Financial Friction and Monetary Union

Simon Gilchrist* Raphael Schoenle † Jae Sim‡ Egon Zakrajšek§

June 2014

Abstract

In contrast to the conventional wisdom that the European crisis grew from unsustainable fiscal imbalances of the peripheral countries of the European Union, we consider an alternative hypothesis: misalignment of real exchange rates and resulting current account deficits of the peripheral countries since the late 1990s are the heart of the current crisis. Building on Gilchrist, Schoenle, Sim, and Zakrajsek [2013], we emphasize the role of heterogeneous financial frictions in a currency union. We show that facing financial distress in the absence of a devaluation option, the firms in financially weak countries countries have an incentive to raise their prices to cope with liquidity shortfalls, while the firms in countries with greater financial slack poach from the customer base of the former countries by undercutting their prices, without internalizing the detrimental effects on union-wide aggregate demand. Thus, a monetary union among countries with heterogeneous degrees of financial frictions may create a tendency toward internal devaluation for core countries with greater financial resources, leading to chronic current account deficits of the peripheral countries. A risk-sharing arrangement between the core and the periphery can potentially undo the distortion brought about by the currency union. However, such risk sharing requires unrealistic amounts of wealth transfers from the core to the periphery. Returning to a floating exchange rate regime may potentially resolve the misalignment, but only if the member countries adopt an activist monetary policy after the break-up.

---

*Boston University and NBER. Email: sgilchri.@bu.edu

†Brandeis University. Email: schoenle@brandeis.edu

‡Federal Reserve Board of Governors. Email: jae.w.sim@frb.gov

§Federal Reserve Board of Governors. Email: egon.zakrajsek@frb.gov

1 Introduction

While the critical moment of the European crisis since 2008 seems to be behind us, a meaningful recovery from the crisis has yet to come as the unemployment rates of the peripheral countries of the European Union are still at unusually elevated levels. The unemployment rates of Greece and Spain are still 15 percentage points highter than their levels in 2002, when the coins and notes of the historic common currency, Euro were brought into use for the first time. Meanwhile, the consumer price indices of PIIGS countries (Portugal, Italy, Ireland, Greece and Spain) have started to show a mild deflationary pressure in 2014 for the first time since the crisis, although the inflation rates were surprisingly and unexpectedly high despite the large widening of resource gaps in these countries during the first a few years of the crisis for a reason made clear below.

