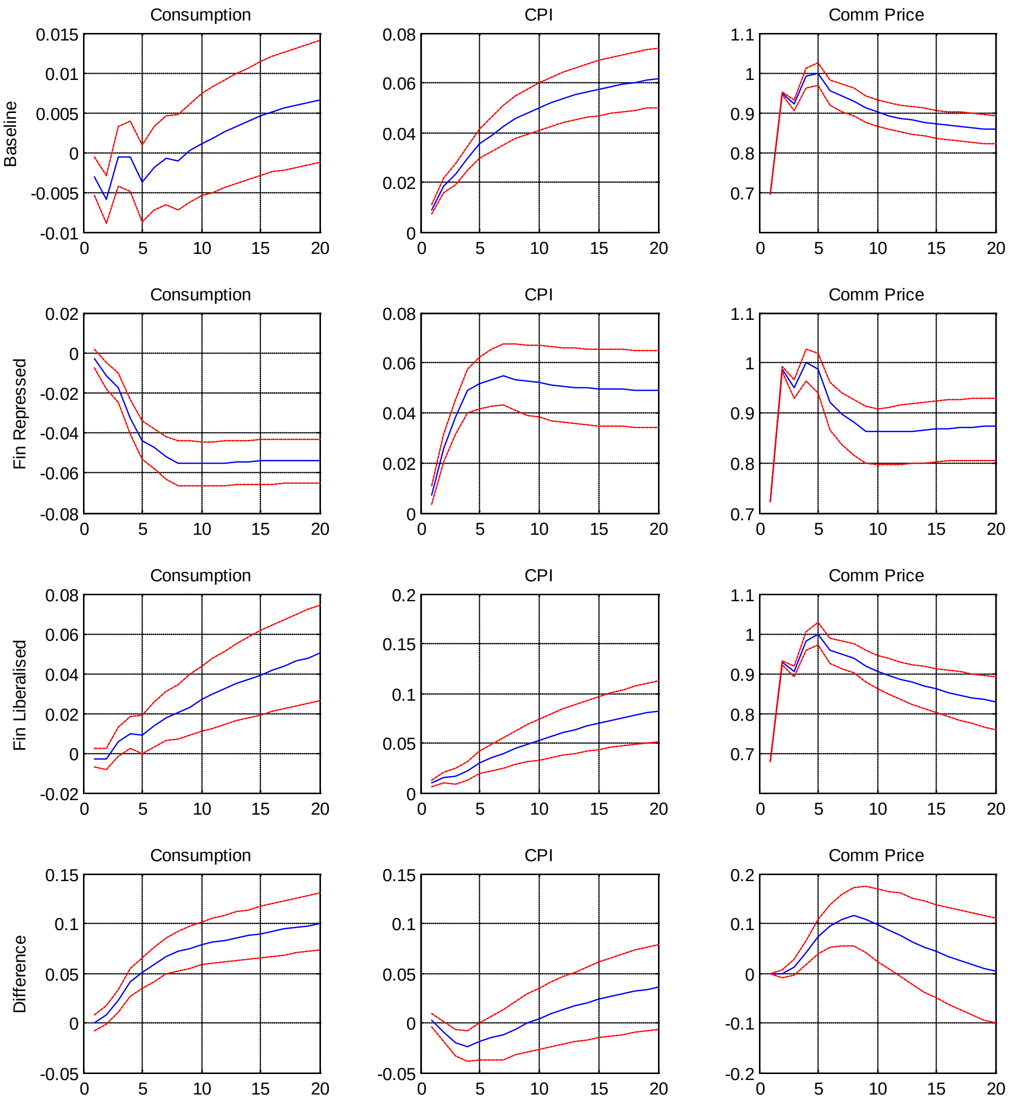
3.3 Do the results change with the IPVAR approach?

Previous work, that has examined to which extent the transmission of commodity price shocks is affected by changes in the economic characteristics of a country, typically adopted the IPVAR approach. In our model, this is equivalent to assuming that the dynamic coefficients are a deterministic function of the country characteristics in question, namely $\boldsymbol{\lambda_B} = \mathbf{0}$.

Figures 5a-8a repeat the analysis of section 3.2, but rather than estimating $\boldsymbol{\lambda_B}$, it is set to zero in the estimation procedure. An examination of the figures 5a-8a reveals two things immediately. First the confidence bands are narrower. This should not be surprising, as an important source of uncertainty has been removed. As a result of this, most of the impulse responses are now statistically significantly different across country characteristic. For example, the evidence that both real consumption and the CPI response change with labour (7a) and product market (8a) deregulation is now much stronger. The previous result, that financial liberalisation affects the real consumption response remains, but the difference seems greater and more statistically significant than before.

