A Macroeconomic Model of Equities and Real, Nominal, and Defaultable Debt

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Abstract

Linkages between the real economy and financial markets are of great interest and importance, as evidenced by the 2007–09 financial crisis. This paper develops a simple, structural macroeconomic model that is consistent with a wide variety of asset pricing facts, such as the size and variability of risk premia on equities, real and nominal government bonds, and corporate bonds, commonly referred to as the equity premium puzzle, bond premium puzzle, and credit spread puzzle, respectively. The paper makes two main contributions: First, I show how standard dynamic macroeconomic models can be brought into general agreement with a range of asset prices, making it possible to use these models to study the linkages between risk premia in financial markets and the real economy. Second, I provide a simple structural framework that unifies a variety of asset pricing puzzles and can help explain the relationships between them.

1. Introduction

Traditional macroeconomic models, such as Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007), ignore asset prices and risk premia and, in fact, do a notoriously poor job of matching the risk premia on assets (e.g., Mehra and Prescott, 1985; Backus, Gregory, and Zin, 1989; Rudebusch and Swanson, 2008). At the same time, traditional finance models, such as Dai and Singleton (2003) and Fama and French (2013), ignore the real economy; even when these models use a stochastic discount factor or consumption rather than latent factors, those economic variables are still taken to be exogenous, reduced-form processes.

Yet the relationship between the real economy and financial markets is enormously interesting and important. During the 2007–09 financial crisis, concerns about asset values caused lending and the real economy to plummet, while at the same time the deteriorating economy led private-sector risk premia to increase and asset prices to spiral further downward (e.g., Mishkin, 2011; Gorton and Metrick, 2012). The crisis and recession also led to dramatic fiscal and monetary policy interventions that were beyond the range of past experience.[1] Reduced-form finance models that perform well based on past empirical correlations may perform very poorly when those past correlations no longer hold, such as when there is a structural break or unprecedented policy intervention of the types observed during the crisis. A structural macroeconomic model is more robust to these changes and can immediately provide answers and insights into their possible effects on risk premia, financial markets, and the real economy. Macroeconomic models can also provide useful intuition about *why* consumption, inflation, and asset prices co-move in certain ways and how that comovement may change in response to policy interventions or structural breaks.

In the present paper, I develop a simple, structural macroeconomic model that is consistent with a wide range of asset pricing facts, such as the size and variability of risk premia on equity and real, nominal, and defaultable debt. Thus, unlike traditional macroeconomic models, the model presented here is able to match asset prices and risk premia remarkably well. And unlike traditional finance models, the model in this paper can be used to assess the effects of policy changes and structural breaks on asset prices, and to provide a unified structural story for the

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[1] For example, the U.S. Treasury bought large equity stakes in automakers and financial institutions, and insured money market mutual funds to prevent them from "breaking the buck." The Federal Reserve purchased very large quantities of longer-term Treasury and mortgage-backed securities and gave explicit forward guidance about the likely path of the federal funds rate for years into the future. See, e.g., Mishkin (2011) and Gorton and Metrick (2012).

behavior of risk premia on a variety of assets.

The model developed here builds on earlier work by Rudebusch and Swanson (2012) and has two essential ingredients: generalized recursive preferences (as in Epstein and Zin, 1989, and Weil, 1989) and nominal rigidities. Generalized recursive preferences allow the model to generate risk premia that are as large as in the data. Nominal rigidities are required for the model to match the behavior of inflation, nominal interest rates, and the risk premia on nominal assets such as Treasuries and corporate bonds.

My results have important implications for both macroeconomics and finance. For macroeconomics, I show how standard dynamic structural general equilibrium (DSGE) models can be modified to bring them into agreement with a wide range of asset pricing facts. I thus address Cochrane's (2008) critique that a total failure of macroeconomic models to match even the most basic of these facts is a sign of fundamental flaws in the model.[2] Moreover, bringing macroeconomic models into better agreement with asset prices makes it possible to use these models to study the linkages between risk premia in financial markets and the real economy.

For finance, I provide a structural framework that unifies a variety of asset pricing puzzles and can be used to study the relationships between them. For example, Backus, Gregory, and Zin (1989), Donaldson, Johnsen, and Mehra (1990), and Den Haan (1995) argue that the yield curve ought to slope downward on average because interest rates tend to be low during recessions, implying that bond prices are high when consumption is low, which would lead to an insurance-like, negative risk premium. According to the macroeconomic model of the present paper, the nominal yield curve can slope upward even though the real yield curve slopes downward if technology shocks (or other supply-type shocks) are an important source of economic fluctuations. Technology shocks cause inflation to rise when consumption falls, so that long-term nominal bonds *lose* rather than gain value in recessions, implying a positive risk premium. These predictions of the macroeconomic model—an upward-sloping nominal yield curve and downward-sloping real yield curve—are consistent with the data. Similarly, the model developed here can be used to study the interesting changes in correlations between stock and bond returns documented by Baele, Bekaert, and Inghelbrecht (2010), Campbell, Sundaram, and Viceira (2013), and others.

Previous macroeconomic models of asset prices have tended to focus exclusively on a single

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[2] As Cochrane (2008) points out, asset markets are the mechansim by which marginal rates of substitution are equated to marginal rates of transformation in a macroeconomic model. If the model is wildly inconsistent with basic asset pricing facts, then by what mechanism does the model equate these marginal rates of substitution and transformation?

type of asset, such as equities (e.g., Boldrin, Christiano, and Fisher, 2001; Tallarini, 2000; Guvenen, 2009; Barillas, Hansen, and Sargent, 2009) or debt (e.g., Rudebusch and Swanson, 2008, 2012; Van Binsbergen et al., 2012; Andreasen, 2012). A disadvantage of this approach is that it is unclear whether the results in each case generalize to other asset classes. For example, Boldrin, Christiano, and Fisher (2001) show that capital immobility in a two-sector DSGE model can fit the equity premium by increasing the volatility of the price of capital and the covariance of capital prices with consumption; however, this mechanism cannot explain risk premia on long-term debt, which involve the valuation of a fixed nominal payment stream. By focusing on multiple asset classes, I impose additional discipline on the model and ensure that its results apply more generally. Matching the behavior of a variety of assets also can help identify model parameters, since different types of assets may be relatively more informative about different aspects of the model. For example, nominal assets are helpful for identifying parameters related to inflation.

A number of recent papers study stock and bond prices jointly in a traditional affine framework (e.g., Eraker, 2008; Bekaert, Engstrom, and Grenadier, 2010; Lettau and Wachter, 2011; Ang and Ulrich, 2013; Koijen, Lustig, and Van Nieuwerburgh, 2013).[3] Some of these studies work with latent factors, ignoring the real economy, while others relate asset prices to the reduced-form behavior of consumption. In either case, the more structural approach of the present paper has the advantages discussed above: namely, the ability to analyze policy interventions and structural breaks, and provide greater insight into the macroeconomic fundamentals driving asset prices. Although reduced-form models often fit the data better than structural macroeconomic models, this can simply be a tautological implication of Roll's (1977) critique (that any mean-variance efficient portfolio perfectly fits the mean returns of all assets), as noted by Cochrane (2008). It is only the correspondence of financial risk factors to plausible economic risks that makes reduced-form financial factors interesting.

Chen, Collin-Dufresne, and Goldstein (2009), Bhamra, Kuehn, and Strebulaev (2010), and Chen (2010) model equity and corporate bond prices jointly in an endowment economy.[4] Those authors undertake a much more detailed, structural analysis of the corporate financing decision than is considered here, but they do so in a much simpler, reduced-form macroeconomic envi-

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[3] See also Campbell, Sundaram, and Viceira (2012), who price stocks and bonds jointly in a quadratic latent-factor framework.

[4] Chen et al. (2009) use a reduced-form process for surplus consumption as in Campbell and Cochrane (1999), while Bhamra et al. (2010) and Chen (2010) use a continuous-time regime-switching model with Epstein-Zin-Weil preferences.

ronment. As above, the advantage of the approach taken in the present paper is its ability to consider the effects of novel policy interventions and structural breaks, which cannot be studied in a reduced-form macroeconomic environment.

The two papers most closely related to the present one are Rudebusch and Swanson (2012) and Campbell, Pflueger, and Viceira (2013).[5] Rudebusch and Swanson (2012) extend a standard New Keynesian DSGE model to incorporate Epstein-Zin-Weil preferences and show that the model can match the behavior of nominal bond yields given a sufficiently high level of risk aversion. Relative to Rudebusch and Swanson (2012), the model here is substantially simplified to clarify its essential features and is extended to study equities and real and defaultable debt. Campbell, Pflueger, and Viceira (2013, henceforth CPV) study stock and bond prices in a reduced-form New Keynesian model. In contrast to the present paper, CPV use a stochastic discount factor that is related to their New Keynesian IS curve, Phillips curve, and monetary policy rule only in an ad hoc, reduced-form manner—in this respect, their analysis is similar to the term-structure studies of Rudebusch and Wu (2007) and Bekaert, Cho, and Moreno (2010). In fact, the ad hoc connection between the stochastic discount factor and the economy is crucial for CPV's results: as shown by Lettau and Uhlig (2000) and Rudebusch and Swanson (2008), CPV's Campbell-Cochrane (1999) habit specification cannot produce significant risk premia when households are able to endogenously smooth consumption (as in a standard macroeconomic model), because households endogenously choose a path for consumption that is so smooth as to stabilize the stochastic discount factor.[6] In the present paper, I undertake a more structural approach, specifying a complete—but simple—macroeconomic model in which the stochastic discount factor is internally consistent with the rest of the model.

The remainder of the paper proceeds as follows. Section 2 presents a simple New Keynesian DSGE model with nominal rigidities and Epstein-Zin preferences, shows how to solve the model, and discusses the calibration of the model and its implications for macroeconomic quantities. Section 3 derives the prices of stocks and real, nominal, and defaultable bonds within the framework of the model, and compares the behavior of those asset prices to the data. Section 4 provides

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[5] See also Van Binsbergen et al. (2012) and Andreasen (2012) for variations on the analysis in Rudebusch and Swanson (2012).

[6] Households with Campbell-Cochrane (1999) habits are extremely averse to high-frequency fluctuations in consumption. In a DSGE model (as opposed to an endowment economy), households can self-insure themselves from these fluctuations by varying their hours of work or savings. In fact, for realistic parameterizations of DSGE models, households endogenously choose a path for consumption that is so smooth the stochastic discount factor does not vary much more than in the model without habits, leading risk premia to be about the same as without habits. See Rudebusch and Swanson (2008) and Lettau and Uhlig (2000) for details.

additional analysis and discussion related to issues raised in Sections 2 and 3. Section 5 concludes. An Appendix presents all the equations of the model and discusses the numerical solution method in more detail.

2. A Simple Macroeconomic Model

This section develops a simple dynamic macroeconomic model with generalized recursive preferences and nominal rigidities. Generalized recursive preferences (e.g., Epstein and Zin, 1989; Weil, 1989) are required for the model to match the size of risk premia in the data.[7] Nominal rigidities are required for the model to match the basic behavior of inflation, nominal interest rates, and the risk premia on nominal assets such as Treasuries and corporate bonds.

In this section, I strive to keep the model as simple as possible while still matching the essential features of the behavior of macroeconomic variables and asset prices. For this reason, the model deliberately follows the very simple New Keynesian structure of Clarida, Galí, and Gertler (1999) and Woodford (2003), extended to the case of Epstein-Zin preferences. In principle, the more realistic, medium-scale New Keynesian models of Christiano et al. (2005) and Smets and Wouters (2007) could also be extended to the case of Epstein-Zin preferences to achieve an even better empirical fit to the data, but at the cost of being much more complicated. The simple model developed here is designed to maximize intuition and insight into the relationships between the macroeconomy and asset prices.

2.1 Households

Time is discrete and continues forever. There is a unit continuum of representative households, each with generalized recursive preferences as in Epstein and Zin (1989) and Weil (1989). In each period t, the representative household receives the utility flow

$$u(c_t, l_t) \equiv \log c_t - \eta \frac{l_t^{1+\chi}}{1+\chi}, \tag{1}$$

where c_t and l_t denote household consumption and labor in period t, respectively, and $\eta > 0$ and $\chi > 0$ are parameters. Note that equation (1) differs from Epstein and Zin (1989) and Weil (1989) in that period utility depends on labor as well as consumption.

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[7] See the previous footnote and Rudebusch and Swanson (2008) for a discussion of why habits in household preferences, such as Campbell and Cochrane (1999), are unable to match the size of risk premia in DSGE models.

The assumption of additive separability in (1) follows Woodford (2003) and simplifies many aspects of the model. For example, the household's intertemporal elasticity of substitution is unity, its Frisch elasticity of labor supply is $1/\chi$, and its stochastic discount factor (defined below) is related to c_{t+1}/c_t, instead of a more complicated expression involving labor. The similarity of the stochastic discount factor to versions of the model without labor also facilitates comparison to the finance literature. In addition, the assumption of logarithmic preferences over consumption ensures that the model is consistent with balanced growth (King, Plosser, and Rebelo, 1988, 2002) and is a standard benchmark in the macroeconomics literature (e.g., King and Rebelo, 1999).