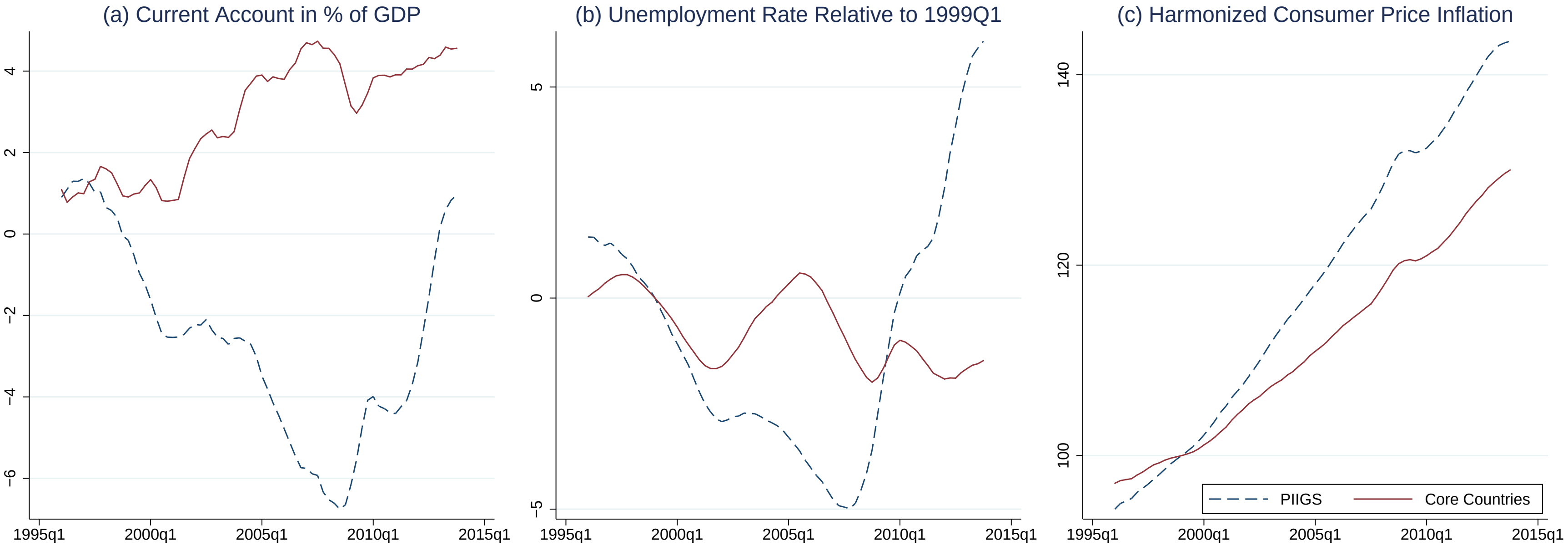
The conventional wisdom held by many policy makers finds the root of the crisis in the unsustainable fiscal imbalances of the peripheral countries. However, as many careful commentators pointed out, the fiscal positions of the peripheral countries today is more likely to be the results of the on-going crisis rather than the cause. The Average fiscal balance of these coutries over the period of 1999-2007 was only mildly negative, and in particular, Spain, the one affected the most by the crisis, held small surplus on average over this period.

In this paper, we consider an alternative hypothesis that the root of the current crisis should be found in another type of unstainability–the misalignment of the real exchange rate and the resulting current account deficits of the peripheral countries since late 1990s. One can never abolish the real exchange rate by adopting a common currency among heterogeneous countries unless the price levels of the member countries move in lockstep.

As shown by panel (c) of figure 1, the growth of the consumer price indices of PIIGS country has far surpassed that of the core northern countries since late 1990s. With the nominal exchange rate held fixed by the union, the difference in the inflation rates between the periphery and core has been directly translated into the appreciation of the real exchange rate. In fact, if we use export price indices, figure 2 shows that the real exchange rates of PIIGS countries with respect to Germany have appreciated 10∼30 percent except Ireland since late 1990s. In contrast, the real exchange rates of the northern countries such as France, Finland and Netherlands have essentially maintained their initial levels with respect to Germany over the same period. The massive real appreciation has eroded the competitiveness of the sourthern countries and deteriorated their current account positions substantially. Panel (a) shows that the current account deficit relative to GDP has increased to more than 6 percent on average while the current account surplus of the core countries has increased to 4 percent of ther GDPs on average over the same period.

In principle, an economic crisis, such as the one that erupted in 2008, could solve the problem caused by the misalignment by creating massive decline in aggregate demand and prices in the periphery. However, the crisis in 2008 moved in the opposite direction, and the real exchange rate misalignment has not been corrected fundamentally until recently even when the resolution of crisis is on the horizon. This is a part of a bigger puzzle facing the European nations: despite the massive resource gaps, why deflation did not happen in the southern European countries until recently?