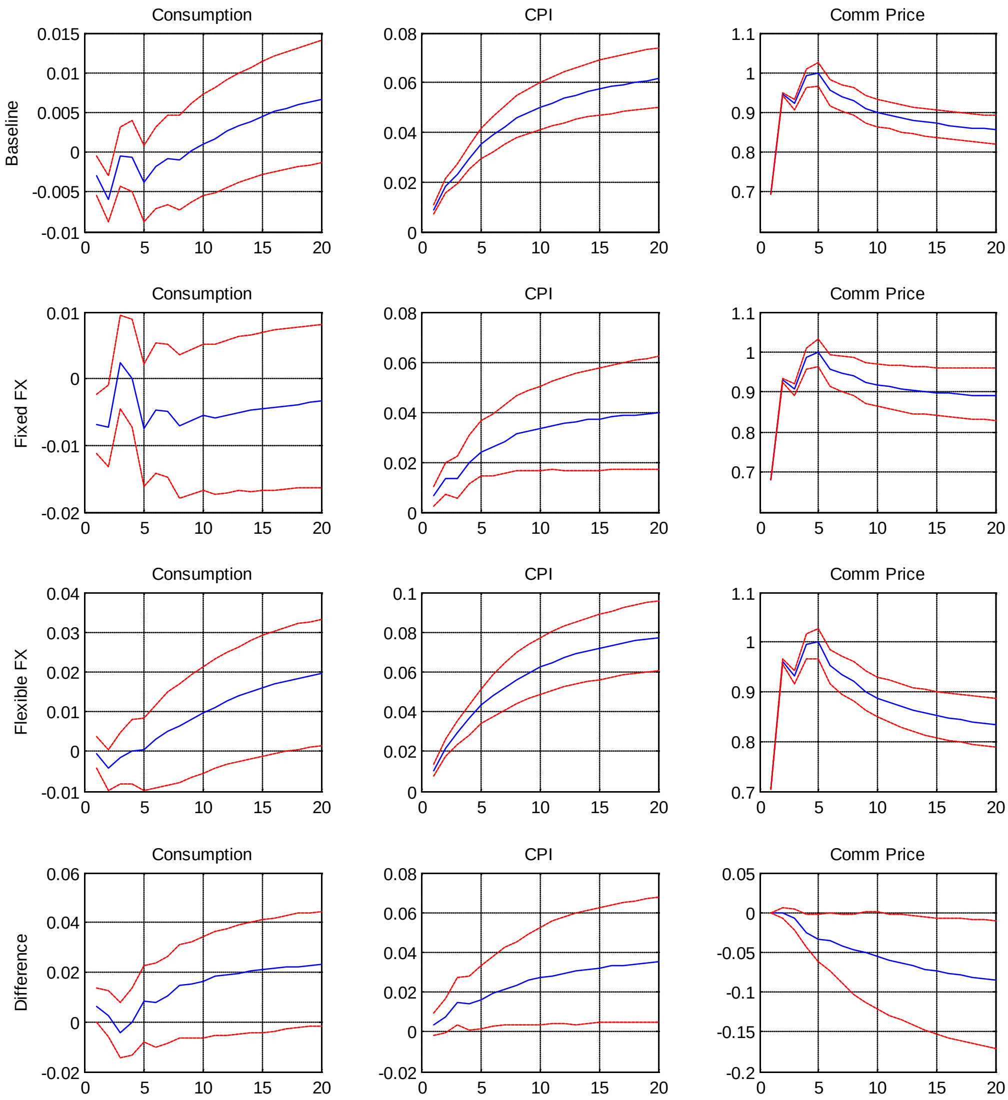
Overall this suggests that, at least in this application, the IPVAR model where the coefficients are a deterministic function of the country characteristics may under-estimate the degree of uncertainty around impulse responses. This can in turn lead an investigator to researchers to conclude that a given country characteristic affects the transmission mechanism when it does not.

Figure 5a: The effect of financial liberalisation on the commodity price transmission mechanism