Households can borrow and lend in a default-free one-period nominal bond market at the continuously-compounded interest rate i_t. The use of continuous compounding allows for greater comparability to the finance literature and also simplifies the bond-pricing equations below. Each period, the household faces a flow budget constraint

$$a_{t+1} = e^{i_t} a_t + w_t l_t + d_t - c_t, \tag{2}$$

where a_t denotes beginning-of-period nominal assets and w_t and d_t denote the nominal wage and exogenous transfers to the household, respectively. The household faces a standard no-Ponzi-scheme constraint,

$$\lim_{T \to \infty} \prod_{\tau=t}^{T} e^{-i_{\tau+1}} a_{T+1} \geq 0. \tag{3}$$

Let $(c^t, l^t) \equiv \{(c_\tau, l_\tau)\}_{\tau=t}^{\infty}$ denote a state-contingent plan for household consumption and labor from time t onward, where the explicit state-dependence of the plan is suppressed to reduce notation. Following Epstein and Zin (1989) and Weil (1989), the household has preferences over state-contingent plans ordered by the recursive functional

$$\widetilde{V}(c^t, l^t) = u(c_t, l_t) + \beta \left[E_t \widetilde{V}(c^{t+1}, l^{t+1})^{1-\alpha} \right]^{1/(1-\alpha)}, \tag{4}$$

where $\beta \in (0, 1)$ and $\alpha \in \mathbb{R}$ are parameters,[8] E_t denotes the mathematical expectation conditional on the state of the economy at time t, and (c^{t+1}, l^{t+1}) denotes the state-contingent plan (c^t, l^t) from date $t + 1$ onward. Equation (4) has the same form as expected utility preferences, but with the expectation operator "twisted" and "untwisted" by the coefficient $1 - \alpha$. When $\alpha = 0$,

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[8] The case $\alpha = 1$ is understood to correspond to $\widetilde{V}(c^t, l^t) = u(c_t, l_t) + \beta \exp\left[E_t \log \widetilde{V}(c^{t+1}, l^{t+1}) \right]$. Note that when $\alpha > 0$, the household prefers early resolution of uncertainty (see Kreps and Porteus, 1978), and when $\alpha < 0$ the household prefers late resolution of uncertainty (assuming $\widetilde{V} \geq 0$). See Swanson (2013) for additional discussion.

(4) reduces to the special case of expected utility. When $\alpha \neq 0$, the intertemporal elasticity of substitution over deterministic consumption paths in (4) is the same as for expected utility, but the household's risk aversion with respect to gambles over future utility flows is amplified (or attenuated) by the additional curvature parameter α. Thus, generalized recursive preferences allow the household's intertemporal elasticity of substitution and coefficient of relative risk aversion to be parameterized independently.

In each period, the household maximizes (4) subject to the budget constraint (2)–(3). The state variables of the household's optimization problem are a_t and Θ_t, where the latter is a vector denoting the state of the aggregate economy at time t. The household's "generalized value function" $V(a_t; \Theta_t)$ satisfies the generalized Bellman equation

$$V(a_t; \Theta_t) \; = \; \max_{(c_t, l_t)} \; u(c_t, l_t) \; + \; \beta \left[E_t \, V(a_{t+1}; \Theta_{t+1})^{1-\alpha} \right]^{1/(1-\alpha)}, \tag{5}$$

Where a_{t+1} is given by (2).

Note that many authors write generalized recursive preferences in terms of a CES aggregate over current and future utility, such as

$$U(a_t; \Theta_t) \; = \; \max_{(c_t, l_t)} \; \left\{ \widetilde{u}(c_t, l_t)^\rho + \beta \left[E_t \, U(a_{t+1}; \Theta_{t+1})^{\widetilde{\alpha}} \right]^{\rho/\widetilde{\alpha}} \right\}^{1/\rho}, \tag{6}$$

where $\rho < 1$. This notation follows Epstein and Zin (1989) closely, where those authors take $\widetilde{u}(c_t, l_t) = c_t$ in their framework without labor. However, setting $V = U^\rho$, $u = \widetilde{u}^\rho$, and $\alpha = 1 - \widetilde{\alpha}/\rho$, this can be seen to correspond exactly to (5).[9] The advantage of using the notation (5) is that it has a much clearer relationship than (6) to standard dynamic programming results, regularity conditions, and first-order conditions. For example, the benefits of additive separability of the period utility function $u(c_t, l_t)$ are readily apparent in (5) but not in (6).[10]

It's straightforward to show (e.g., Rudebusch and Swanson, 2012), that the household's stochastic discount factor for the additively separable period utility function (1) is given by

$$m_{t+1} \; \equiv \; \frac{c_t}{c_{t+1}} \left(\frac{V_{t+1}}{\left(E_t V_{t+1}^{1-\alpha} \right)^{1/(1-\alpha)}} \right)^{-\alpha}. \tag{7}$$

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[9] For the case $\rho < 0$, set $V = -U^\rho$ and $u = -\widetilde{u}^\rho$. The case $\rho = 0$ corresponds to multiplier preferences. See Swanson (2013) for additional discussion.

[10] See also the discussion in Swanson (2013). In either (5) or (6), parameter values must be chosen to ensure that u or $\widetilde{u}$ is positive for all admissible values of (c_t, l_t), or negative for all admissible values, in order to avoid complex numbers in the twisted expectations operator. When $u \leq 0$, it is natural to let $V \leq 0$ and interpret (5) as $V(a_t; \Theta_t) = \max_{(c_t, l_t)} u(c_t, l_t) - \beta(E_t(-V(a_{t+1}; \Theta_{t+1}))^{1-\alpha})^{1/(1-\alpha)}$.

Let r_t denote the one-period continuosly-compounded risk-free real interest rate. Then

$$e^{-r_t} = E_t m_{t+1}. \tag{8}$$

2.2 Firms

The economy also contains a continuum of infintely-lived monopolistically competitive firms indexed by $f \in [0, 1]$, each producing a single differentiated good. Firms hire labor from households in a competitive market and have identical Cobb-Douglas production functions,

$$y_t(f) = A_t k^{1-\theta} l_t(f)^\theta, \tag{9}$$

where $y_t(f)$ denotes firm f's output, A_t is aggregate productivity affecting all firms, k and $l_t(f)$ denote the firm's capital and labor inputs at time t, respectively, and $\theta > 0$ is a parameter. For simplicity, and following Woodford (2003), firms' capital is assumed to be fixed, so that labor is the only variable input to production. Intuitively, movements in the capital stock are small at business-cycle frequencies and are dominated by fluctuations in labor.[11]

Technology, A_t, follows an exogenous AR(1) process,

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^A, \tag{10}$$

where $\rho_A \in (-1, 1]$, and ε_t^A denotes an i.i.d. white noise process with mean zero and variance σ_A^2. For simplicity and comparability to models in finance, I set $\rho_A = 1$ in the baseline calibration of the model, discussed below.

Firms set prices optimally subject to nominal rigidities in the form of Calvo (1983) price contracts, which expire with probability $1 - \xi$ each period, $\xi \in (0, 1)$. Each time a Calvo contract expires, the firm sets a new contract price $p_t^*(f)$ freely, which then remains in effect over the life of the new contract, with indexation to the (continuously-compounded) steady-state inflation rate $\bar{\pi}$ each period.[12] In each period $\tau \geq t$ that the contract remains in force, the firm must

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[11] Woodford (2003, p. 167) compares a model with fixed firm-specific capital to a model with endogenous capital and investment adjustment costs and finds that the basic business-cycle features of the two models are very similar. In models with endogenous capital (e.g., Christiano et al., 2005; Smets and Wouters, 2007; Altig et al., 2011), investment adjustment costs are typically included to keep the capital stock stable at higher frequencies. Thus, one can think of the fixed-capital assumption as a simple way of achieving the same result. Woodford (2003) and Altig et al. (2011) also show that firm-specific capital stocks help generate inflation persistence that is consistent with the data (see particularly Woodford, 2003, pp. 163-173).

[12] The assumption of indexation keeps the model well-behaved with respect to changes in steady-state inflation. The continuous compounding is notationally simpler for some of the equations below.

supply whatever output is demanded at the contract price $p_t^*(f)e^{(\tau - t)\overline{\pi}}$, hiring labor $l_\tau(f)$ from households at the market wage w_τ.

Firms are jointly owned by households and distribute all profits and losses back to households each period in an aliquot, lump-sum manner. When a firm's price contract expires, the firm chooses the new contract price $p_t^*(f)$ to maximize the value to shareholders of the firm's cash flows over the lifetime of the contract,

$$\max_{p_t(f)} \ E_t \sum_{j=0}^{\infty} m_{t,t+j}(P_t/P_{t+j})\xi^j \left[p_t(f)e^{j\overline{\pi}}y_{t+j}(f) - w_{t+j}l_{t+j}(f) \right], \tag{11}$$

where $m_{t,t+j} \equiv \prod_{i=1}^{j} m_{t+i}$ denotes shareholders' stochastic discount factor from period $t+j$ back to t, P_t the aggregate price level (defined below), and w_t the nominal wage at time t.[13]

The output of each firm f is purchased by a perfectly competitive final goods sector, which aggregates the continuum of differentiated firm goods into a single final good using a CES production technology,

$$Y_t = \left[\int_0^1 y_t(f)^{(\epsilon-1)/\epsilon}df \right]^{\epsilon/(\epsilon-1)}, \tag{12}$$

where Y_t denotes the quantity of the final good and $\epsilon > 1$ is a parameter. Each intermediate firm f thus faces a downward-sloping demand curve for its product,

$$y_t(f) = \left(\frac{p_t(f)}{P_t} \right)^{-\epsilon} Y_t, \tag{13}$$

where P_t is the CES aggregate price of the final good,

$$P_t \equiv \left[\int_0^1 p_t(f)^{1-\epsilon}df \right]^{1/(1-\epsilon)}, \tag{14}$$

and $p_t(f)$ denotes the price in effect for firm f at time t (so $p_t(f) = p_\tau^*(f)$, letting $\tau \leq t$ denotes the most recent period in which firm f reset its contract price).

Differentiating (11) with respect to $p_t(f)$ and setting the derivative equal to zero yields the standard New Keynesian price optimality condition,

$$p_t^*(f) = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{j=0}^{\infty} m_{t,t+j}(P_t/P_{t+j})\xi^j y_{t+j}(f)\mu_{t+j}(f)}{E_t \sum_{j=0}^{\infty} m_{t,t+j}(P_t/P_{t+j})\xi^j y_{t+j}(f)}, \tag{15}$$

where $\mu_t(f)$ denotes the marginal cost for firm f at time t,

$$\mu_t(f) \equiv \frac{w_t l_t(f)}{\theta y_t(f)}. \tag{16}$$

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[13] Equivalently, the firm can be thought of as choosing a state-contingent plan for prices that maximizes the value of the firm to shareholders.

That is, the firm's optimal contract price $p_t^*(f)$ is a monopolistic markup $\epsilon/(\epsilon-1)$ over a discounted weighted average of expected future marginal costs over the lifetime of the contract.

2.3 Aggregate Resource Constraints and Government

Let L_t denote the aggregate quantity of labor demanded by firms,

$$L_t = \int_0^1 l_t(f)df. \tag{17}$$

Then L_t satisfies

$$Y_t = \Delta_t^{-1} A_t K L_t^\theta, \tag{18}$$

where $K = k$ denotes the aggregate capital stock and

$$\Delta_t \equiv \int p_t(f)df = \left[(1-\xi)\sum_{j=0}^\infty \xi^j \left(\frac{p_{t-j}^*(f)e^{j\overline{\pi}}}{P_t}\right)^{-\epsilon/\theta}\right]^\theta \tag{19}$$

measures the cross-sectional dispersion of prices across firms. A greater degree of cross-sectional price dispersion increases Δ_t and reduces the economy's efficiency at producing final output.

Labor market equilibrium requires that $L_t = l_t$, firms' labor demand equals the aggregate labor supplied by households. Equilibrium in the final goods market requires $Y_t = C_t$, where $C_t = c_t$ denotes aggregate consumption. For simplicity, there are no government purchases or investment in the model.

Finally, there is a monetary authority that sets the one-period nominal interest rate i_t according to a Taylor (1993)-type policy rule,

$$i_t = r + \pi_t + \phi_\pi(\pi_t - \overline{\pi}) + \frac{\phi_y}{4}(y_t - \overline{y}_t), \tag{20}$$

where $r = 1/\beta$ denotes the steady-state real interest rate, $\pi_t \equiv \log(P_t/P_{t-1})$ denotes the inflation rate, $\overline{\pi}$ the monetary authority's inflation target, $y_t \equiv \log Y_t$,

$$\overline{y}_t \equiv \rho_{\overline{y}}\overline{y}_{t-1} + (1 - \rho_{\overline{y}})y_t \tag{21}$$

denotes a trailing moving average of log output, and $\phi_\pi, \phi_y \in \mathbb{R}$ and $\rho_{\overline{y}} \in [0,1)$ are parameters.[14] The term $(\pi_t - \overline{\pi})$ in (20) represents the deviation of inflation from policymakers' target and $(y_t - \overline{y}_t)$ is a measure of the "output gap" in the model.

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[14] Note that interest rates and inflation in (20) are at quarterly rather than annual rates, so ϕ_y corresponds to the sensitivity of the annualized short-term interest rate to the output gap, as in Taylor (1993). I also exclude a lagged interest rate "smoothing" term on the right-hand side of (20) to keep the model as simple as possible and keep the number of state variables to a minimum. Rudebusch (2002) argues that the degree of federal funds rate smoothing from one quarter to the next is essentially zero, and that instead the Federal Reserve's deviations from the Taylor rule (20) are serially correlated due to factors outside the rule being persistent. In other words, Rudebusch argues that the residuals ε_t^i in the empirical version of (20) are serially correlated.

2.4 Solution Method

The model above is solved by writing each equation in recursive form, dividing nonstationary variables (Y_t, C_t, w_t, etc.) by the level of technology A_t, and using the method of local approximation around the nonstochastic steady state, or perturbation methods.[15] The complete set of recursive equations that define the model are standard and are reported in the Appendix, along with the asset pricing equations discussed below.

Macroeconomic models similar to the one developed above are typically solved using a first-order approximation (a linearization or log-linearization), but this solution method reduces all risk premia in the model to zero.[16] A second-order approximation to the model produces risk premia that are nonzero but constant over time (a constant function of the variance σ_A^2). In order for risk premia in the model to vary with the state of the economy, the model must be solved to at least third order around the steady state. Note that second- and third-order terms in the model solution can be non-negligible as long as the model is sufficiently "curved", which is the case when risk aversion (related to the Epstein-Zin parameter α) is sufficiently large.