Figure 1: **Eurozone Current Account, Unemployment and Inflation**

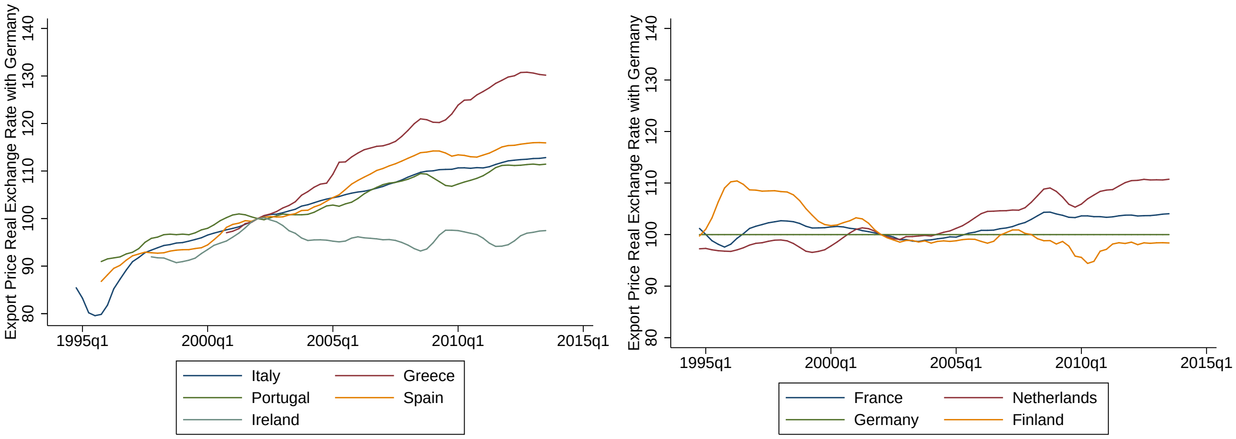


(a) Current Account in % of GDP

(b) Unemployment Rate Relative to 1999Q1

(c) Harmonized Consumer Price Inflation

Figure 2: Bilateral Real Exchange Rate With Respect To Germany



Note: Blue, solid line is the peripheral country and red, dash-dotted line is the central country. The shock assumes that the dilution cost for the peripheral countries go up by 100

To answer this question properly, one needs a theory that can shed light on two issues: the nature of international macroeconomic adjustment under a currency union; pricing strategy of firms under financial market friction. Analyzing the first issue involves understanding the macroeconomic consequences of lack of independent monetary policy under a currency union among countries with heterogeneous financial market development. Studying the second issue requires to introduce the financial market friction to the optimization problem of pricing firms. In this sense, we fundamentally extend the analysis of financial accelerator literature (for example Bernanke, Gertler, and Gilchrist [1999]) by considering optimal pricing decision and financing decision together in a unified framework rather than considering them separately. Our analysis also expands the span of New Keynesian theory on the real exchange rate dynamics such as Obstfeld and Rogoff [2000] and Steinsson [2008] by analyzing the issue under both complete and incomplete risk sharing arrangements in an environment where pricing firms face meaningful financial market friction in their optimal pricing decisions.

To this end, we build upon our earlier study, Gilchrist, Schoenle, Sim, and Zakrajsek [2013], and extend the analysis in a two-countries international business cycle framework with nominal rigidity both under a currency union and under a floating exchange rate regime. Our theory predicts that facing a financial strain in the absence of devaluation option, the countries under greater financial frictions have incentive to raise their prices to cope with liquidity shortfalls, whereas the countries with greater financial slack try to to steal the customer base of the former countries by undercutting their prices, without internalizing the detrimental effects on the union-wide aggregate demand. Thus, a monetary union among countries with heterogeneous degrees of financial market friction may create a tendency toward internal devaluation for *core* countries with greater financial resources, not for financially vulnerable, peripheral countries. The mechanism explains why the peripheral countries may manage to avoid outright debt deflation in the middle of the financial crisis even with unemployment rates of 20-25 percent, but face a chronic stagnation instead with persistent erosion of competitiveness.

Using macroeconomic data of the member countries of the European Union, we find supporting evidence for the link between the financial market friction and pricing pressure. In particular, we estimate both backward looking and forward looking Phillips curves of the member countries using only the data prior to the crisis, and construct prediction errors of inflation rates during the crisis. We then regress these errors onto CDS spreads of the member countries, natural measures of the heterogeneous degrees of financial market friction. We have found a strong evidence for positive relations between them: the countries with greater financial market friction experience higher inflation rates after we control other fundamental determinants of empirical Phillips curves. In contrast, we have found no evidence for such relationships among the core European countries. This confirms our new New Keynesian pricing theory and explains why the real exchange rate misalignment was not corrected, and in fact, was aggravated by the crisis.