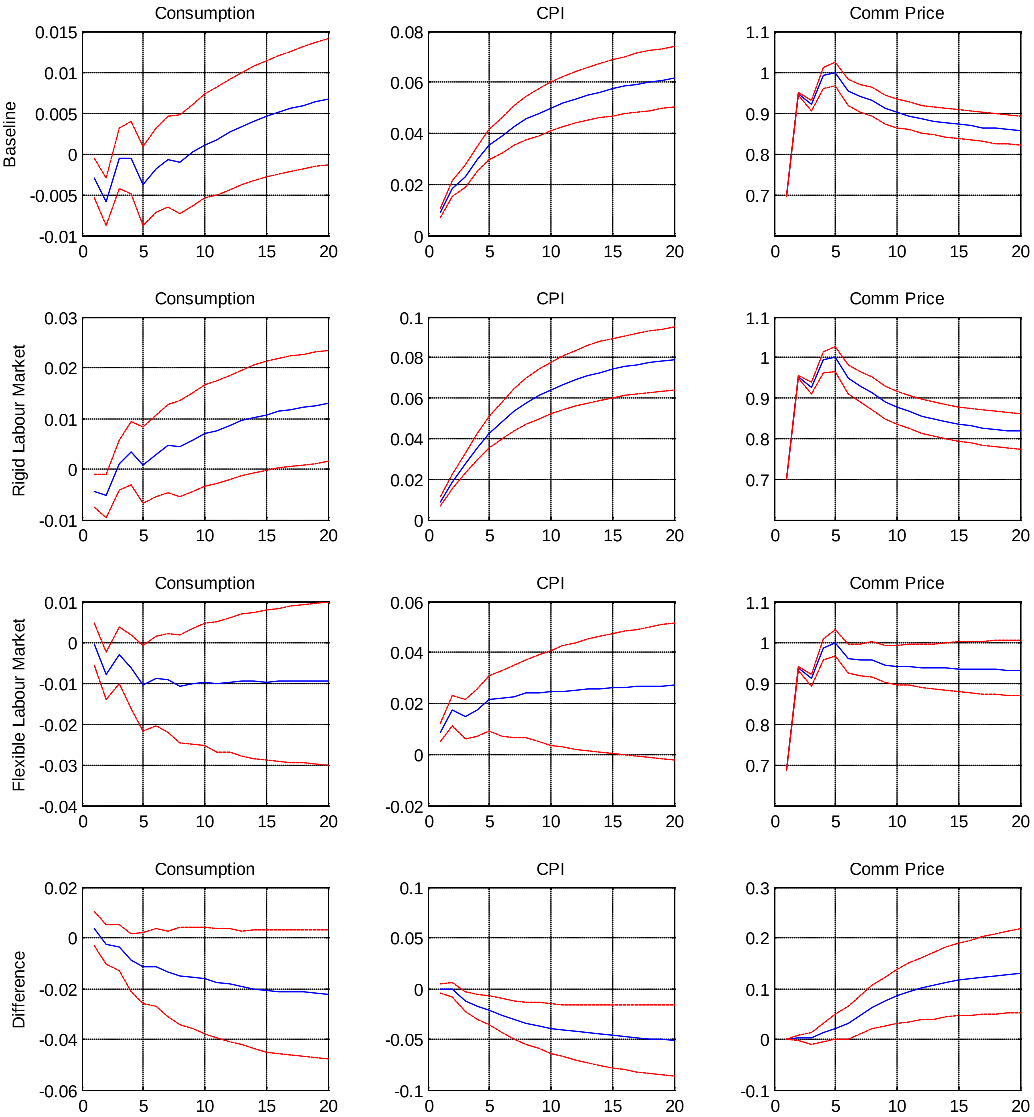
Note: Figure 5a shows the effect of financial liberalisation on the transmission of an unexpected 1% rise in real imported commodity prices. Column one, two and three show impulse responses to a 1% rise in real commodity prices of real consumption, the CPI and real imported commodity prices. Cumulated impulse responses are shown, as all variables enter the model in log differences. Row one shows the responses when all of the the exchange rate, financial, labour and product market indices have been evaluated at the sample medians. Row two shows the responses when the financial liberalisation index has been evaluated at the 10th percentile of values realised at the sample, with all the other indices evaluated at their medians. Row three repeats this exercise, with the financial liberalisation index now evaluated at the 90th percentile of the values in the sample. Row four reports the median and 68% quantiles based on the difference in impulse responses that were used to obtain the corresponding statistics for rows two and three, respectively.

Figure 6a: The effect of Exchange Rate Regime flexibility on the commodity price transmission mechanism