Third- and higher-order solutions of the model are computed using the Perturbation AIM algorithm of Swanson, Anderson, and Levin (2006), which can compute general nth-order Taylor series approximate solutions to discrete-time recursive rational expectations models. The model above has three state variables (Δ_t, $\overline{y}_t$, and A_t) and a single shock (ε_{t+1}^A) and thus can be solved to third order very quickly, in just a few seconds on a standard laptop computer.[17] To obtain greater accuracy over a wider range of values for the state variables, the model can be solved to higher order; the results reported below are for the fifth-order solution unless stated otherwise. (Results for fourth- and sixth-order solutions are very similar, suggesting that the Taylor series has essentially converged over the relevant range for the state variables.) Aruoba et al. (2006) compare a variety of numerical solution techniques for standard macroeconomic models and find

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[15] The equity price p_t^e is normalized by A_t^ν rather than A_t, where ν denotes the degree of leverage (see below). The value function V_t is normalized by defining $\tilde{V}_t \equiv V_t - (1-\beta)^{-1} \log A_t$. Note that this transformation makes the model stationary to first order around the nonstochastic steady state, but second- and higher-order terms are (slightly) nonstationary, as discussed in the Appendix and the asset pricing results below. The Epstein-Zin coefficient α in (5) prevents the normalization of V_t from canceling out for terms beyond first order.

[16] In the finance literature, it is standard to log-linearize the model and then take expectations of all variables assuming joint lognormality. This approximate solution method produces nonzero (but constant) risk premia, but effectively treats higher-order moments of the lognormal distribution on par with first-order economic terms. Standard perturbation methods (e.g., Judd, 1998; Swanson, Anderson, and Levin, 2006) explicitly relate higher-order moments of the shock distribution to the corresponding order of the state variables (so variance is a second-order term, skewness a third-order term, etc.), because their magnitudes are the same in theory.

[17] Despite the normalization by A_t above, it remains a state variable. The lagged growth rate A_t/A_{t-1} appears in several normalized equations, and the level of A_t appears in the normalized equation for V_t, as discussed above.

TABLE 1: PARAMETER VALUES, BASELINE CALIBRATION

β	0.99	θ	0.6	ϕ_π	0.5
χ	2	ξ	0.75	ϕ_y	0.75
η	0.545	ϵ	10	$\overline{\pi}$	0.01
RRA (R^c)	60	ρ_A	1	$\rho_{\bar{y}}$	0.9
		σ_A	0.007		
		$K/(4Y)$	2.5		

that higher-order perturbation solutions are among the most accurate globally as well as being the fastest to compute. Swanson, Anderson, and Levin (2006) provide details of the algorithm and discuss the global convergence properties of nth-order Taylor series approximations.

A noteworthy feature of the nonlinear solution algorithm used here, relative to the loglinear-lognormal approximation typically used in finance, is that second- and higher-order terms of the Taylor series display endogenous conditional heteroskedasticity. Letting x_t denote a generic state variable and ε_{t+1} a generic shock, the second-order Taylor series solution has terms of the form $x_t\varepsilon_{t+1}$, which have a one-period-ahead conditional variance that depends on the economic state x_t (that is, $\text{Var}_t(x_t\varepsilon_{t+1})$ depends on x_t). Thus, even though the model's exogenous driving shocks ε_{t+1}^A are homoskedastic, the nonlinear solution algorithm used here preserves the endogenous conditional heteroskedasticity that is naturally generated by the nonlinearities in the model.

2.5 Calibration

The model described above is meant to be illustrative rather than provide a comprehensive empirical fit to the data, so I calibrate rather than estimate its key parameters. The baseline calibration is reported in Table 1, and is meant to be standard, following along the lines of parameter values estimated by Christiano et al. (2005), Smets and Wouters (2007), and Levin et al. (2006) using quarterly U.S. data.

The household's discount factor, β, is set to .99, implying a nonstochastic steady-state real interest rate of about 4 percent per year. Although this might seem a bit high, households' risk aversion will drive the expected risk-free real rate close to 2 percent in the stochastic case.

The assumption of logarithmic preferences over consumption implies an intertemporal elasticity of substitution of unity, which is higher than estimates based on aggregate data (e.g., Hall, 1988), but similar to estimates based on household-level data (e.g., Vissing-Jorgensen, 2002). Logarithmic preferences over consumption are also a standard benchmark in macroeconomics (e.g., King and Rebelo, 1999). Bansal and Yaron (2004) and Dew-Becker (2012) argue that estimates

based on aggregate data are biased downward, suggesting that the value of unity assumed here is reasonable.[18]

The calibrated value of $\chi = 2$ implies a Frisch elasticity of labor supply of $1/2$, consistent with estimates in Levin et al. (2006) and estimates from household data (e.g., Pistaferri, 2003). The parameter η is set so as to normalize $L = 1$ in steady state.

I set the parameter α to imply a coefficient of relative risk aversion $R^c = 60$ in steady state, using the closed-form expressions derived in Swanson (2013) for models with labor.[19] Although this value is high, it is standard in the literature and is largely a byproduct of the model's simplicity.[20] Households in the model have perfect knowledge of all the model's equations, parameter values, and shock processes, so the quantity of risk in the model is far smaller than in the actual U.S. economy. As a result, the household's aversion to risk in the model must be correspondingly larger to fit the risk premia seen in the data. Barillas, Hansen, and Sargent (2009) formalize this intuition by showing that high risk aversion in an Epstein-Zin specification is isomorphic to a model in which households have low risk aversion but a moderate degree of uncertainty about the economic environment. Campanale, Castro, and Clementi (2010) echo this point, emphasizing that the quantity of consumption risk in a standard DSGE model is very small, and thus the risk aversion required to match asset prices must be correspondingly larger.[21] As an alternative to high risk aversion, one could increase the quantity of risk in the model instead, such as by introducing long-run risk as in Bansal and Yaron (2004), or disaster risk as in Rietz (1988) and

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[18] The results of the paper are not sensitive to setting the IES equal to unity. For example, specifications with $u(c_t, l_t) = c_t^{1-\gamma}/(1 - \gamma) - \eta l^{1+\chi}/(1 + \chi)$ or $u(c_t, l_t) = (c_t^{1-\gamma} - 1)/(1 - \gamma) - \eta l^{1+\chi}/(1 + \chi)$ (which are not exactly equivalent when $\alpha \neq 0$) produce very similar results when γ is set to 0.9 or 1.1. Of course, these specifications do not satisfy balanced growth and are first-order nonstationary in response to permanent technology shocks.

[19] Swanson (2013) derives the coefficient of relative risk aversion for generalized recursive preferences with flexible labor and arbitrary period utility function $u(c_t, l_t)$. For additively separable period utility (1) with $l = 1$ in steady state, risk aversion is given by

$$R^c = \frac{1}{1 + \frac{\eta}{\chi}} + \alpha \frac{1}{\log c - \frac{\eta}{1+\chi}}.$$

See Swanson (2013) for the derivation and details. In general, risk aversion is lower when labor supply can vary because the household is better able to insure itself from shocks.

[20] For example, Piazzesi and Schneider (2006) estimate a value of 57, Rudebusch and Swanson (2012) a value of 110, Van Binsbergen et al. (2012), Andreasen (2012), and Campbell and Cochrane (1999) a value of about 80, and Tallarini (2000) a value of about 50. The nonstationarity of technology implied by $\rho_A = 1$ in the present paper increases the quantity of risk in the model here relative to Rudebusch and Swanson (2012), which allows the coefficient of relative risk aversion here to be smaller.

[21] The simplifying assumption of a representative household also plays a role. Mankiw and Zeldes (1991), Parker (2001), and Malloy, Moskowitz, and Vissing-Jorgensen (2009) show that the consumption of stockholders is more volatile (and more correlated with the stock market) than the consumption of nonstockholders, so the required level of risk aversion in a representative-agent model is higher than it would be in a model that recognized that stockholders have more volatile consumption (Guvenen, 2009).

Barro (2006).

Turning to the production side of the economy, I set the elasticity of output with respect to labor $\theta = 0.6$. I calibrate the Calvo contract parameter $\xi = 0.75$, implying an average contract duration of four quarters, consistent with Christiano et al. (2005), Levin et al. (2006), and Altig et al. (2010). The elasticity of demand ϵ faced by the monopolistically competitive intermediate goods firms is calibrated to a value of 10, implying a steady-state markup of about 11 percent, consistent with estimates in Christiano et al. (2005) and Altig et al. (2010). The technology process A_t is assumed to be a random walk in the baseline calibration, so $\rho_A = 1$. The standard deviation of technology shocks, σ_A, is set to .007, following estimates in King and Rebelo (1999). The steady-state ratio of the capital stock to annualized output is calibrated to 2.5.

The response of monetary policy to inflation, ϕ_π, is set to 0.5, as in Taylor (1993, 1999). I set $\phi_y = 0.75$, between the values of 0.5 and 1 used by Taylor (1993) and Taylor (1999). I set the monetary authority's inflation target $\overline{\pi}$ to 1 percent per quarter, implying a nonstochastic steady-state inflation rate of about 4 percent per year. As with the real interest rate, households' risk aversion will drive the expected inflation rate somewhat below this in the stochastic case. Also note that many central banks' current official inflation targets of 2 percent are not high enough to explain the historical average level of nominal yields in those countries (e.g., the U.S. and U.K.), even over relatively recent samples such as 1990–2007, as will be seen below. Finally, I calibrate $\rho_{\bar{y}} = 0.9$, implying that the monetary authority uses the deviation of current output from its average level over the past roughly 2.5 years to approximate the output gap.

2.6 Impulse Response Functions

Figure 1 plots first-order impulse response functions of the model to a one-standard-deviation technology shock, under the baseline calibration described above. Although the model can easily be solved to higher than first order using the methods above, the impulse response functions for the macroeconomic variables reported in Figure 1 are all dominated by their first-order terms, so the responses in the figure are sufficiently accurate to convey all the intuition for the behavior of these variables.

The top left panel of Figure 1 reports the impulse response of technology, A_t, to the shock. Since $\rho_A = 1$, technology jumps on impact and remains permanently at the higher level.

The response of consumption, C_t, is plotted in the top right panel. Consumption jumps upward on impact, because higher productivity both increases the supply of output and makes
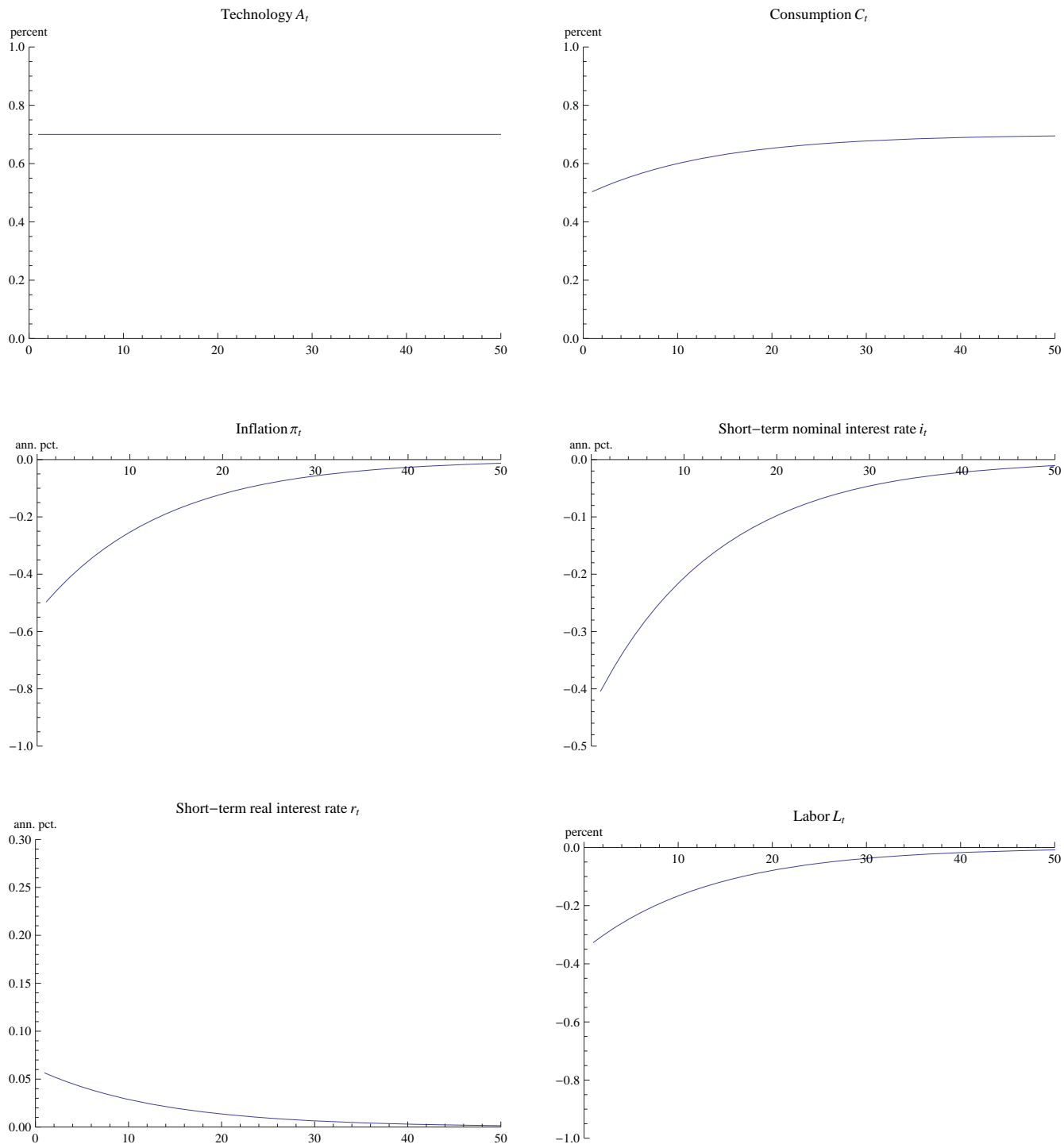
Figure 1. First-order impulse response functions for technology A_t, consumption C_t, inflation π_t, short-term nominal interest rate i_t, short-term real interest rate r_t, and labor L_t to a one-standard-deviation (0.7 percent) technology shock in the model. See text for details.

households wealthier in present-value terms, increasing consumption demand. However, the increase in real interest rates (described shortly) implies that consumption does not jump all the way to its new, higher level on impact. Instead, consumption continues to increase gradually over time to approach the new steady state.

The middle left panel reports the impulse response for inflation, π_t. The higher level of technology reduces firms' marginal costs of production. Firms are monopolistic and set their price equal to a constant markup over expected future marginal costs, whenever they are able to reset their price. Inflation falls on impact (by about 0.5 percent at an annualized rate) as those firms who are able to reset their prices do so. The response of inflation is persistent, however, as firms' price contracts expire only gradually.

