Efficient Perturbation Methods for Solving Regime-Switching DSGE Models

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Why Regime-Switching DSGE Models?

- Economic structures break: e.g. changes in policy regimes
- Variances change : Stock and Watson (2003), Sims and Zha (2006), Justiniano and Primiceri (2008)
- Policy behavior changes : Bernanke et al. (1999), Lubik and Schorfheide (2004), Davig and Leeper (2007)
- Conventional policies weaken
- Distributions shift
- Past events sometimes re-occur

Models and theories have to adapt if they are to continue being useful for policy and for forecasting

The literature

- **Nonlinear switching DSGE models solved "globally"**: Bi and Traum(2013), Richter, Throckmorton and Walker (2014), Davig, Leeper and Walker(2010).
- **Markov-switching linear Rational Expectation models**: Farmer, Waggoner and Zha(2009,2011), Svensson and Williams(2007), Cho(2011,2014), Blake and Zampolli (2006).
- **Nonlinear switching DSGE models solved with perturbations**: Foerster, Rubio-Ramirez, Waggoner and Zha(2013,2014)

This paper

- Higher-order perturbations
- Flexible choice of approximation point
- no partitioning of switching parameters
- Endogenous probabilities
- Anticipated shocks (different from News shocks!!!): Maih(2010), Juillard and Maih(2010) : increased number of state variables
- No explosion of the number of cross terms in higher-order approximation (Levintal, 2014)
- Efficient methods for solving the Quadratic matrix polynomial arising from first-order approximation
- RISE: An object-oriented toolbox to implement it all

The Regime-Switching DSGE model

- **The problem to solve**

$$E_t \sum_{r_{t+1}=1}^{h} \pi_{r_t,r_{t+1}}\left(\mathcal{I}_t\right) d_{r_t}\left(x_{t+1}\left(r_{t+1}\right), x_t\left(r_t\right), x_{t-1}, \varepsilon_t, \theta_{r_{t+1}}\right) = 0$$

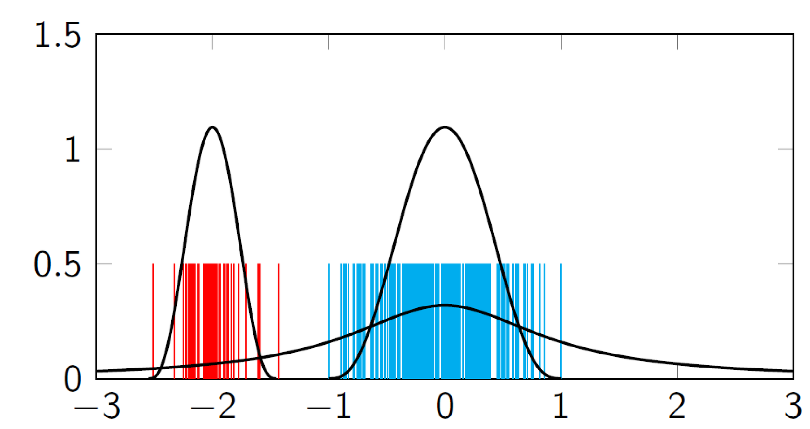
- **The perturbation solution**

$$x_t\left(r_t\right) \simeq \bar{x}\left(r_t\right) + \mathcal{T}_z^{r_t}\left(z_t - \bar{z}_{r_t}\right) + \frac{1}{2}\mathcal{T}_{zz}^{r_t}\left(z_t - \bar{z}_{r_t}\right)^{\otimes 2} + \frac{1}{3!}\mathcal{T}_{zzz}^{r_t}\left(z_t - \bar{z}_{r_t}\right)^{\otimes 3} + \frac{1}{4!}\mathcal{T}_{zzzz}^{r_t}\left(z - \bar{z}_{r_t}\right)^{\otimes 4} + \cdots + \frac{1}{p!}\mathcal{T}_{z\ldots z}^{r_t}\left(z - \bar{z}_{r_t}\right)^{\otimes p}$$

- **The state variables with anticipated shocks**

$$z_t \equiv \left[\, x_{t-1}' \;\; \sigma \;\; \varepsilon_t' \;\; \varepsilon_{t+1}' \;\; \cdots \;\; \varepsilon_{t+k}' \,\right]'$$

Illustration of switching dynamics



If data generated by different distributions

- we cannot **use models designed for one distribution to explain or make predictions about another**
- **where is the steady state?**

If rational agents assign a positive probability to the reoccurrence of past events, **future regimes affect current behavior**

- **de-trend or chop off data: we lose the expectational effects of shifts**

Proposed solution methods for Quadratic Matrix Polynomial

- Functional iterations (possibly with exploitation of sparsity): solves fast when converge
- Newton with Kroneckers (possibly with exploitation of sparsity): stable around solutions
- Newton without Kroneckers (possibly with exploitation of sparsity): fastest on large models

How we tested

Replicated results in

- Farmer, Waggoner and Zha (2011)
- Cho (2014)
- Foerster, Rubio-Ramirez, Waggoner and Zha (2014)

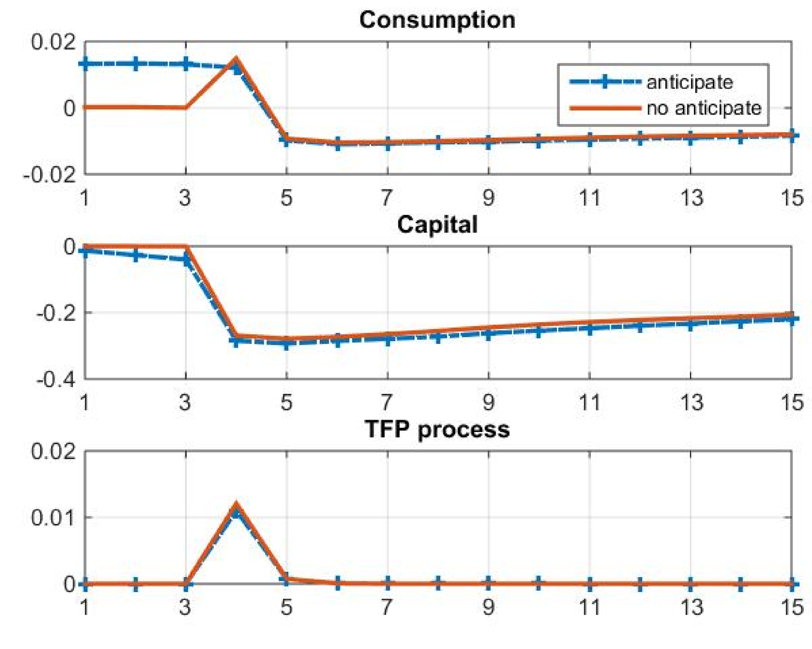
Higher-order perturbations in constant-parameter DSGE: Results identical with

- Dynare
- dynare++
- codes by Binning(2013a,2013b)

Successfully solved a second-order perturbation of a version of NEMO with 272 equations

Generalized IRFs in the Foerster et al. (2014) model

Figure: Anticipated vs Unanticipated technology shock (third-order perturbation)



What RISE does for you

- DSGE : switching+ Optimal policy + OSR + DSGE-VAR, etc.
- VAR : switching + restrictions + identification
- SVAR : switching + restrictions
- Forecasting + Conditional forecasting (linear and nonlinear)
- Deterministic + Stochastic simulation
- Symbolic, numerical and algorithmic/automatic differentiation
- MLE + Bayesian estimation + MCMC
- Time series + Reporting
- Derivative-free Optimization
- HDMR + MCF

Grab a copy at **https://github.com/jmaih/RISE_toolbox/**