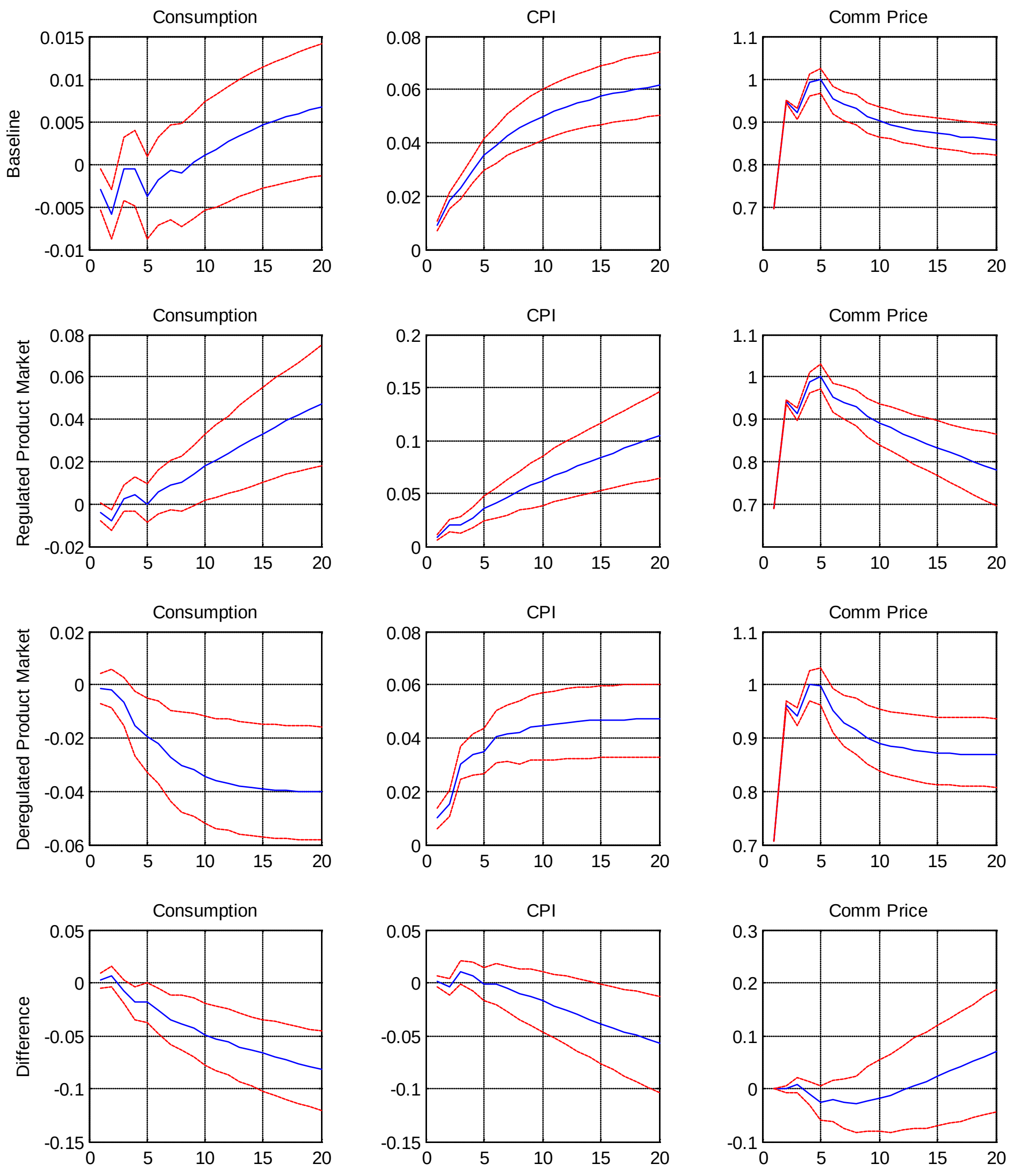
Note: Figure 6a shows the effect of exchange rate regime liberaliation on the transmission of an unexpected 1% rise in commodity prices. Column one, two and three show impulse responses to a 1% rise in real imported commodity prices of real consumption, the CPI and real imported commodity prices.. Cumulated impulse responses are shown, as all of the variables enter the model in log differences. Row one shows the responses when all of the the exchange rate, financial, labour and product market indices have been evaluated at the sample medians. Row two shows the responses when the FX regime index has been evaluated at the 10^{th} percentile of values realised at the sample, with all the other indices evaluated at their medians. Row three repeats this exercise, with the FX regime index now evaluated at the 90^{th} percentile of the values in the sample. Row four reports the median and 68% quantiles based on the difference in impulse responses that were used to obtain the corresponding statistics for rows two and three, respectively.

Figure 7a: The effect of labour market liberalization on the commodity price transmission mechanism



Note: Figure 7a shows the effect of labour market liberaliation on the transmission of an unexpected 1% rise in real imported commodity prices. Column one, two and three show impulse responses to a 1% rise in real commodity prices of real consumption, the CPI and real imported commodity prices. Cumulated impulse responses are shown, as all of the variables enter the model in log differences. Row one shows the responses when all of the the exchange rate, financial, labour and product market indices have been evaluated at the sample medians. Row two shows the responses when the labour market liberalisation index has been evaluated at the 10th percentile of values realised at the sample, with all the other indices evaluated at their medians. Row three repeats this exercise, with the labour market liberalisation index now evaluated at the 90th percentile of the values in the sample. Row four reports the median and 68% quantiles based on the difference in impulse responses that were used to obtain the corresponding statistics for rows two and three, respectively.