The nominal interest rate i_t, in the middle right panel, is set by the monetary authority as a function of output and inflation according to the policy rule (20). Interest rates respond more strongly to inflation than output, causing nominal rates to decline in response to the shock. The nominal interest rate drops about 40 basis points (at an annual rate) on impact and gradually returns to steady state.

The bottom left panel plots the response of the real interest rate, r_t. Inflation falls by more than the nominal interest rate after the shock, causing the real rate to rise by about 5 basis points (at an annual rate) on impact.[22] The real rate then gradually falls back to steady state.

The response of labor, L_t is graphed in the bottom right panel. After the technology shock, households are wealthier in present value terms and want to consume more leisure. This tends to push labor downward. Because prices are sticky and firms are monopolistic, firms hire whatever labor is necessary to satisfy output demand. This tends to push labor upward, but for the very simple model developed here, the first effect dominates. (This is common in simple New Keynesian models, as pointed out by Galí, 1999.) As a result, labor declines slightly on impact, by about 0.3 percent, and gradually returns to steady state. The sign of this response isn't crucial for the asset pricing results, below, and in more complicated models, such as Altig et al. (2011), increased demand for investment following the technology shock is typically enough to make the second effect dominate. (Alternatively, a stronger monetary policy response that would drive the short-term real interest rate down in response to the shock, would cause consumption to jump above 0.7 percent on impact and lead to an increase in labor.)

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[22]Note that $r_t = i_t - E_t\pi_{t+1}$ to first order, according to the timing convention for the interest rate subscripts.

3. Asset Prices and Risk Premia

The stochastic discount factor implied by the simple macroeconomic model above can be used to price any asset in the model. In this section, I derive the implications of the model for equities and real, nominal, and defaultable debt.

3.1 Equity

An equity security in the model is defined to be a levered claim on the aggregate consumption stream, so that each period, equity pays a dividend equal to C_t^ν, where ν denotes the degree of leverage. (Results are very similar if an equity security is defined to be a claim on the monopolistic intermediate firm sector, with fixed costs in that sector generating leverage.) Consistent with Abel (1999), Bansal and Yaron (2004), and Campbell et al. (2013), I calibrate $\nu = 3$. Note that any fixed costs of production create operational leverage for firms, so that ν can be interpreted as representing operational as well as financial leverage (see Gourio, 2012, and Campbell et al., 2013).

Let p_t^e denote the ex-dividend time-t price of an equity share. In equilibrium,

$$p_t^e = E_t m_{t+1}(C_{t+1}^\nu + p_{t+1}^e). \tag{22}$$

Let R_{t+1}^e denote the realized gross return on equity,

$$R_{t+1}^e \equiv \frac{C_{t+1}^\nu + p_{t+1}^e}{p_t^e}. \tag{23}$$

I define the equity premium at time t, ψ_t^e, to be the expected excess return to holding equity for one period,

$$\psi_t^e \equiv E_t R_{t+1}^e - e^{r_t}. \tag{24}$$

Note that

$$
\begin{aligned}
\psi_t^e &= \frac{E_t m_{t+1} E_t(C_{t+1}^\nu + p_{t+1}^e) - E_t m_{t+1}(C_{t+1}^\nu + p_{t+1}^e)}{p_t^e E_t m_{t+1}} \\
&= \frac{-\mathrm{Cov}_t(m_{t+1}, R_{t+1}^e)}{E_t m_{t+1}} \\
&= -\mathrm{Cov}_t\left(\frac{m_{t+1}}{E_t m_{t+1}}, R_{t+1}^e\right),
\end{aligned} \tag{25}
$$

where Cov_t denotes the covariance conditional on information at time t.[23]

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[23] If m_{t+1} and R_{t+1}^e are jointly lognormally-distributed, as is typically assumed in finance, then the equation $E_t m_{t+1} R_{t+1}^e = 1$ implies $E_t r_{t+1}^e - r_t^f = -\mathrm{Cov}_t(\log m_{t+1}, r_{t+1}^e) - \frac{1}{2}\mathrm{Var}_t r_{t+1}^e$, where $r_{t+1}^e \equiv \log R_{t+1}^e$. Equation (25) says essentially the same thing without assuming lognormality.

TABLE 2: EQUITY PREMIUM AS A FUNCTION OF RISK AVERSION AND SHOCK PERSISTENCE

Risk aversion R^c	Shock persistence ρ_A	Equity premium ψ^e
10	1	0.60
30	1	1.96
60	1	4.39
90	1	7.29
60	.995	1.99
60	.99	1.19
60	.98	0.61
60	.95	0.21

Model-implied equity premium ψ^e, in annualized percentage points, for different values of relative risk aversion R^c and technology shock persistence ρ_A, holding the other parameters of the model fixed at their baseline calibrated values. State variables of the model are evaluated at the nonstochastic steady state. See text for details.

The recursive equity pricing and equity premium equations (22)–(25) can be appended to the equations of the macroeconomic model in the previous section. The equity premium (24) can then be solved numerically as described above. For the baseline calibration of the model solved to fifth order, the expected excess return to holding the equity security is 1.1 percent per quarter (or 4.39 percent at an annualized rate), evaluating the model's state variables at their nonstochastic steady-state values. Empirical estimates of the equity premium typically range from about 3 to 6.5 percent for quarterly excess returns at an annual rate (e.g., Campbell, 1999, Fama and French, 2002), so the equity premium implied by the model is consistent with the data.

The model-implied equity premium is very sensitive to both the level of risk aversion R^c and the persistence of the technology shock ρ_A. Table 2 reports values for the equity premium ψ^e for several different values of R^c and ρ_A, holding the other parameters of the model fixed at their baseline calibrated values from Table 1.

The equity premium increases about linearly along with the household's coefficient of relative risk aversion, R^c, consistent with the analysis in Swanson (2013).[24] Perhaps more surprising is the substantial drop in the equity premium for values of ρ_A that are only slightly less than unity—for example, reducing ρ_A from 1 to .995 reduces the equity premium in the model by more than half. There are two reasons why the premium ψ^e is so sensitive to ρ_A: First, equity is very long-lived, so it is sensitive to changes in consumption even at distant horizons. Second, the household's value

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[24] The equity premium increases linearly with risk aversion to second order around the nonstochastic steady state. The equity premium in Table 2 is computed to fifth order and thus is not strictly linear in risk aversion, but the intuition from the analysis in Swanson (2013) still holds.
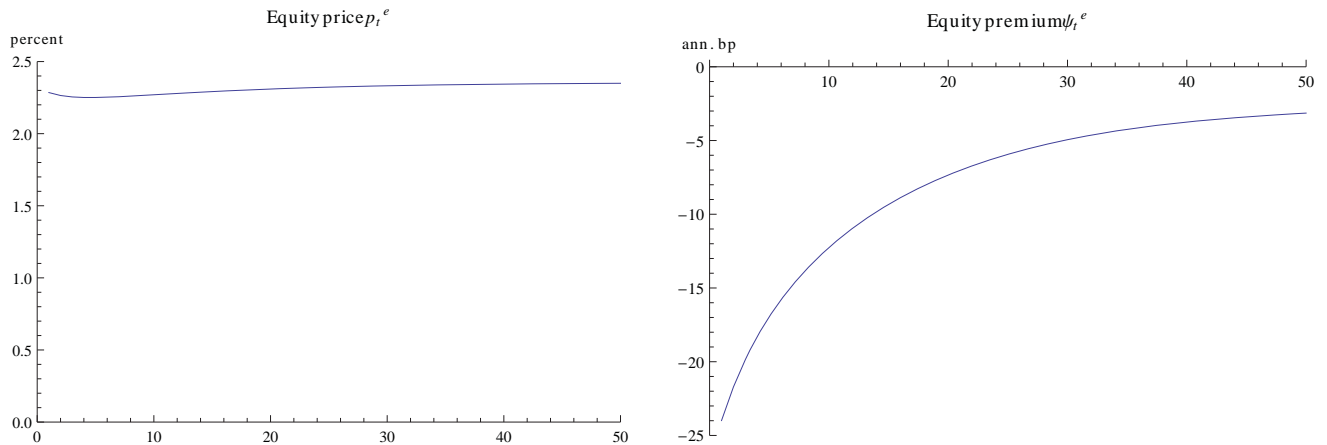
Equity price p_t^e

percent

Equity premium ψ_t^e

ann. bp

Figure 2. Nonlinear impulse response functions for equity price p_t^e and equity premium ψ_t^e to a one-standard-deviation (0.7 percent) technology shock in the model, with state variables initialized to their nonstochastic steady state values. See text for details.

function V_t, which enters into the stochastic discount factor (7), is also sensitive to consumption at long horizons. Reductions in ρ_A below unity have a very large effect on consumption at distant horizons, and thus have a large effect on the contemporaneous response of both the equity price and the stochastic discount factor to shocks, reducing the equity premium. The long-run risks literature, beginning with Bansal and Yaron (2004), takes advantage of this fact to increase the equity premium by making long-run consumption even more volatile than is implied by the random-walk technology process used here; as a result, they are able to generate a large equity premium with a lower value for risk aversion.

The equity premium in the model also varies substantially over time. Figure 2 plots the impulse responses of the equity price (22) and the equity premium (24) to the technology shock. In contrast to the responses reported in Figure 1, here the impulse responses are for the full nonlinear solution to the model, starting from an initial condition in which all of the state variables are at their nonstochastic steady-state values.[25]

The left-hand panel of Figure 2 graphs the response of the equity price, which jumps about

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[25] The impulse responses in Figure 2 are computed using the fifth-order solution to the model, as follows. The state variables of the model are initialized to their nonstochastic steady-state values. The impulse response function is computed as the period-by-period difference between a "one-shock" and a "no-shock" (baseline) scenario. In the one-shock scenario, ε_t is set equal to .007 in period 0, and equal to 0 from period 1 onward. In the no-shock scenario, ε_t is set equal to 0 in every period. Agents in the model do not have perfect foresight, so they still act in a precautionary manner even though the realized shocks turn out to be deterministically equal to 0 from period 1 onward ex post. In principle, the nonlinear impulse response functions graphed in Figure 2 can vary as one varies the initial point of the simulation, or may scale nonlinearly as one varies the size of the shock ε_t. In practice, however, reasonable variations of the initial point did not lead to economically meaningful variation in the size or shape of the impulse functions, and variations in the shock size did not lead to meaningful nonlinearities in the scale of the response or its shape.

2.3 percent on impact as the expected path of future dividends increases. The impulse response is remarkably flat after impact, due to two offsetting forces: First, the equity price is pushed upward over time as consumption increases and the short-term real interest rate r_t decreases (see Figure 1). However, the equity price is also pushed downward over time as the equity premium—graphed in the right-hand panel of Figure 2—rises back toward its initial level. (All else equal, as the equity premium rises, the price of the equity tends to fall.)

On impact, the equity premium in the left-hand panel of Figure 2 drops about 25 basis points (bp) at an annual rate. It then rises slowly back toward its initial level. Over the course of a year, the standard deviation of the equity premium (the *expected* excess return on equity) in the model is about 42 bp (obtained by summing the squares of the first four quarters of the impulse response and taking the square root).

To compare this estimate to the data, it is useful to express it in terms of the Sharpe ratio, $\psi_t^e / \sqrt{\mathrm{Var}_t r_{t+1}^e}$, which is the standard measure used in the empirical literature. The average quarterly, non-annualized Sharpe ratio implied by the model is $1.1/2.3 = 0.48$, which is a bit higher than the typical estimates of 0.2 to 0.4 in the literature (e.g., Campbell and Cochrane, 1999; Lettau and Ludvigson, 2010). This is not surprising, since the model here is driven by a single shock and thus understates the overall volatility of equity prices; adding a monetary policy shock to the model, for example, would increase the volatility of equity without much altering its excess return (because monetary policy shocks are much less persistent than technology shocks), and lead to a lower Sharpe ratio more in line with the data.

The quarterly standard deviation of the (non-annualized) Sharpe ratio in the model is about $0.25/2.3 = 0.11$. This is very similar to the standard deviation of 0.09 in Campbell and Cochrane (1999), for example, but is substantially less than the (very high) empirical estimate of 0.47 in Lettau and Ludvigson (2010, Table 11.7). Indeed, the latter authors emphasize how volatile their estimate of the time-varying Sharpe ratio is. While the model presented here does not match this higher value, it is consistent with other calibrations and results in the literature, such as those cited by Lettau and Ludvigson (2010).

The quarterly standard deviation of *realized* excess returns to holding equity is essentially 2.3 percent per quarter, or 4.6 percent at an annual rate. This is substantially less than the estimated value of about 6 percent in Lettau and Ludvigson (2010), but again is not surprising given that the simple model here is driven entirely by a single type of shock. Adding additional shocks to the model, as is standard in the medium-scale New Keynesian DSGE literature (e.g.,

Smets and Wouters, 2007), would bring equity price volatility closer to the data.

Looking back at equation (25), the decline in the equity premium in Figure 2 must be due to a drop in the conditional covariance of the equity price with the stochastic discount factor. In other words, the model generates *endogenous* conditional heteroskedasticity in response to shocks, even though the exogenous technology shocks that drive the model are homoskedastic. This is a striking and very important feature of the model. The mechanism works as follows: when consumption increases in response to a technology shock, the value function (5) is shifted upward additively rather than multiplicatively, because of the household's logarithmic preferences over consumption and additive separability in the period utility function (1). But an additive increase in V_{t+1} causes the conditional volatility of $V_{t+1}/\left(E_t V_{t+1}^{1-\alpha}\right)^{1/(1-\alpha)}$ to decline, because it increases the size of the denominator without increasing the size of shocks to the numerator.[26] The end result is that the stochastic discount factor displays endogenous conditional heteroskedasticity, with an increase in consumption leading to a decrease in conditional volatility, and vice versa.[27]

Note that additive separability of consumption in the period utility function (1) is the main driver of endogenous conditional heteroskedasticity in the model. Without additive separability of consumption, preferences would be closer to homothetic and the model would be more homogeneous—and homoskedastic—in response to shocks. In a perfectly homogeneous, homoskedastic model—such as the ones typically used in finance that have no labor—the only way to generate a time-varying equity premium is for the exogenous driving shock itself to be conditionally heteroskedastic (see, e.g., Bansal and Yaron, 2004).