We show how a complete risk sharing arrangement among the member countries can remedy the distortion brought about by the fixed exchange rate. Such a risk sharing arrangement, in principle, can be achieved by developing a fiscal union. However, our analysis indicates that the risk sharing arrangement requires unrealistic amounts of wealth transfers from the north to the south, putting in doubt the realism of such risk sharing. Finally, we consider the dynamics of international macroeconomic adjustment under a floating exchange rate regime as an ultimate correction of the distortion brought about by the union. We show that returning back to a floating exchange rate regime does not necessarily fix the problem depending on what type of monetary policy regime countries choose after the breakup of the union. If the countries insist upon non-activist monetary policies, they may have macroeconomic outcomes that are similar to those under the currency union. Only with sufficiently strong monetary policy activism, the countries can fundamentally improve upon the outcomes under the currency union.

The rest of the paper is organized as follows. Section 2 develops our model in a two-countires international business cycle framework mong heterogenous countries. Section 3 analyzes the effects of financial and real shocks under four international risk sharing arrangements: a floating exchange rate under a complete risk sharing; a monetary union under a complete risk sharing; a floating exchange rate under an incomplete risk sharing; a monetary union under an incomplete risk sharing. The same section then discuss about possible remedies to the distortion caused the currency union, including the option to return to a floating exchange rate regime under various monetary policy regimes. Section 4 summarizes our empirical findings. Section 5 concludes.

2 Model

2.1 Preferences

We restict our attention to a two-country setting, where foreign country variables are denoted by asterisks. In each country, there exists a continuum of households, indexed by $j \in N_c \equiv [0,1]$. Each household consumes two types, h and f of different variety of consumption goods, indexed by $i \in N_h \equiv [1,2]$ in home country and by $i \in N_f \equiv [2,3]$ in foreign country. As is standard, we assume that home country produces only $h-$ type and foregin country produces only $f-$ type. For instance, $c^j_{i,f,t}$ denotes home country consumer j's consumption of product i of type f whereas $c^{j*}_{i,f,t}$ denotes the foreign counterpart. Note that $c^j_{i,f,t}$ is consumption of an imported good by a home country consumer and $c^{j*}_{i,f,t}$ is consumption of a domestically produced good by a foreign consumer.

For simplicity, we assume perfect immobility of labor. Under this assumption, the preferences are specified as

The preferences of home country are given by

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s U(x^j_{t+s} - \delta_{t+s}, h^j_{t+s}) \text{ for } j \in [0,1]. \tag{1}$$

The static utility function $U(\cdot,\cdot)$ is strictly increasing and concave in the first argument, and is strictly decreasing and concave in the second argument. The consumption/habit aggregator x^j_t is defined as an Armington aggregator,

$$x^j_t \equiv \left\{ \sum_{k=h,f} \omega_k \left[\int_{N_k} (c^j_{i,k,t}/s^{\theta}_{i,k,t-1})^{1-1/\eta} dk \right]^{\frac{1-1/\varepsilon}{1-1/\eta}} \right\}^{1/(1-1/\varepsilon)}, \sum_{k=h,f} \omega^{\varepsilon}_k = 1 \tag{2}$$

where η and ε are the elasticities of substitution within a type and between types. ω_k is a parameter that governs the home bias in consumption basket in the steady state. $s_{i,k,t}$ is good-specific habit, which evolves according to

$$s_{i,k,t} = \rho s_{i,k,t-1} + (1-\rho) c_{i,k,t} \text{ for } k = h, f, \tag{3}$$

where $c_{i,k,t}$ is defined as an average consumption level of good i, i.e., $c_{i,k,t} \equiv \int_0^1 c^j_{i,k,t} dj$ for $k = h, f$. In other words, it is not the individual level of consumption that matters for the habit, but the level of average consumption. Hence, the preferences can be considered 'Catching Up with Joneses' at the good level rather than at the aggregate consumption level. δ_t is a demand shock that alters the marginal utility of consumption.