Figure 8a: The effect of product market liberalization on the commodity price transmission mechanism



Note: Figure 8a shows the effect of product market liberalisation on the transmission of an unexpected 1% rise in real imported commodity prices. Column one, two and three show impulse responses to an unexpected 1% rise in real imported commodity prices of real consumption and the CPI. Cumulated impulse responses are shown, as all the variables enter the model in log differences. Row one shows the responses when all of the the exchange rate, financial, labour and product market indices have been evaluated at the sample medians. Row two shows the responses when the product liberalisation index has been evaluated at the 10th percentile of values realised at the sample, with all the other indices evaluated at their medians. Row three repeats this exercise, with the product market liberalisation index now evaluated at the 90th percentile of the values in the sample. Row four reports the median and 68% quantiles based on the difference in impulse responses that were used to obtain the corresponding statistics for rows two and three, respectively.

4. Conclusion

In recent years, several studies have used interacted panel VAR (IPVAR) models to examine the impact of a given country characteristic on the transmission mechanism of an economic shock of interest. These models typically assume that the VAR coefficients are a deterministic function of the country characteristics. The varying coefficient Bayesian panel VAR model introduced in this paper extends this to the stochastic case. As an application, I study whether the transmission of commodity price shocks is affected by exchange rate regime, financial, labour or product market liberalisation in 18 OECD countries from 1976Q1 to 2006Q4.

The results suggest that commodity price shocks lead to a fall in real consumption and rise in the CPI, regardless of whether coefficients are modelled as a deterministic or stochastic function of country characteristics. But in the deterministic case, confidence bands around impulse responses are narrower than in the stochastic case. As a result most of the country characteristics affect the transmission of commodity price shocks on real consumption and CPI. In the stochastic case, confidence bands are wider and as a result only financial liberalisation affects the transmission of commodity price shocks. In particular, commodity price shocks do not affect the response of real consumption to the same size shock in a financially liberalised economy. Theoretically, this is consistent with greater risk-sharing across countries. From a pragmatic perspective this suggests that results from IPVAR models should be treated with caution. Future research may want to take this into account.

Appendix

In this short note, I show how to expand the Bayesian panel VAR model in Jarocinski (2012, JAE) to shrink the coefficients to a vector of observables variables, as opposed to just a common mean.

0.1 Model

Consider the following, time-varying coefficient panel VAR model:

$$Y_{c,t} = X_{c,t}B_{c,\tau} + E_{c,t} \qquad E_{c,t} \sim N(0, A'_{c,\tau}\Sigma_c A_{c,\tau})$$

$$y_{c,t} \equiv vec(Y_{c,t}) : N \; x \; 1$$

$$\beta_{c,\tau} \equiv vec(B_{c,\tau}) : \mathrm{K\,.\,N\;x\;1}$$

$$a_{c,\tau} \equiv vec(A_{c,\tau}) : K.N \; x \; 1$$

$$e_{c,t} \equiv vec(E_{c,t}) : N \; x \; 1$$

The corresponding likelihood function is proportional to:

$$\prod_t \prod_c \left| A'_{c,\tau}\Sigma_c A_{c,\tau} \right| \exp \left\{ \begin{matrix} -\frac{1}{2}\sum_t \sum_c \left(y_{c,t} - (I_N \otimes X_{c,t})\beta_{c,\tau}\right)\left(A'_{c,\tau}\Sigma_c A_{c,\tau}\right)^{-1} \\ * \left(y_{c,t} - (I_N \otimes X_{c,t})\beta_{c,\tau}\right) \end{matrix} \right\}$$

To estimate this model, we need to make a prior assumption about the time-varying coefficients, $B_{c,\tau}$ and $a_{c,\tau}$. Previous work has typically assumed that these coefficients evolve according to a stochastic random walk. In this paper I make the prior assumption that they vary as a function of observables:

$$\beta_{c,\tau} \,|y_{c,t}, X_{c,t}, a_{c,\tau}, \Sigma_c \sim N(D_{c,\tau}\delta_B, \Lambda_{Bc})$$

$$a_{c,\tau} \,|y_{c,t}, X_{c,t}, \beta_{c,\tau}, \Sigma_c \sim N(D_{c,\tau}\delta_A, \Lambda_{Ac})$$

The tightness of each prior is determined by Λ_{Bc} and Λ_{Ac} . This can be factored into $\Lambda_{Bc} = \lambda_B L_{Bc}$, where L_c is a matrix of scaling parameters, which are obtained in a similar fashion, as in the Litterman or Sims-Zha prior. λ_A and λ_B are shrinkage parameters, which, following a hierarchical modelling approach are also determined by the data. For these parameters, I follow the approach in Jarocinski (2010) and assume an inverted Gamma density for both of them. In particular:

$$\lambda_B|y_{c,t}, X_{c,t}, \beta_{c,\tau}, \Sigma_c \sim IG_2 \propto \lambda_B^{-\frac{v+2}{2}} exp\left(-\frac{1}{2}\frac{s}{\lambda_B}\right)$$

$$\lambda_A|y_{c,t}, X_{c,t}, a_{c,\tau}, \Sigma_c \sim IG_2 \propto \lambda_A^{-\frac{v+2}{2}} exp\left(-\frac{1}{2}\frac{s}{\lambda_A}\right)$$