Another interesting feature of the equity premium impulse response in Figure 2 is that it remains permanently lower in response to the shock, by about 3 bp. That is, the equity premium in the model is slightly nonstationary, due to the logarithmic form of household's preferences (1) combined with (4). The reason for this is essentially the same as above: the permanently higher level of consumption in response to the shock leads to a permanent additive increase in the level

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[26] In other words, a multiplicative shock that raises or lowers V_t by a factor of 2 has no effect on the conditional volatility of $V_{t+1}/\left(E_t V_{t+1}^{1-\alpha}\right)^{1/(1-\alpha)}$ with respect to additional multiplicative shocks, because the numerator and denominator of the ratio are scaled equally. In contrast, a positive additive shock to V_t of 1 util reduces the conditional volatility of the ratio, because it makes the denominator larger without affecting the size of additional additive shocks to the numerator. Similarly, a negative shock to V_t of 1 util increases the volatility of the ratio.

[27] Stock prices, in contrast, are roughly homoskedastic in response to shocks, as consumption itself is essentially homoskedastic. (Neither consumption nor stock prices are perfectly homoskedastic in the model, as the nonlinear solutions for these variables include higher-order terms of the form $x_t \varepsilon_{t+1}$, which as discussed previously, are conditionally heteroskedastic; nevertheless, the response of consumption to technology shocks in the model is close to log-linear, so the higher-order terms have only small effects on these variables.) Thus, the conditional heteroskedasticity in consumption and stock prices is small, and the change in conditional volatility of the stochastic discount factor described above passes through essentially one-for-one to a change in the conditional covariance (25).

of V_t, due to the households' logarithmic preferences over consumption in period utility (1). The additively higher level of V_t reduces the volatility of the stochastic discount factor with respect to future additive shocks. This reduced volatility makes the equity premium permanently lower, according to (25).[28]

Of course, whether the equity premium in the model is literally nonstationary or not depends on whether $\rho_A = 1$ or $\rho_A < 1$. The value $\rho_A = 1$ was chosen for simplicity and consistency with much of the finance literature, but empirically there is little reason to prefer $\rho_A = 1$ as opposed to values that are slightly less than unity (see, e.g., Christiano and Eichenbaum, 1990). The main point that should be taken away from Figure 2 is not that the equity premium in the model is literally nonstationary, since that depends on very low-frequency properties of the model that are essentially unobservable empirically. Instead, the interesting feature of Figure 2 is that the very simple macroeconomic model developed above naturally generates an equity premium that varies endogenously and substantially in response to shocks. Moreover, consistent with the conventional wisdom in the literature (e.g., Fama and French, 1989; Campbell and Cochrane, 1999), the equity premium in the model is countercyclical.

3.2 Real and Nominal Default-Free Bonds

A default-free zero-coupon real bond in the model pays one unit of consumption at maturity. Let $p_t^{(n)}$ denote the price of an n-period zero-coupon real bond, with $p_t^{(0)} \equiv 1$. Then for $n \geq 1$,

$$p_t^{(n)} \;=\; E_t \, m_{t+1} p_{t+1}^{(n-1)} \tag{26}$$

in each period t. In particular, $p_t^{(1)} = e^{-r_t}$.

A default-free zero-coupon nominal bond pays one nominal dollar at maturity. Letting $p_t^{\$(n)}$ denote the price of an n-period zero-coupon nominal bond, with $p_t^{\$(0)} \equiv 1$, then

$$p_t^{\$(n)} \;=\; E_t \, m_{t+1} e^{-\pi_{t+1}} p_{t+1}^{\$(n-1)} \tag{27}$$

in each period t. In particular, $p_t^{\$(1)} = e^{-i_t}$.

More generally, let $r_t^{(n)}$ denote the n-period continuously-compounded yield to maturity on a real zero-coupon bond, and $i_t^{(n)}$ the corresponding yield on an n-period nominal bond. Then

$$r_t^{(n)} \;=\; -\frac{1}{n} \, \log p_t^{(n)}, \tag{28}$$

---

[28] See Section 2.4 and the Appendix for additional discussion. Also note that the shape of the impulse response function, which is large on impact and diminishes over time, is related to the path of the real interest rate, since equation (25) implies $\psi_t^e = -e^{r_t} \mathrm{Cov}_t(m_{t+1}, r_{t+1}^e)$.

$$i_t^{(n)} = -\frac{1}{n} \log p_t^{\$(n)}. \tag{29}$$

Even though the bonds in this section are free from default, they are risky in the sense that their prices can fluctuate in response to shocks, for $n > 1$. The risk premium on a bond is typically written as a *term premium,* the difference between the yield to maturity on the bond and the hypothetical, risk-neutral yield to maturity on the same bond. For example, the risk-neutral price $\hat{p}_t^{(n)}$ of an n-period zero-coupon real bond is given by

$$\hat{p}_t^{(n)} = e^{-r_t} E_t \, \hat{p}_{t+1}^{(n-1)}, \tag{30}$$

where $\hat{p}_t^{(0)} \equiv 1$. Thus,

$$
\begin{aligned}
\hat{p}_t^{(n)} - p_t^{(n)} &= E_t m_{t+1} E_t \hat{p}_{t+1}^{(n-1)} - E_t m_{t+1} p_{t+1}^{(n-1)} \\
&= -\mathrm{Cov}_t\big(m_{t+1}, p_{t+1}^{(n-1)}\big) + e^{-r_t} E_t\big(\hat{p}_{t+1}^{(n-1)} - p_{t+1}^{(n-1)}\big) \\
&= -E_t \sum_{j=0}^{n-1} e^{-r_{t,t+j}} \mathrm{Cov}_{t+j}\big(m_{t+j+1}, p_{t+j+1}^{(n-j-1)}\big),
\end{aligned}
\tag{31}
$$

where $r_{t,t+j} \equiv \sum_{\tau=t+1}^{t+j} r_\tau$, and the last line of (31) follows from forward recursion. Equation (31) shows that, even though the bond price depends only on the one-period-ahead covariance between the stochastic discount factor and next period's bond price, the risk premium on the bond depends on this covariance over the entire lifetime of the bond.

Let $\psi_t^{(n)}$ denote the term premium on the bond. Then

$$
\begin{aligned}
\psi_t^{(n)} &\equiv \frac{1}{n}\big(\log \hat{p}_t^{(n)} - \log p_t^{(n)}\big) \\
&\approx \frac{1}{n\overline{p}^{(n)}}\big(\hat{p}_t^{(n)} - p_t^{(n)}\big) \\
&= -\frac{1}{n\overline{p}^{(n)}} E_t \sum_{j=0}^{n-1} e^{-r_{t,t+j}} \mathrm{Cov}_{t+j}\big(m_{t+j+1}, p_{t+j+1}^{(n-j-1)}\big),
\end{aligned}
\tag{32}
$$

where $\overline{p}^{(n)}$ denotes the steady-state bond price.[29] Intuitively, the term premium is larger the more negative the covariance between the stochastic discount factor and the price of the bond over the lifetime of the bond. The formula for the term premium on a nominal n-period bond, $\psi_t^{\$(n)}$, is analogous.

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[29] The first-order approximation on the first line of (32) is useful for gaining intuition. However, when I solve for bond prices and risk premia in the model numerically, the solution will always include second-, third-, and higher-order terms as well as first-order terms.

Table 3: Real Zero-Coupon Bond Yields, Data vs. Model

	2-yr.	3-yr.	5-yr.	7-yr.	10-yr.	(10y)−(2y)
US TIPS, 1999–2013[a]			1.45	1.71	1.97	
US TIPS, 2004–2013[a]	0.29	0.41	0.72	1.01	1.34	1.05
US TIPS, 2004–2007[a]	1.39	1.52	1.74	1.91	2.09	0.70
UK indexed gilts, 1983–1995[b]	6.12	5.29	4.34		4.12	−2.00
UK indexed gilts, 1985–2013[c]	2.19	2.15	2.26	2.35	2.44	0.25
UK indexed gilts, 1990–2007[c]	2.82	2.77	2.78	2.79	2.80	−0.02
macroeconomic model	1.92	1.89	1.84	1.81	1.77	−0.15

[a]Gürkaynak, Sack, and Wright (2010) online dataset.
[b]Evans (1999).
[c]Bank of England web site.
Estimated zero-coupon real yields from inflation-indexed bonds in the U.S. and U.K., and zero-coupon real yields implied by the macroeconomic model presented above. The last column reports the difference between the 10-year and 2-year yields in each row. See text for details.

The bond pricing and bond yield equations (26)–(30) are recursive and can be appended to the macroeconomic model above and solved numerically along with the macroeconomic variables, equity price, and equity premium. (Note that, to consider a bond with n periods to maturity, $n-1$ bond pricing equations must be appended to the model, one for each maturity from 2 to n.)

Table 3 reports the real yield curve implied by the model, along with the corresponding average real yields estimated from inflation-indexed government bonds in the U.S. and U.K. over different sample periods. Data for U.S. inflation-indexed Treasuries (TIPS) are taken from Gürkaynak, Sack, and Wright (2010). The first TIPS were issued in 1998, and a yield curve for maturities of 5 years or more can be estimated beginning in 1999. The first row of Table 3 thus reports average TIPS yields from 1999 to 2013. Real yields over this sample averaged about 1.5 to 2 percent per year. Zero-coupon yields for shorter-maturity TIPS (down to 2 years; neither Gürkaynak et al., 2010, nor the Bank of England report zero-coupon real yields with a maturity less than 2 years) can be estimated beginning in 2004, and are reported in the second row of Table 3, along with the average yields for longer maturities over the same sample. This sample also excludes the period of lower TIPS liquidity in the first few years after they were issued. Over this sample, average real yields are lower, between about 0.3 and 1.3 percent. However, the period from 2008–13 is unusual in that the financial crisis and severe recession led the Federal Reserve to reduce short-term interest rates to record lows, and to some extent we might expect this to show up in shorter-term real yields as well, both as a lower level of yields and as a steeper yield curve slope. Thus, the third row of Table 3 reports results from 2004–07, a short sample, but

one that avoids both the low liquidity of TIPS in its first few years and the financial crisis and recession. Over this sample, real yields average between about 1.5 and 2 perent.

However, this is a short sample and the period from 2004 to 2005 was also characterized by very easy monetary policy and a very low level of short-term U.S. yields as the Federal Reserve worked to facilitate recovery from the 2001 recession. Thus, the next three rows of Table 3 report average real yields on inflation-indexed gilts in the U.K. Indexed gilts have traded since at least the early 1980s, so we have a much longer sample of data with which to estimate real U.K. yields. Evans (1999) estimates real zero-coupon U.K. yields from 1983 to 1995, reported in the fourth row of Table 3, which average between about 4 and 6 percent over that sample. Interestingly, the real U.K. gilt yield curve slopes *downward* rather than upward over this period, by about 200 basis points. However, as in the U.S., the early years of the U.K. index-linked gilt market may have suffered from low liquidity and correspondingly higher yields. Thus, the fifth row of Table 3 reports estimated real yields from 1985 to the present, from the Bank of England's web site. Over this longer sample, real U.K. yields average about 2.2 to 2.4 percent, and the yield curve sloped upward by about 25 bp. The sixth row of Table 3 reports results for the U.K. excluding both the early years of the sample and the financial crisis and recession period, for the same reasons as for the U.S. Over this sample, 1990–2007, real yields in the U.K. are a bit higher, averaging about 2.8 percent, and the yield curve is about flat, sloping downward by 2 bp.

While the level of real yields and slope of the real yield curve is somewhat sensitive to sample period and whether one looks at the U.S. or U.K., the macroeconomic model presented above is able to fit the basic patterns of real yields seen in the data. Real yields in the model average a bit less than 2 percent under the baseline calibration, evaluating the model's state variables at the nonstochastic steady state.[30] The model also implies that the real yield curve is about flat or even slightly downward-sloping, with the spread between the 10-year and 2-year real bond averaging about −15 bp.

The downward-sloping real yield curve implied by the model is a standard feature of traditional real-business-cycle studies, such as Backus, Gregory, and Zin (1989), Donaldson, Johnsen, and Mehra (1990) and Den Haan (1995). Intuitively, if short-term real interest rates fall in recessions, then the price of a long-term real bond will tend to rise in recessions, which is exactly when

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[30]Note that β alone would imply a real yield of almost 4 percent in the nonstochastic steady state. However, the real yield $r_t = 1/E_t m_{t+1}$, and $E_t m_{t+1}$ is substantially greater than $1/\beta$ due to Jensen's inequality terms. This is true even though r_t itself is a risk-free interest rate. Intuitively, households' aversion to risk drives up their demand for the riskless asset, lowering the risk-free rate below its nonstochastic steady-state value.
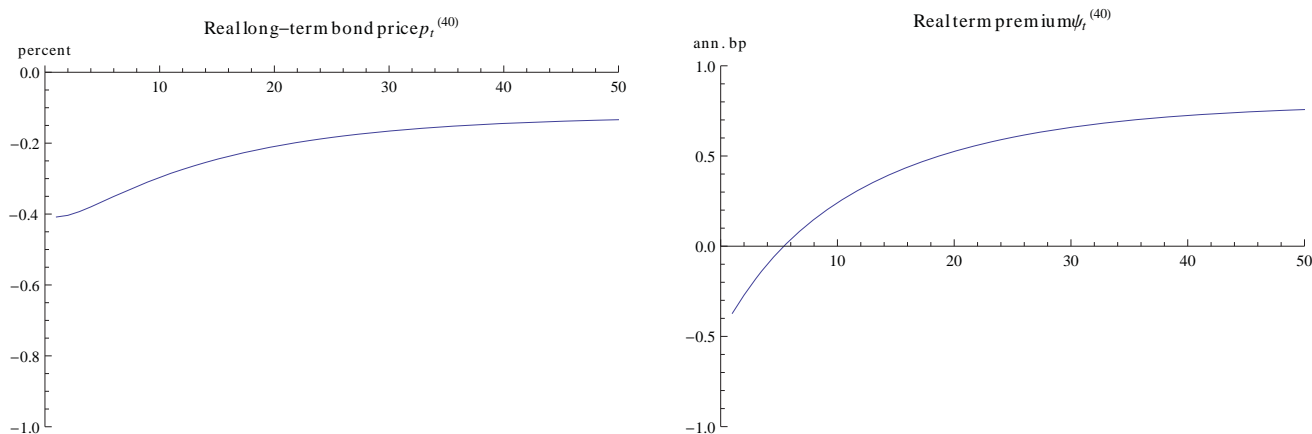
Figure 3. Nonlinear impulse response functions for real long-term bond price $p_t^{(40)}$ and term premium $\psi_t^{(40)}$ to a one-standard-deviation (0.7 percent) technology shock in the model, with state variables initialized to their nonstochastic steady state values. See text for details.

households value consumption the most. Thus, long-term real bonds act somewhat like recession insurance, and should carry a negative risk premium. In the macroeconomic model developed here, the response of the short-term real interest rate to the shock is fairly small (see Figure 1), only about 5 bp. As a result, the price of a real long-term bond is not very countercyclical and the insurance properties of the bond are relatively minor, resulting in only a small negative risk premium.