The cost minimization associated with (1) implies the following demand functions:

$$c^j_{i,k,t} = \left(\frac{P_{i,k,t}}{\tilde{P}_{k,t}} \right)^{-\eta} s^{\theta(1-\eta)}_{i,k,t-1} x^j_{k,t} \tag{4}$$

where the welfare-based price index and consumption basket are defined as

$$\tilde{P}_{k,t} \equiv \left[\int_{N_k} (P_{i,k,t} s^{\theta}_{i,k,t-1})^{1-\eta} di \right]^{1/(1-\eta)} \tag{5}$$

$$\text{and } x^j_{k,t} \equiv \left[\int_{N_k} (c_{i,k,t} / s^{\theta}_{i,k,t-1})^{1-1/\eta} di \right]^{1/(1-1/\eta)} \tag{6}$$

for $k = h, f$. $\tilde{P}_{k,t}$ and $x^i_{k,t}$ should be interepreted as habit adjusted price index, and habit adjusted consumption basket of goods of type k in home country. The appendix shows the details of the derivation of (4)∼(6). The appendix also shows that the consumption basket (6) in equilibrium is determined as

$$x^j_{k,t} = \omega^{\varepsilon}_k \left(\frac{\tilde{P}_{k,t}}{\tilde{P}_t} \right)^{-\varepsilon} x^j_t \text{ for } k = h, f \tag{7}$$

$$\text{where } \tilde{P}_t = \left[\sum_{k=h,f} \omega_k \tilde{P}^{1-\varepsilon}_{k,t} \right]^{1/(1-\varepsilon)}, \tag{8}$$

the welfare based aggregate price index of home country. Due to the symmetric structure of the two countries, the foreign counterparts can be expressed simply with asterisks placed in (4)∼(8). For later use, we also define CPI as

$$P_t \equiv \left[\sum_{k=h,f} \omega_k P^{1-\varepsilon}_{k,t} \right]^{1/(1-\varepsilon)} \tag{9}$$

where

$$P_{k,t} \equiv \left[\int_{N_k} (P_{i,k,t})^{1-\eta} di \right]^{1/(1-\eta)} \text{ for } k = h, f$$

is defined as a type-specific CPI.

2.2 Technology

Following our earlier study (Gilchrist, Schoenle, Sim, and Zakrajsek [2013]), we assume that the production technology is specified as

$$y_{i,t} = \left(\frac{A_t}{a_{i,t}} h_{i,t} \right)^{\alpha} - \phi, \quad 0 < \alpha \leq 1 \tag{10}$$

$$\text{and } y^*_{i,t} = \left(\frac{A^*_t}{a^*_{i,t}} h^*_{i,t} \right)^{\alpha} - \phi^*, \quad 0 < \alpha \leq 1 \tag{11}$$

where A_h and A^*_t are country specific aggregate technology shocks, possibly correlated with each other, $a_{i,t}$ and $a^*_{i,t}$ are idiosyncratic technology shocks to home and foreign firms. We assume that the idiosyncratic shocks follow symmetric iid log-normal distributions, $\log a_{i,t} \sim N(-0.5\sigma^2, \sigma^2)$ and $\log a^*_{i,t} \sim N(-0.5\sigma^2, \sigma^2)$.

ϕ and ϕ^* are country-specific fixed costs of operation, which make it possible for the firms to have negative incomes, and hence a liquidity problem if external financing is costly. As shown by (Gilchrist, Schoenle, Sim, and Zakrajsek [2013]), the fixed costs can work as a parimonious proxy to heterogeneous degree of financial obligation, including fixed payments to long term bonds, etc. However, to make ϕ and ϕ^* important for liquidity condition for firms, we need to introduce some frictions to the flow of funds constraint of the firms.