With these prior assumptions, the likelihood function of this model is then proportional to:

$$\prod_c \prod_t |\Sigma_c| \exp\left(-\frac{1}{2}\sum_t \sum_c (y_{c,t} - \tilde{X}_{c,t}\beta_{c,\tau})'\left(A_{c,\tau}^{'}\Sigma_c A_{c,\tau}\right)^{-1}(y_{c,t} - \tilde{X}_{c,t}\beta_{c,\tau})\right)$$

$$\lambda_B^{-\frac{TCNK}{2}} exp(-\frac{1}{2}\sum_c \sum_\tau (\beta_{c,\tau} - \bar{\beta}_{c,\tau})' L_{Bc}^{-1} \lambda_B^{-1}(\beta_{c,\tau} - \bar{\beta}_{c,\tau}) \prod_c \prod_t |\Sigma_c|^{-\frac{N+1}{2}} \lambda_B^{-\frac{v+2}{2}} exp(-\frac{1}{2}\frac{s}{\lambda_B})$$

$$\lambda_A^{-\frac{TN(N-1)}{2}} exp(-\frac{1}{2}\sum_c \sum_\tau (a_{c,\tau} - a_{c,\tau})' L_{Ac}^{-1} \lambda_A^{-1}(a_{c,\tau} - \bar{a}_{c,\tau}) \prod_c \prod_\tau |\Sigma_c|^{-\frac{N+1}{2}} \lambda_A^{-\frac{v+2}{2}} exp(-\frac{1}{2}\frac{s}{\lambda_A})$$

where $\tilde{X}_{c,t} \equiv I_N \otimes X_{c,t}$, $y_{c,t} \equiv vec(Y_{c,t})$, $\beta_{c,\tau} \equiv vec(B_{c,\tau})$, $\bar{\beta}_{c,\tau} \equiv vec(D_{c,\tau}\delta_B)$, $a_{c,\tau} \equiv vec(A_{c,\tau})$ and $\bar{a}_{c,\tau} \equiv vec(D_{c,\tau}\delta_A)$

In the above, I assumed that $A'_{c,\tau}$ is lower triangular, with ones on the diagonal, following the approach in Primiceri (2005). As a result, $|A_{c,\tau}| = \Pi a_{c,\tau,ii} = 1$ and $|A'_{c,\tau}\Sigma_c A_{c,\tau}| = |\Sigma_c|$. From the above, it is easy to derive the conditional posteriors for $\delta_B, \delta_A, \beta_{c,\tau}, a_{c,\tau}$ and Σ_c.

Conditional Posterior for δ_B:

The conditional posterior, which only involves δ_B is proportional to the following:

$$\propto exp\left\{-\frac{1}{2}\sum_c \sum_\tau (\beta_{c,\tau} - D_{c,\tau}\delta_B)' \Lambda_{Bc}^{-1}(\beta_{c,\tau} - D_{c,\tau}\delta_B)\right\}$$

$$\propto exp\left\{-\frac{1}{2}\sum_c \sum_\tau (\beta_{c,\tau} - (D_{c,\tau}\delta_B)')\Lambda_{Bc}^{-1}(\beta_{c,\tau} - D_{c,\tau}\delta_B)\right\}$$

$$\propto exp\left\{-\frac{1}{2}\sum_c \sum_\tau (\beta_{c,\tau}\Lambda_{Bc}^{-1} - \delta'_B D'_{c,\tau}\Lambda_{Bc}^{-1})(\beta_{c,\tau} - D_{c,\tau}\delta_B)\right\}$$

$$\propto exp\left\{-\frac{1}{2}\sum_c \sum_\tau (\beta'_{c,\tau}\Lambda_{Bc}^{-1}\beta_{c,\tau} - \delta'_B D^{'}_{c,\tau}\Lambda_{Bc}^{-1}\beta_{c,\tau} - \beta'_{c,\tau}\Lambda_{Bc}^{-1}D_{c,\tau}\delta_B + \delta'_B D'_{c,\tau}\Lambda_{Bc}^{-1}D_{c,\tau}\delta_B)\right\}$$

$$\propto exp\left\{-\frac{1}{2}\left(\begin{array}{c} \sum_c \sum_\tau \left[\beta'_{c,\tau}\Lambda_{Bc}^{-1}\beta_{c,\tau}\right] - \sum_c \sum_\tau \left[\delta'_B D^{'}_{c,\tau}\Lambda_{Bc}^{-1}\beta_{c,\tau}\right] \\ - \sum_c \sum_\tau \left[\beta'_{c,\tau}\Lambda_{Bc}^{-1}D_{c,\tau}\delta_B\right] + \sum_c \sum_\tau \left[\delta'_B D'_{c,\tau}\Lambda_{Bc}^{-1}D_{c,\tau}\delta_B\right] \end{array}\right)\right\}$$