Figure 3 reports nonlinear impulse response functions for the long-term (10-year) real bond price and term premium, computed the same way as for the equity price and equity premium. The impulse response of the bond price confirms the intuition in the preceding paragraph, falling only about 0.4 percent, reflecting the small change in short-term real rates in Figure 1. The real term premium also move very little, falling about one-half of one basis point on impact. However, like the equity premium, the real term premium in the model is slightly nonstationary, rising permanently by about 0.8 bp after the shock. The reason here is essentially the same as for the equity premium: the permanently higher level of consumption in response to the shock leads to a permanent additive increase in the level of V_t, which reduces the volatility of the stochastic discount factor with respect to future additive shocks. The reduced volatility of the stochastic discount factor makes the negative real long-term bond premium slightly less negative, leading to the small permanent increase seen in Figure 3.

Table 4 reports the level of nominal yields in the data and implied by the model. Gürkaynak, Sack, and Wright (2007) estimate zero-coupon nominal Treasury yields for the U.S. going back to

TABLE 4: NOMINAL ZERO-COUPON BOND YIELDS, DATA VS. MODEL

	1-yr.	2-yr.	3-yr.	5-yr.	7-yr.	10-yr.	(10y)−(1y)
US Treasuries, 1961–2013[a]	5.44	5.66	5.84	6.11	6.23		
US Treasuries, 1971–2013[a]	5.64	5.87	6.07	6.38	6.62	6.89	1.25
US Treasuries, 1990–2007[a]	4.56	4.84	5.06	5.41	5.68	5.98	1.42
UK gilts, 1970–2013[b]	7.22	7.39	7.55	7.79	7.97	8.15	0.93
UK gilts, 1990–2007[b]	6.20	6.30	6.38	6.48	6.51	6.50	0.30
macroeconomic model	4.52	4.79	5.01	5.36	5.60	5.84	1.32

[a]Gürkaynak, Sack, and Wright (2007) online dataset.
[b]Bank of England web site.
Empirical estimates of zero-coupon nominal yields from government bonds in the U.S. and U.K., and zero-coupon nominal yields implied by the macroeconomic model presented above. The last column reports the difference between the 10-year and 1-year yield in each row. See text for details.

1961 for maturities out to 7 years, and 1971 for maturities out to 10 years. Over the 1961–2013 sample, nominal yields averaged about 5.5 to 6.25 percent. From 1971 to 2013, the average is a bit higher, about 5.5 to 7 percent, with an average yield curve slope of about 125 bp. Just as for real yields, though, the period from 2008–13 may be atypical in that short-term interest rates hit record lows in response to the financial crisis and recession. The "Great Inflation" period of the 1970s and early 1980s may also be problematic in that monetary policy may have experienced a structural break since that period and is now conducted in a more aggressive anti-inflationary manner (e.g., Clarida, Galí, and Gertler, 1999). Thus, the third row of Table 4 reports average yields from 1990 to 2007, a period that excludes both the Great Inflation and recent Great Recession periods. Over this sample, nominal Treasury yields averaged about 4.5 to 6 percent, with a yield curve slope of about 140 bp.

The Bank of England also reports zero-coupon yield curve estimates for the U.K. going back to 1970. From 1970 to 2013, nominal gilt yields in the U.K. averaged between about 7.2 and 8.2 percent, with a yield curve slope of about 93 bp, as reported in the fourth row of Table 4. Restricting attention to the period from 1990 to 2007, for the same reasons as above, average U.K. nominal yields are a bit lower, about 6.2 to 6.5 percent, with a slope of just 30 bp.

Again, the model is able to reproduce these features of the data quite well. Evaluating the model's state variables at their nonstochastic steady state values, the average level of nominal yields is between about 4.5 and 5.8 percent, with a yield curve slope of about 130 bp. Interestingly, while the model-implied real yield curve slopes downward, the model-implied nominal yield curve slopes upward substantially. As discussed at length by Rudebusch and Swanson (2012), this is

because technology shocks in the model make nominal bonds risky. A negative technology shock causes inflation to rise persistently at the same time that consumption falls. As a result, long-term nominal bonds in the model *lose* value in recessions, exactly the opposite of long-term real bonds. This implies that long-term nominal bonds should carry a substantial risk premium, about 150 bp over the corresponding risk-neutral yield. Thus, the simple model presented here provides a straightforward answer to the puzzle posed by Backus, Gregory, and Zin (1989), Donaldson, Johnsen, and Mehra (1990), and Den Haan (1995): namely, why does the nominal yield curve slope upward? The answer is technology shocks, or more generally, any type of "supply shock" that causes inflation to move inversely with output (such as an oil price shock or markup shock, which are not modeled here).

Of course, the larger and more important are technology or supply shocks in the model, the larger the term premium on nominal bonds will be. Thus, if supply shocks were relatively larger in the 1970s and early 1980s than in the 1960s or more recently, we should see a larger term premium on nominal bonds in those periods when supply shocks were larger. And in fact, this prediction seems to be consistent with the data: Rudebusch, Sack, and Swanson (2007) graph several measures of the term premium—from a VAR, affine no-arbitrage models with latent or observable factors, or the Cochrane-Piazzesi (2005) "tent-shaped" predictor of excess returns—and for all of these measures, the estimated term premium on long-term nominal bonds in the U.S. is higher in the 1970s and early 1980s than in the 1960s or more recently.

Campbell, Sundaram, and Viceira (2013) also document changing correlations between stock and nominal bond returns over time. Although the simplified model of the present paper considers only technology shocks, extending the model to allow for other shocks, such as fiscal shocks or monetary policy shocks, is straightforward, and would bring the model closer to standard medium-scale New Keynesian DSGE models such as Smets and Wouters (2007) and Levin et al. (2005), which consider a variety of shocks. In these models, if the relative importance of technology or supply shocks—which move inflation inversely to consumption—is varied, then size of the term premium and the correlation of excess bond returns with excess stock returns will vary as well. Thus, changing correlations of stock and bond returns can be mapped back to more fundamental features of the model.

Figure 4 reports the nonlinear impulse response functions for the long-term (10-year) nominal bond price and term premium to a one-standard-deviation technology shock, computed in the same way as for the real bond and equity. As discussed earlier, a positive technology shock causes
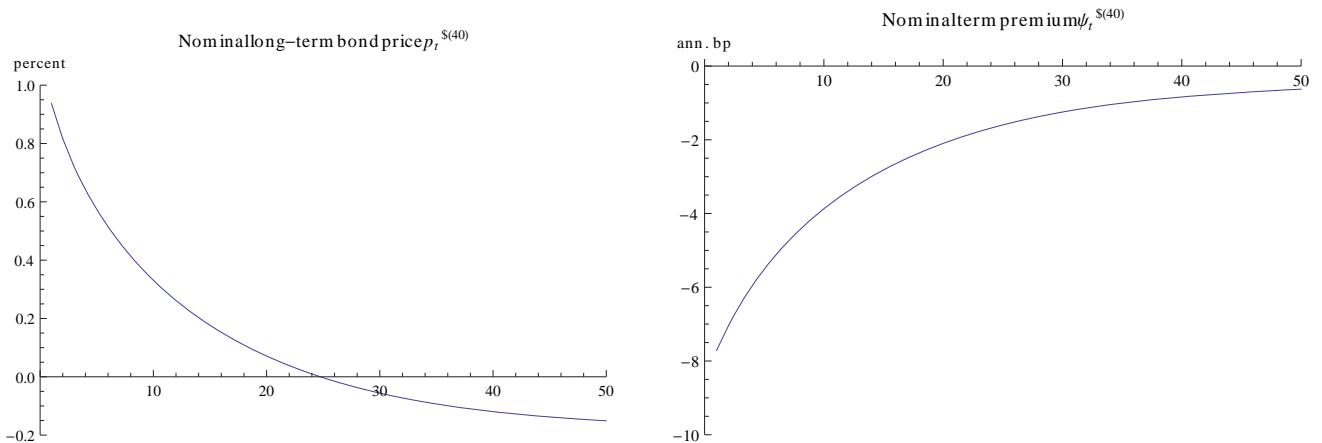
Figure 4. Nonlinear impulse response functions for nominal long-term bond price $p_t^{\$(40)}$ and term premium $\psi_t^{\$(40)}$ to a one-standard-deviation (0.7 percent) technology shock in the model, with state variables initialized to their nonstochastic steady state values. See text for details.

inflation and the short-term nominal interest rate to fall (Figure 1) and the nominal long-term bond price to rise substantially (Figure 4), almost 1 percent on impact. The nominal term premium falls about 8 bp. The reason for this drop is essentially the same as for the equity premium: the rise in consumption leads to an additive increase in the household's value function V_t, which reduces the volatility of the stochastic discount factor, making the bond less risky. After the initial impact, the term premium gradually rises back toward its initial level.[31] Over the course of a year, the standard deviation of the term premium is about 14 bp.

Estimates of the quarterly standard deviation of the term premium in the data range between about 8 and 40 bp—see, e.g., the survey of empirical estimates in Rudebusch, Sack, and Swanson (2007)—so the time-variation in Figure 4 is at the lower end of this range, but is arguably consistent. Standard three-latent-factor affine arbitrage-free models, such as Kim and Wright (2005), imply a quarterly standard deviation of about 30–35 bp, but Rudebusch and Wu (2007) argue that these highly-parameterized models tend to overfit the high-frequency fluctuations in long-term yields, and that fluctuations in the term premium are smaller, only about 8 bp from quarter to quarter.

Like the equity premium, the nominal term premium in the model is slightly nonstationary, remaining permanently below its initial level by about 0.5 bp. The intuition is the same as for the equity premium: the permanent increase in consumption after the shock causes a permanent

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[31] As with the equity premium, the shape of the impulse response function here is related to the short-term nominal interest rate through the nominal version of equation (32). In particular, as the short-term nominal interest rate rises back toward steady state, the covariance terms in (32) are discounted less, allowing the overall sum in (32) to rise.

additive increase in V_t, which reduces the conditional volatility of the stochastic discount factor. The lower volatility of the stochastic discount factor reduces the covariance terms in (32) and leads to a permanently lower term premium.

As with the equity premium, the important point to take away from Figure 4 is that the very simple macroeconomic model developed here is able to generate a term premium that varies endogenously and substantially in response to shocks. Consistent with the evidence in Fama and French (1989) and conventional wisdom in the literature (e.g., Campbell and Cochrane, 1999), the term premium in the model is countercyclical.

3.3 Defaultable Bonds

The simple macroeconomic model above is capable of matching the risk premium on defaultable bonds as well. For simplicity, I model a defaultable bond as a slowly depreciating consol that has some probability of defaulting each period. The *credit spread* in the model is the difference in yield between the defaultable consol and an otherwise identical consol that is free from default. I consider two cases in the analysis below: first, where the probability of default is constant over time, and second, where the probability of default varies countercyclically.

A default-free consol is an infinitely-lived bond that pays a geometrically declining coupon of δ^n nominal dollars in each period $n = 1, 2, \ldots$ after issuance. The ex-coupon price p_t^c of the bond in period t is given in equilibrium by

$$p_t^c = E_t \, m_{t+1} e^{-\pi_{t+1}} (1 + \delta p_{t+1}^c), \tag{33}$$

where the size of the next coupon payment is normalized to one dollar. The very simple recursive structure of (33) makes this type of long-term bond extremely convenient to work with and generalizes naturally to the case where the bond may default, considered shortly.[32] When $\delta = 0$, the consol reduces to a one-period zero-coupon bond, and when $\delta = 1$, it behaves like a traditional nondepreciating consol. By choosing δ appropriately, the depreciating consol can be given any desired Macauley duration and made to behave very similarly to the corresponding zero-coupon bond.

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[32] In the finance literature, Leland (1994), Duffie and Lando (2001), and Chen (2010) use a nondepreciating consol to model corporate bonds, while Leland (1998) uses a depreciating consol. Rudebusch and Swanson (2008) use a (default-free) depreciating consol to study the long-term bond premium puzzle. The behavior of the depreciating consol in the simple model above and in Rudebusch and Swanson (2008) is very similar to that of a zero-coupon bond with the same Macauley duration. Nevertheless, the preceding section (and Rudebusch and Swanson, 2012) use zero-coupon bonds rather than depreciating consols to maximize comparability to the finance literature.

The continuously-compounded yield to maturity, i_t^c, for the consol satisfies

$$p_t^c = \frac{1}{e^{i_t^c}} + \frac{\delta}{e^{2i_t^c}} + \frac{\delta^2}{e^{3i_t^c}} + \cdots, \tag{34}$$

implying

$$i_t^c = \log\left(\frac{1}{p_t^c} + \delta\right). \tag{35}$$

The Macauley duration of the consol is given by

$$-\frac{d\log p_t^c}{di_t^c} = 1 + \delta p_t^c. \tag{36}$$

When calibrating the model below, I set δ so that the consol has a Macauley duration of 10 years, corresponding to the approximate duration of the longer-term coupon bonds in Moody's indexes.