2.3 Pricing Frictions and Financial Distortions

To allow for nominal rigidities, we assume that the firms face a quadratic cost to adjusting nominal prices as specified in (Rotemberg [1982]):

$$\frac{\gamma}{2}\left(\frac{P_{i,h,t}}{P_{i,h,t-1}}-\bar{\pi}\right)^2 c_t+\frac{\gamma^*}{2}\frac{S_tP_t^*}{P_t}\left(\frac{P_{i,h,t}^*}{P_{i,h,t-1}^*}-\bar{\pi}\right)^2 c_t^* \tag{12}$$

where S_t is the nominal exchange rate. We allow the degree of nominal rigidity to differ in home and foreign countries as indicated by separate notations for γ and γ^*. We assume that the price adjustment costs are proportional to local consumptions, i.e., c_t and c_t^*. After dividing the numerators and denominators by type specific price indices $P_{h,t}$ and $P_{h,t}^*$, we can express the price adjustment costs as

$$\frac{\gamma}{2}\left(\frac{p_{i,h,t}}{p_{i,h,t-1}}\pi_{h,t}-\bar{\pi}\right)^2 c_t+\frac{\gamma^*}{2}q_t\left(\frac{p_{i,h,t}^*}{p_{i,h,t-1}^*}\pi_{h,t}^*-\bar{\pi}^*\right)^2 c_t^*.$$

where $p_{i,h,t}\equiv P_{i,h,t}/P_{h,t}$, $p_{i,h,t}^*\equiv P_{i,h,t}^*/P_{h,t}^*$ and $q_t\equiv S_tP_t^*/P_t$, the real exchange rate.

To introduce financial frictions in a tractable manner we assume that the firms must commit to pricing and hence output decisions based on all aggregate information available within the period but prior to the realization of their firm-specific idiosyncratic shock to productivity. Based on this aggregate information, the firms post prices, take orders from customers and plan production based on expected marginal cost. The firms then realize actual marginal cost and hire labor to meet demand. Expost, profits may be too low to cover the total cost of production in which case the firms must raise external finance. Without loss of generality, we focus on equity finance.[1] We assume that expost-equity finance involves a constant per-unit dilution cost φ, $\varphi^*\in(0,1)$.

---

[1] As shown by Gomes [2001] and Stein [2003], other forms of costly external financing can be replicated by properly parameterized equity dilution costs.

2.4 Profit Maximization Problem

The firm problem is to maximize the present value of dividend flows, $\mathbb{E}_t[\sum_{s=0}^{\infty} m_{t,t+s} d_{i,t+s}]$ where $d_{i,t} \equiv D_{i,t}/P_t$ denotes real dividend payouts when positive, and equity issuance when negative. We assume that the firms are owned by the households, and discount future cashflows with the stocahstic discounting factor of a representative household, $m_{t,t+s}$. The dilusion cost implies that when a firm issues a notional amount of equity $d_{i,t}(< 0)$, actual cash inflow from the issuance is reduced to $-(1-\varphi)d_{i,t}$. An implicit assumption is that the equity markets in the two countries are segmented: only domestic households invest in the shares of domestic firms.

This assumption can be justified if the information asymmetry underlying the dilution phenomenon is disproportionately larger for cross-border equity holdings. The home bias in equity holdings is well documented by empirical researchers in fiance: French and Poterba [1991], Tesar and Werner [1995] and Obstfeld and Rogoff [2000]. In fact, in our model, even an arbitrarily small disadvantage associated with cross-border financing is large enough to generate a perfect home bias because the perceived supply of of domestic funding at the given cost of dilusion is infinitely elastic. Hence the stream of dividends is discounted with the stochastic discounting factor of the representative household in local country.