Now using exp{a = b} = exp {a}.exp{b} we can ignore the first term since it does not involve δ_B.

$$\propto\ exp\left\{-\frac{1}{2}\left[-\sum_c\sum_\tau\left[\delta'_B D^{'}_{c,\tau}\Lambda_{Bc}^{-1}\beta_{c,\tau}\right]\ -\sum_c\sum_\tau\left[\beta'_{c,\tau}\Lambda_{Bc}^{-1}D_{c,\tau}\delta_B\right]\ +\sum_c\sum_\tau\left[\delta'_B D'_{c,\tau}\Lambda_{Bc}^{-1}D_{c,\tau}\delta_B\right]\right]\right\}$$

$$\propto\ exp\left\{-\frac{1}{2}\left[-\delta'_B\left(\sum_c\sum_\tau D^{'}_{c,\tau}\Lambda_{Bc}^{-1}\beta_{c,\tau}\right)\ -\left(\sum_c\sum_\tau\beta'_{c,\tau}\Lambda_{Bc}^{-1}D_{c,\tau}\right)\delta_B\right.\right.$$

$$\left.\left.+\ \delta'_B\left(\sum_c\sum_\tau D'_{c,\tau}\Lambda_{Bc}^{-1}D_{c,\tau}\right)\delta_B\right]\right\}$$

Rearranging we get

$$\propto\ exp\left\{-\frac{1}{2}\left[\delta'_B\left(\sum_c\sum_\tau D'_{c,\tau}\Lambda_{Bc}^{-1}D_{c,\tau}\right)\delta_B\ -\left(\sum_c\sum_\tau\beta'_{c,\tau}\Lambda_{Bc}^{-1}D_{c,\tau}\right)\delta_B-\ \delta'_B\left(\sum_c\sum_\tau D^{'}_{c,\tau}\Lambda_{Bc}^{-1}\beta_{c,\tau}\right)\right]\right\}$$

Note this looks like the Kernel of a normal and we just need to complete the square to put in the familiar form $\propto \exp\left\{-\frac{1}{2}(\delta-\ \mu)'\Omega^{-1}(\delta-\ \mu)\right\}$ and then solve for the exact forms of the mean (μ) and the variance (Ω). Manipulating the Kernel of a multivariate normal

$$\propto \exp\left\{-\tfrac{1}{2}(\delta_B-\ \mu)'\Omega^{-1}(\delta_B-\ \mu)\right\}$$

$$\propto \exp\left\{-\frac{1}{2}(\delta'_B\Omega^{-1}-\ \mu'\Omega^{-1})(\delta_B-\ \mu)\right\}$$

$$\propto \exp\left\{-\frac{1}{2}(\delta'_B\Omega^{-1}\delta_B-\ \mu'\Omega^{-1}\delta_B-\ \delta'_B\Omega^{-1}\mu+\ \mu'\Omega^{-1}\mu)\right\}$$

Now comparing equations 1 and 2 we note that $\Omega^{-1}=\ \left(\sum_c\sum_t D'_{c,\tau}\Lambda_{Bc}^{-1}D_{c,\tau}\right)$ and $\mu'\Omega^{-1}=\left(\sum_c\sum_t\beta'_{c,\tau}\Lambda_{Bc}^{-1}D_{c,\tau}\right)$. Thus we get the posterior mean and variance

$$\delta_B\sim N(\mu,\Omega)$$

$$\mu=\ \left(\sum_c\sum_t D'_{c,\tau}\Lambda_{Bc}^{-1}D_{c,\tau}\right)^{-1}\ \left(\sum_c\sum_t\sum_t\beta'_{c,\tau}\Lambda_{Bc}^{-1}D_{c,\tau}\right)'$$

$$or\ \mu=\ \left(\sum_c\sum_t D'_{c,\tau}\Lambda_{Bc}^{-1}D_{c,\tau}\right)^{-1}\ \left(\sum_c\sum_t D^{'}_{c,\tau}\Lambda_{Bc}^{-1}\beta_{c,\tau}\right)$$

$$\Omega=\ \left(\sum_c\sum_t D'_{c,\tau}\Lambda_{Bc}^{-1}D_{c,\tau}\right)^{-1}$$