A defaultable consol pays a nominal coupon each period in the same way as a default-free consol, but in addition there is a chance each perid that the bond will default and cease paying interest forever. In the event of default, bondholders receive some recovery rate times the previous value of the bond, which can be calibrated to the data. Thus, the defaultable consol price p_t^d satisfies

$$p_t^d = E_t\, m_{t+1} e^{-\pi_{t+1}}\left[(1 - \mathbf{1}_{t+1}^d)(1 + \delta p_{t+1}^d) + \mathbf{1}_{t+1}^d\, \omega_{t+1}\, p_t^d\right], \tag{37}$$

where $\mathbf{1}_t^d$ is an indicator variable equal to 1 if the bond defaults in period t and 0 otherwise, and ω_t denotes the recovery rate on the bond in the event of default. The yield to maturity i_t^d and duration of the defaultable bond are defined by equations (35)–(36), with p_t^d in place of p_t^c. The credit spread is the yield differential, $i_t^d - i_t^c$.

Since δ is set to match the maturity of the bond, it remains to calibrate $\Pr_t\{\mathbf{1}_{t+1}^d = 1\}$ and ω_t in (37). The average rate of default for bonds initially rated Baa or BBB is about 0.6 percent per year (e.g., Moody's, 2006; Standard & Poor's, 2014), and the average recovery rate on defaulted bonds is about 42 percent (Chen, Collin-Dufresne, and Goldstein, 2009; Chen, 2010).[33] As a first calibration, then, I set $\Pr_t\{\mathbf{1}_{t+1}^d = 1\}$ to an exogenous, constant rate of 0.15 percent per quarter, and ω_t to an exogenous constant of 42 percent.

The credit spread implied by the model for this calibration is reported in the first row of Table 5. With an average annual default probability of 0.6 percent that is constant over time, the

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[33] The default rate on bonds *currently* rated Baa/BBB is much lower, about 0.15 percent per year on average. However, these bonds also lose value when they are downgraded, which happens with much higher probability. Rather than keep track of credit ratings, the probability of downgrades, and capital losses in the event of downgrade, I simply keep track of the default rate for bonds *initially* rated Baa/BBB.

TABLE 5: MODEL-IMPLIED CREDIT SPREAD ON DEFAULTABLE BONDS

average ann. default prob.	cyclicality of default prob.	average recovery rate	cyclicality of recovery rate	credit spread (bp)
.006	0	.42	0	34.4
.006	−0.3	.42	0	125.5
.006	−0.3	.42	2.5	136.8
.006	−0.15	.42	2.5	77.1
.006	−0.6	.42	2.5	345.7
.006	−0.3	.42	1.25	131.2
.006	−0.3	.42	5	148.1

Model-implied credit spread $i_t^d - i_t^c$ for defaultable vs. default-free depreciating consols with Macauley duration of 10 years. Average annualized default probability is calibrated to bonds initially rated Baa. Cyclicality of default probability and recovery rate are the loadings on the output gap, $y_t - \overline{y}_t$. See text for details.

model-implied credit spread is about 34.4 bp. This is essentially the risk-neutral expected loss each period from default, $(.006)(.58) = 34.8$ bp, and is far less than the historical average credit spread on Baa-rated bonds of about 120 bp (e.g., Chen, Collin-Dufresne, and Goldstein, 2009; Chen, 2010).[34] Intuitively, if the risk of default in the model is uncorrelated with the stochastic discount factor, there is no additional risk premium attached to losses from default.

Empirically, however, corporate bond defaults are highly countercyclical and recovery rates highly procyclical (see, e.g., Chen, 2010; Giesecke, Longstaff, Schaefer, and Strebulaev, 2011; Standard & Poor's, 2011). For example, in Figure 1 of Chen (2010), the default rate averages about 0.9 percent over the postwar period, but spikes to about 3.7 percent in 1990, 4 percent in 2001, and 5.5 percent in 2009, with smaller spikes in earlier recessions (and a spike to 8.5 percent in 1933). In boom years, the default rate falls to essentially zero. Recovery rates average about 42 percent after 1982, the period for which we have data, but drop to about 20 or 25 percent in 1990, 2001, and 2009, while in boom years, recovery rates are 50 or 60 percent.

Thus, the next rows of Table 5 consider cases where the default rate, recovery rate, or both are correlated with the output gap in the model, $y_t - \overline{y}_t$. I calibrate the cyclicality of the model's annualized default rate to a value of −0.3, which implies a drop in output of 5 percent below

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[34] This is the average difference between the yield on Moody's Baa and Aaa seasoned corporate bond indexes from 1921–2013. The average spread over alternative sample periods is similar. The spread between Baa-rated corporate bonds and U.S. Treasuries is even larger, about 185 bp. However, U.S. Treasuries carry an additional premium for their extreme liquidity and beneficial tax treatment, so the Baa-Aaa spread is often used in the literature to measure the credit spread (since Aaa corporate bonds are similar in liquidity and tax treatment to Baa-rated bonds and the probability of default on Aaa-rated bonds is still extremely low; see, e.g., Chen, Collin-Dufresne, and Goldstein, 2009).

trend is associated with an increase in the default rate of about 1.5 percentage points. While this cyclicality is lower than in Chen (2010), my focus here is on bonds initially rated Baa/BBB, while the data in Chen (2010) is for all bonds, which includes many that were issued at ratings below investment grade.[35]

The second row of Table 5 reports the credit spread in the model when the default rate is countercyclical, holding the recovery rate constant over time. This greatly increases the model-implied credit spread, to about 125 bp, consistent with the observed spread in the data.

The third row considers the case where the recovery rate is also cyclical. I calibrate the cyclicality of the recovery rate in the model to 2.5, so that a fall in output of 5 percent below trend is associated with a roughly 12.5-percentage-point decrease in the recovery rate on defaulted corporate bonds, similar to the fluctuations reported in Chen (2010). Given this degree of cyclicality, the credit spread in the model increases a bit further, to 137 bp, still close to (and even a bit above) the value of 120 bp in the data.

The last four rows of Table 5 vary these cyclicality parameters to check their influence on the results. In the fourth row, I cut the default rate cyclicality in half to −0.15, which reduces the credit spread substantially, to 77 bp. Doubling the default cyclicality to −0.6 more than doubles the credit spread, to about 346 bp. In the last two rows, I cut the cyclicality of the recovery rate in half to 1.25, and double it to 5. The model-implied credit spread is much less sensitive to these changes, varying by just 5 and 13 bp, respectively. Intuitively, a marginal increase in the probability of default is much more costly to households, because it implies an increase in the chance of a large loss. In contrast, a marginal fall in the recovery rate implies only a very small chance (0.15 percent per quarter) of a small increase in the loss. Thus, the cyclicality of recovery rates can essentially be ignored in the model.

Figure 5 reports the nonlinear impulse response functions for the defaultable bond price and credit spread to a positive one-standard-deviation technology shock, computed in the same way as for the equity and default-free bonds. On impact, the defaultable bond price jumps about 1.5 percent, in between the response of the default-free nominal bond price and the equity claim in the initial period.

The credit spread, depicted in the right-hand panel of Figure 5, drops about 9.5 bp on impact, a little less than half as much as the fall in the equity premium. In the data, the standard

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[35] See the discussion in footnote 33. Also, to prevent the default rate in the model from becoming negative, I model it in logarithms rather than in levels. That is, the cyclicality of the log default rate is set to −50, which, when multiplied by the average default rate of .006 per year produces −0.3.
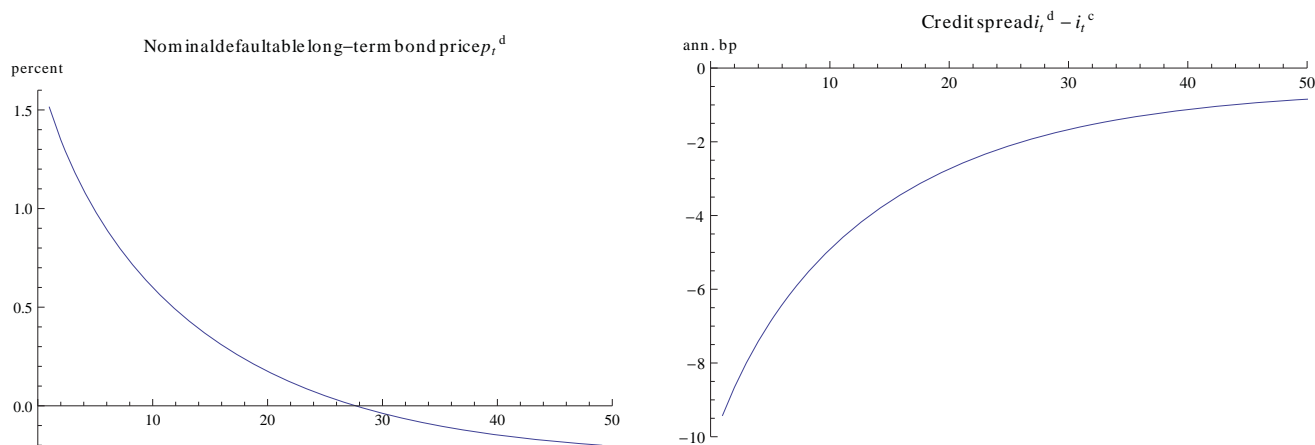
Figure 5. Nonlinear impulse response functions for defaultable long-term bond price p_t^d and credit spread $i_t^d - i_t^c$ to a one-standard-deviation (0.7 percent) technology shock in the model, with state variables initialized to their nonstochastic steady state values. See text for details.

deviation of the post-war quarterly change in the Baa-Aaa spread is about 20 bp, larger than in the model but not dramatically so. Given the simplicity of the model (and the fact that it has only one driving shock) and the very stylized definition of defaultable bonds, the variation in the credit spread fits the data remarkably well.

Over time, both the defaultable bond price and the credit spread return back toward their baseline levels, but do not return all the way to baseline owing to the slight nonstationarity of the model. As with the equity premium and default-free long-term bond premium, the permanently higher level of household wealth after the shock causes households to be slightly less risk averse.

To some extent, the model's ability to jointly fit equity and corporate bond yield data is not surprising, since Chen, Collin-Dufresne, and Goldstein (2009), Bhamra, Kuehn, and Strebulaev (2010), and Chen (2010) all achieve a similar simultaneous fit in an endowment economy setting. Nevertheless, this is the first paper to jointly match these data in a fully-specified macroeconomic model. The distinction is important because results in an endowment economy often do not carry over to the case where households can choose their consumption stream endogenously. For example, the extremely strong habit specification of Campbell and Cochrane (1999)—which is also used by Chen et al. (2009)—matches the behavior of asset prices very well in an endowment economy, but fails completely when households with these extreme preferences are given any ability to smooth consumption endogenously (see Lettau and Uhlig, 2000, and Rudebusch and Swanson, 2008).

Like the present paper, Bhamra et al. (2010) and Chen (2010) use Epstein-Zin preferences,

albeit in an endowment economy. Inflation in Bhamra et al. (2010) and Chen (2010) is also taken to be an exogenous, reduced-form process. The advantage of the present paper's structural macroeconomic approach is its ability to consider the effects of novel policy interventions and structural breaks, which cannot be studied in a reduced-form macroeconomic environment. The more serious modeling of inflation in the present paper also provides insight into issues related to the behavior of nominal vs. real assets. The advantages of the simpler, more reduced-form macroeconomic structure of the other authors is that it permits them to undertake a more detailed, structural analysis of firms' corporate financing and endogenous default decisions. In other words, I have adopted a very simplistic, reduced-form model of the firm in order to better focus on the structural behavior of the macroeconomy, while Bhamra et al. (2010) and Chen (2010) have adopted a very simplistic, reduced-form model of the macroeconomy to better focus on the structural finance behavior of the firm.

4. Discussion

In this section, I discuss the model's relationship to the literature in greater detail. First, I compare the model's assumption of a unitary intertemporal elasticity of substitution (IES) to the typical assumption that the IES $\gg 1$ in the long-run risks literature. [Additional discussion sections to be added.]

4.1 The Intertemporal Elasticity of Substitution and Stochastic Volatility

In the long-run risks literature, such as Bansal and Yaron (2004), the intertemporal elasticity of substitution is typically assumed to be substantially greater than unity. There are two main reasons for that calibration: first, an IES > 1 implies that an increase in consumption causes stock prices to rise rather than fall; and second, that an exogenous decrease in volatility causes stock prices to rise rather than fall.

The details of the macroeconomic model developed here differ from the standard long-run risks specification—for example, technology growth shocks here are i.i.d. rather than persistent, and households can vary their labor supply as well as savings if they wish to change consumption—so it is no longer necessary for the IES to be greater than unity to satisfy the two criteria mentioned above. For example, even though the IES is equal to unity, Figures 1 and 2 show that a positive shock to consumption (through technology) causes stock prices to rise, consistent with the first

criterion.[36]

To investigate the second criterion, I extend the macroeconomic model of the present paper to include exogenous stochastic volatility in the technology shock. In particular, let the standard deviation of the technology shock each period, $\sigma_{A,t}$, follow the autoregressive process

$$\log \sigma_{A,t} = (1 - \rho_\sigma) \log \bar{\sigma}_A + \rho_\sigma \log \sigma_{A,t-1} + \varepsilon_t^\sigma, \tag{38}$$

where $\bar{\sigma}_A = .007$, as in Table 1. Following Bansal and Yaron (2004), I calibrate $\rho_\sigma = 0.98$ and $\mathrm{Var}(\varepsilon_t^\sigma) = (0.1)^2$.[37]

The model's nonlinear impulse responses to a positive one-standard-deviation shock to ε_t^σ are computed the same way as in previous figures and are reported in Figure 6. Volatility σ_t^A increases to about .0077 on impact and slowly declines back toward its initial level of .007. Consumption drops by about 0.3 percent on impact, as households increase precautionary savings, and inflation falls about 0.4 percent in response to the decrease in demand. The increase in the conditional volatility of consumption greatly increases the volatility of the stochastic discount factor (since α is large) which causes a large, 100 bp jump in the equity premium. (The nominal term premium also responds subsantially to the volatility shock, increasing by about 25 bp.) The large and persistent rise in the equity premium implies that the equity price must fall dramatically on impact, about 6 percent.[38] Thus, the model satisfies the second criterion discussed above—namely, that an exogenous increase in volatility causes stock prices to decline rather than rise—without the need for an IES > 1.

[Additional discussion sections to be added.]