Another important assumption we adopt in this paper is that the two countries are different in terms of the degree of capital market imperfection. In particular, we assume that the dilution cost is strictly greater in home country than in foreign country: $0 \leq \varphi^* < \varphi$. Despite this difference, we assume that the firms in home country cannot avoid the higher external financing cost by issuing equities overseas. Together with the higher operating costs in home country, the higher external financing cost expose the firms in home country to a liquidity risk to a greater degree. We explore the implication of this financial vulnerability on the product pricing under various macroeconomic environment such as fixed vs floating exchange rate on the one hand, complete and incomplete risk sharing arrangement between the two countries on the other hand. To save space, we describe the firm problem from the viewpoint of home country.

The firm problem is subject to the following flow of funds constraint:

$$\begin{aligned} d_{i,t} = p_{i,h,t} p_{h,t} c_{i,h,t} + q_t p^*_{i,h,t} p^*_{h,t} c^*_{i,h,t} - w_t h_{i,t} + \varphi \min\{0, d_{i,t}\} \\ -\frac{\gamma}{2}\left(\frac{p_{i,h,t}}{p_{i,h,t-1}}\pi_{h,t} - \bar{\pi}\right)^2 c_t - \frac{\gamma^*}{2} q_t \left(\frac{p^*_{i,h,t}}{p^*_{i,h,t-1}}\pi^*_{h,t} - \bar{\pi}^*\right)^2 c^*_t \end{aligned} \tag{13}$$

where $p_{h,t} \equiv P_{h,t}/P_t$, $p^*_{h,t} \equiv P^*_{h,t}/P^*_t$, $c_{i,h,t} = \int_{N_c} c^j_{i,h,t} dj$ and $c^*_{i,h,t} = \int_{N_c} c^{j*}_{i,h,t} dj$. $w_t \equiv W_t/P_t$ is real wage rate. In a symmetric equilibrium, all households in the same country make the same consumption decision, and hence, $c^j_{i,h,t} = c_{i,h,t}$ and $c^{j*}_{i,h,t} = c^*_{i,h,t}$ for all j. The firm problem is also subject to a demand constraint:

$$\left(\frac{A_t}{a_{i,t}} h_{i,t}\right)^{\alpha} - \phi \geq c_{i,h,t} + c^*_{i,h,t} \tag{14}$$

The firm problem can then be expressed as the following Lagrangian:

$$\begin{aligned}
\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} m_{0,t} \Bigg\{ & d_{i,t} + \kappa_{i,t} \left[\left(\frac{A_t}{a_{i,t}} h_{i,t} \right)^{\alpha} - \phi - (c_{i,h,t} + c^*_{i,h,t}) \right] \\
& + \xi_{i,t} \Bigg[p_{i,h,t} p_{h,t} c_{i,h,t} + q_t p^*_{i,h,t} p^*_{h,t} c^*_{i,h,t} - w_t h_{i,t} - d_{i,t} + \varphi \min\{0, d_{i,t}\} \\
& \qquad - \frac{\gamma}{2} \left(\frac{p_{i,h,t}}{p_{i,h,t-1}} \pi_{h,t} - \bar{\pi} \right)^2 c_t - \frac{\gamma^*}{2} q_t \left(\frac{p^*_{i,h,t}}{p^*_{i,h,t-1}} \pi^*_{h,t} - \bar{\pi}^* \right)^2 c^*_t \Bigg] \\
& + \nu_{i,h,t} \left[(p_{i,h,t})^{-\eta} \tilde{p}^{\eta}_{h,t} s^{\theta(1-\eta)}_{i,h,t-1} x_{h,t} - c_{i,h,t} \right] \\
& + \nu^*_{i,h,t} \left[(p^*_{i,h,t})^{-\eta} \tilde{p}^{*\eta}_{h,t} s^{*\theta(1-\eta)}_{i,h,t-1} x^*_{h,t} - c^*_{i,h,t} \right] \\
& + \lambda_{i,h,t} \left[\rho s_{i,h,t-1} + (1-\rho) c_{i,h,t} - s_{i,h,t} \right] \\
& + \lambda^*_{i,h,t} \left[\rho s^*_{i,h,t-1} + (1-\rho) c^*_{i,h,t} - s^*_{i,h,t} \right] \Bigg\}
\end{aligned}$$

where $\tilde{p}_{h,t} \equiv \tilde{P}_{h,t}/P_{h,t}$, $\tilde{p}^*_{h,t} \equiv \tilde{P}^*_{h,t}/P^*_{h,t}$, and $\kappa_{i,t}$, $\xi_{i,t}$, $\nu_{i,h,t}$, $\nu^*_{i,h,t}$, $\lambda_{i,h,t}$ and $\lambda^*_{i,h,t}$ are shadow values of the constraints (14), (13), (4) and (3).