δ_B can therefore be drawn from:

$$p\left(\delta_B|\beta_{c,\tau}\,,\Lambda_{Bc}^{-1}\right)\sim$$

$$N\left(\left(\sum_c\sum_t D'_{c,\tau}\Lambda_{Bc}^{-1}D_{c,\tau}\right)^{-1}\left(\sum_c\sum_t D^{'}_{c,\tau}\Lambda_{Bc}^{-1}\beta_{c,\tau}\right),\left(\sum_c\sum_t D'_{c,\tau}\Lambda_{Bc}^{-1}D_{c,\tau}\right)^{-1}\right)$$

In analogous fashion, the conditional posterior for δ_A is:

$$p(\delta_A | a_{c,\tau}, \Lambda_{Ac}^{-1}) \sim$$

$$N\left(\left(\sum_c \sum_t D'_{c,\tau} \Lambda_{Ac}^{-1} D_{c,\tau}\right)^{-1} \left(\sum_c \sum_t D'_{c,\tau} \Lambda_{Ac}^{-1} a_{c,\tau}\right), \left(\sum_c \sum_t D'_{c,\tau} \Lambda_{Ac}^{-1} D_{c,\tau}\right)^{-1}\right)$$

To derive the conditional posterior for $\beta_{c,\tau}$, start from the likelihood function ??, use exp{a + b} = exp {a}.exp{b} and write out the summations to obtain:

$$\prod_c \prod_t |\Sigma_c| \, exp \begin{Bmatrix} \ldots (y_{c,t} - \tilde{X}_{c,t}\beta_{c,\tau})\left(A'_{c,\tau}\Sigma_c A_{c,\tau}\right)^{-1}(y_{c,t} - \tilde{X}_{c,t}\beta_{c,\tau}) \\ \ldots + \ (\beta_{c,\tau} - \ D_{c,\tau}\delta_B)'\Lambda_{Bc}^{-1}(\beta_{c,\tau} - D_{c,\tau}\delta_B) + \cdots \end{Bmatrix}$$

From this, it is possible to use standard results (see chapter 8 in Kim and Nelson (2000)) to derive the conditional posterior for

$$p\left(\beta_{c,\tau} | \delta_B, \Lambda_{Bc}^{-1}, \left(A'_{c,\tau}\Sigma_c A_{c,\tau}\right)^{-1}\right) =$$

$$N((G_c)^{-1}\left(\left(A'_{c,\tau}\Sigma_c A_{c,\tau}\right)^{-1} \otimes X'_{c,t}\right) y_{c,t} + \Lambda_{Bc}^{-1} D_{c,t}\delta_B, (G_c{}^{-1}))$$

Similarly, to derive the conditional posterior for $a_{c,\tau}$, note that $e_{c,t} = \ y_{c,t} - \ \tilde{X}_{c,t}\beta_{c,\tau}$ which means that the likelihood function can be written as

$$\prod_c \prod_t |\Sigma_c| \, exp \begin{Bmatrix} \ldots (e_{c,t})\left(A'_{c,\tau}\Sigma_c A_{c,\tau}\right)^{-1}(e_{c,t}) \\ \ldots + \ (a_{c,t} - \ D_{c,\tau}\delta_A)'\Lambda_{Ac}^{-1}(a_{c,t} - D_{c,\tau}\delta_A) + \cdots \end{Bmatrix}$$

Which can then be written as:

$$\prod_c \prod_t |\Sigma_c| \, exp \begin{Bmatrix} \ldots (A'_{c,\tau}e_{c,t})(\Sigma_c)^{-1}(A'_{c,\tau}e_{c,t}) \\ \ldots + \ (a_{c,t} - \ D_{c,\tau}\delta_A)'\Lambda_{Ac}^{-1}(a_{c,t} - D_{c,\tau}\delta_A) + \cdots \end{Bmatrix}$$

Following Primiceri (2005) and treating the endogenous variables in the following equation as predetermined, $A'_{c,\tau}e_{c,t} = \ e_{c,t} - \ Z_{c,t}a_{c,\tau}$, the likelihood function is:

$$\prod_c \prod_t |\Sigma_c| \, exp \begin{Bmatrix} \ldots (e_{c,t} - \ Z_{c,t}a_{c,\tau})(\Sigma_c)^{-1}(e_{c,t} - \ Z_{c,t}a_{c,\tau}) \\ \ldots + \ (a_{c,t} - \ D_{c,\tau}\delta_A)'\Lambda_{Ac}^{-1}(a_{c,t} - D_{c,\tau}\delta_A) + \cdots \end{Bmatrix}$$

From that it is easy to see that $a_{c,t}$ can be drawn as:

$$p(a_{c,t} | \delta_A, \Lambda_{Ac}^{-1}, \Sigma_c{}^{-1}) = N\ (F_c{}^{-1}(\Sigma_c{}^{-1} \otimes EJ'_{c,t})e_{c,t} + \Lambda_{Ac}^{-1} D_{c,t}\delta_A, F_c{}^{-1}$$

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