5. Conclusions

The simple macroeconomic model developed in this paper is consistent with a wide variety of asset pricing facts, such as the equity premium puzzle, long-term bond premium puzzle, and

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[36] This remains true even when the IES < 1. [Add some explanation as to why.]

[37] Bansal and Yaron (2004) assume a more complicated (square-root rather than logarithmic) process for $\sigma_{A,t}$ than (38), but the magnitudes in (38) are essentially comparable to theirs.

[38] In order to generate an equity premium of 100 bp in the first period, stock prices must fall by about 1 percent below their second-period value. Similarly, in order to generate an equity premium in each subsequent period, equity prices must continue to rise. This requires a large initial fall in the equity price so that in each subsequent period equity prices can rise in line with the implied equity premium.
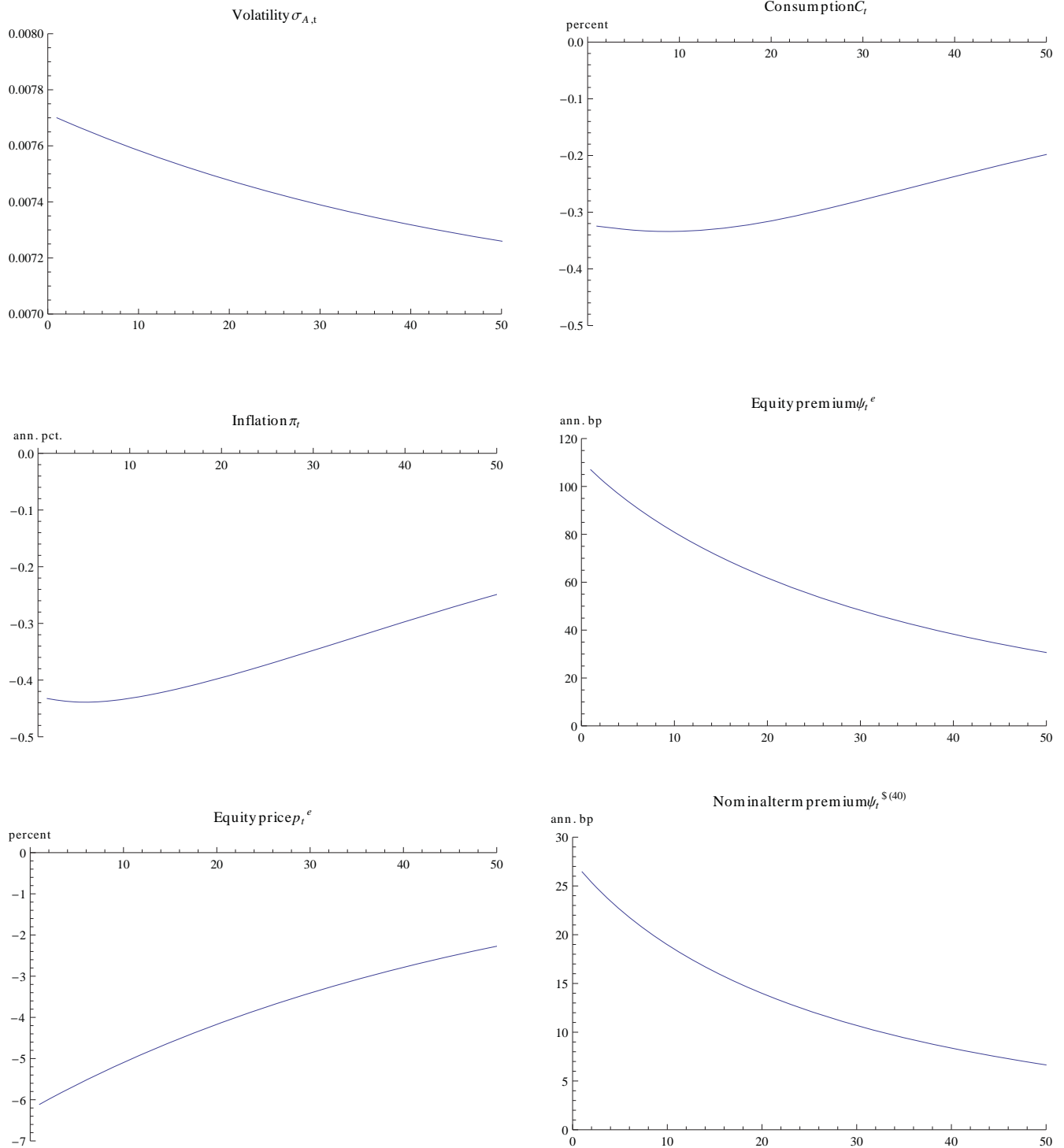
Figure 6. Nonlinear impulse response functions for volatility σ_t^A, consumption C_t, inflation π_t, the equity premium ψ_t^e, equity price p_t^e, and nominal term premium $\psi_t^{\$(40)}$ to a one-standard-deviation (0.1 percent, or .0007) volatility shock in the extended model. See text for details.

credit spread puzzle. A key feature of the model is generalized recursive preferences with a high degree of risk aversion. Thus, the paper shows formally that a wide variety of asset pricing puzzles that are typically studied separately can be thought of as a single, unified puzzle—namely, why does risk aversion in financial markets seem to be so high?

I do not provide an answer to this last puzzle in the paper, but there are a number of other studies in the literature that have made great progress on this issue. For example, simple macroeconomic models such as the one considered here may substantially understate the true level of risk in the economy because of uncertainties about the laws of motion for the economy or its parameters (e.g., Barillas, Hansen, and Sargent, 2009), or the presence of long-run risks (e.g., Bansal and Yaron, 2004) or rare disasters (e.g., Rietz, 1988; Barro, 2006). Moreover, the consumption of stock- and bond-holders is more cyclical than that of non-asset-holders (e.g., Mankiw and Zeldes, 1991; Parker, 2001; Malloy, Moskowitz, and Vissing-Jorgensen, 2009), so the required level of risk aversion in a simple representative-agent model such as the one in the present paper is higher than it would be in a model that recognized this heterogeneity (Guvenen, 2009). Related to this, Adrian, Etula, and Muir (2014) provide evidence that the marginal investor is closely tied to the financial intermediary sector, whose principals' consumption is likely extremely highly correlated with market fluctuations. All of these results help to explain why the very simple macroeconomic model developed here requires such a high coefficient of relative risk aversion to match these asset pricing puzzles. Extending the model to incorporate additional features along the lines of those described here should allow it to explain all of the asset pricing puzzles above with substantially less risk aversion.

An advantage of the simple, structural model developed here compared to the reduced-form approaches of typical studies in finance is that the structural model provides an intuitive framework for thinking about asset prices and asset pricing puzzles. Rather than studying each puzzle in isolation, the model here can provide a reasonable description of the structural linkages across major asset classes. For example, the joint behavior of real and nominal long-term bonds in the model helps to provide intuition for the bond premium puzzle. Similarly, the joint behavior of defaultable and default-free long-term nominal debt can provide intuition for the credit spread puzzle, and the joint behavior of defaultable bonds and equity can provide intuition for the credit spread and equity premium puzzles.

Finally, the present paper opens the door for studying the feedback between those risk premia and the macroeconomy by showing how a standard macroeconomic model can be made

consistent with the behavior of risk premia in financial markets. As evidenced by the recent financial crisis and "Great Recession", these feedback effects can be extremely interesting and important. In the present paper, equities and real, nominal, and defaultable debt can all be priced, but those asset prices have no feedback to the real economy. This is one of the costs of keeping the macroeconomic model as simple as possible, since adding feedback effects from asset prices to the real economy would complicate the model substantially and obscure the intuition underlying the model's asset-pricing results. Nevertheless, it would be very interesting to combine the asset-pricing framework of the present paper with a macroeconomic model that includes a financial accelerator, such as Bernanke, Gertler, and Gilchrist (1999), Kiyotaki and Moore (1997), Gertler and Kiyotaki (2014), and many others. In general, these models abstract from risk aversion and asset pricing and focus instead on the effect of agency problems and collateral constraints on lending and investment. In a combined framework, shocks that cause the economy to deteriorate would lead to an increase in risk premia and a concomitant fall in asset prices, further amplifying the collateral constraint on firms and financial intermediaries. This channel appears to have been an important amplification mechanism in the recent crisis.

Appendix: Model Equations

The equations of the macroeconomic model in Section 2 can be written in recursive form as follows. (Equations for equity and debt are essentially the same as in Section 3 and are not reproduced here.)
Value function:

$$V_t = \log C_t - \eta \frac{L_t^{1+\chi}}{(1+\chi)} + \beta V_t^{\text{twist}}, \tag{A1}$$

$$V_t^{\text{twist}} = (V_t^e)^{1/(1-\alpha)}, \tag{A2}$$

$$V_t^e = E_t V_{t+1}^{1-\alpha}. \tag{A3}$$

Risk-free real rate and Euler equations:

$$e^{-r_t} = \beta E_t (C_{t+1}/C_t)^{-1} (V_{t+1}/V_t^{\text{twist}})^{-\alpha}, \tag{A4}$$

$$C_t^{-1} = \beta E_t e^{i_t - \pi_{t+1}} C_{t+1}^{-1} (V_{t+1}/V_t^{\text{twist}})^{-\alpha}. \tag{A5}$$

Optimal price setting by firms:

$$(p_t^*)^{(1+\epsilon(1-\theta)/\theta)} = \frac{\epsilon}{\epsilon-1} \frac{z_t^n}{z_t^d}, \tag{A6}$$

$$z_t^n = \mu_t Y_t + \beta\xi E_t (C_{t+1}/C_t)^{-1} (V_{t+1}/V_t^{\text{twist}})^{-\alpha} (e^{\pi_{t+1}-\bar{\pi}})^{\epsilon/\theta} z_{t+1}^n, \tag{A7}$$

$$z_t^d = Y_t + \beta\xi E_t (C_{t+1}/C_t)^{-1} (V_{t+1}/V_t^{\text{twist}})^{-\alpha} (e^{\pi_{t+1}-\bar{\pi}})^{\epsilon-1} z_{t+1}^d, \tag{A8}$$

$$(e^{\pi_t-\bar{\pi}})^{1-\epsilon} = (1-\xi)(p_t^* e^{\pi_t-\bar{\pi}})^{1-\epsilon} + \xi. \tag{A9}$$

Marginal cost and real wage:

$$\mu_t = \frac{w_t Y_t^{(1-\theta)/\theta}}{\theta A_t^{1/\theta} K^{(1-\theta)/\theta}}, \tag{A10}$$

$$\eta L_t^\chi / C_t^{-1} = w_t. \tag{A11}$$

Production and resource constraint:

$$Y_t = A_t K^{1-\theta} L_t^\theta / \Delta_t, \tag{A12}$$

$$\Delta_t^{1/\theta} = (1-\xi)(p_t^*)^{-\epsilon/\theta} + \xi(e^{\pi_t-\bar{\pi}})^{\epsilon/\theta} \Delta_{t-1}^{1/\theta}, \tag{A13}$$

$$Y_t = C_t. \tag{A14}$$

Monetary policy rule:

$$i_t = \log(1/\beta) + \pi_t + \phi_\pi(\pi_t - \bar{\pi}) + \frac{\phi_y}{4} \log(Y_t/\bar{Y}_t), \tag{A15}$$

$$\log \bar{Y}_t = \rho_{\bar{y}} \log \bar{Y}_{t-1} + (1-\rho_{\bar{y}}) \log Y_t. \tag{A16}$$

Technology shock:

$$\log A_t = \log A_{t-1} + \varepsilon_t^A. \tag{A17}$$

The value function is broken into more than one equation to correspond to the syntax of Perturbation AIM and other rational expectations equation solvers, which typically require the model to be written as a system of equations in a form similar to $E_t F(X_{t-1}, X_t, X_{t+1}; \varepsilon_t) = 0$. The auxiliary variable V^{twist} is useful for writing the stochastic discount factor.

As discussed in the text, the variables Y_t, C_t, w_t, $\bar{Y}_t$, z_t^n, and z_t^d are all transformed by dividing through by A_t. The value function V_t is transformed by defining $\tilde{V}_t \equiv V_t - A_t/(1-\beta)$ and $\tilde{V}_t^{\text{twist}} \equiv V_t^{\text{twist}} - A_t/(1-\beta)$. In the case $\alpha = 0$, corresponding to expected utility preferences, these transformations render the model stationary, and technology only appears in the model as a growth rate, such as A_t/A_{t-1}. In the more general case $\alpha \neq 0$, the first-order approximation to the model retains these properties. However, when $\alpha \neq 0$, the nonlinear model equations (and their second- and higher-order approximations)

depend on the value of A_{t-1} independent of the growth rate A_t/A_{t-1}. As a result, the model's nonlinear solution (and second- and higher-order approximate solutions) are (slightly) nonstationary.

In this case, a nonstochastic steady state for the model still exists—in fact, any initial condition for A_0 produces a nonstochastic steady state where technology remains at A_0—and I choose units to normalize $A_0 = 1$. The model can be linearized around this point, and is first-order stable and stationary around that point. Similarly, second- and higher-order approximate solutions can be computed around that nonstochastic steady state. These solutions are highly accurate in a neighborhood of the steady state, and become increasingly accurate over larger regions of the state space as the order of approximation n becomes large (see Swanson, Anderson, and Levin, 2006, for details and discussion).

An important technical condition, discussed by Swanson (2013), is that the value function V_t be restricted so that either $V_t \geq 0$ or $V_t \leq 0$ over all relevant states of the economy, in order to avoid complex numbers in the expectations (A2)–(A3). (If period utility is everywhere negative, then it is natural to define $V_t \leq 0$, $V_t^e = E_t(-V_{t+1})^{1-\alpha}$, and $V_t^{\text{twist}} = -(V_t^e)^{1/(1-\alpha)}$, as discussed in Swanson (2013).) A technical disadvantage of logarithmic preferences in the period utility function (1) is that period utility is neither everywhere positive nor everywhere negative for $c \in \mathbb{R}_+$. However, under the baseline calibration of the model, the nonstochastic steady-state value of V is about 135.3, while the response of V_t to a one-standard-deviation technology shock (as in Figure 1) is less than 0.7 (about 100 times the roughly .007 increase in $\log c$), so there is essentially zero probability of $V_{t+1} < 0$ in the expectations (A2)–(A3).

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