2.5 Efficiency Conditions

The efficiency conditions for the firm problem in home country are given by the followings:

$$d_{i,t} : \xi_{i,t} = \begin{cases} 1 & \text{if } d_{i,t} \geq 0 \\ 1/(1-\varphi) & \text{if } d_{i,t} < 0 \end{cases} \tag{15}$$

$$h_{i,t} : \xi_{i,t} w_t = \alpha \kappa_{i,t} \left(\frac{A_t}{a_{i,t}} h_{i,t} \right)^{\alpha - 1} \tag{16}$$

$$\text{where} \quad h_{i,t} = \frac{a_{i,t}}{A_t} \left(\phi + c_{i,h,t} + c^*_{i,h,t} \right)^{1/\alpha} \tag{17}$$

$$c_{i,h,t} : \nu_{i,h,t} = \mathbb{E}^a_t[\xi_{i,t}] \; p_{i,h,t} p_{h,t} - \mathbb{E}^a_t[\kappa_{i,t}] + (1-\rho) \lambda_{i,h,t} \tag{18}$$

$$c^*_{i,h,t} : \nu^*_{i,h,t} = \mathbb{E}^a_t[\xi_{i,t}] \; q_t p^*_{i,h,t} p^*_{h,t} - \mathbb{E}^a_t[\kappa_{i,t}] + (1-\rho) \lambda^*_{i,h,t} \tag{19}$$

$$\begin{aligned} s_{i,h,t} : \lambda_{i,h,t} = {} & \rho \mathbb{E}_t[m_{t,t+1} \lambda_{i,h,t+1}] \\ & + \theta(1-\eta) \mathbb{E}_t \left\{ m_{t,t+1} \mathbb{E}^a_{t+1} \left[\nu_{i,h,t+1} \frac{c_{i,h,t+1}}{s_{i,h,t}} \right] \right\} \end{aligned} \tag{20}$$

$$\begin{aligned} s^*_{i,h,t} : \lambda^*_{i,h,t} = {} & \rho \mathbb{E}_t[m_{t,t+1} \lambda^*_{i,h,t+1}] \\ & + \theta(1-\eta) \mathbb{E}_t \left\{ m_{t,t+1} \mathbb{E}^a_{t+1} \left[\nu^*_{i,h,t+1} \frac{c^*_{i,h,t+1}}{s^*_{i,h,t}} \right] \right\} \end{aligned} \tag{21}$$

$$\begin{aligned} p_{i,h,t}: 0 = {} & \mathbb{E}^a_t[\xi_{i,t}]\left[p_{h,t}c_{i,h,t} - \gamma\frac{\pi_{h,t}}{p_{i,h,t-1}}\left(\pi_{h,t}\frac{p_{i,h,t}}{p_{i,h,t-1}} - \bar{\pi}\right)c_t\right] - \eta\frac{\nu_{i,h,t}}{p_{i,h,t}}c_{i,h,t} \\ & + \gamma\mathbb{E}_t\left[m_{t,t+1}\mathbb{E}^a_{t+1}[\xi_{i,t+1}]\pi_{h,t+1}\frac{p_{i,h,t+1}}{p^2_{i,h,t}}\left(\pi_{h,t+1}\frac{p_{i,h,t+1}}{p_{i,h,t}} - \bar{\pi}\right)c_{t+1}\right] \end{aligned} \tag{22}$$

$$\begin{aligned} p^*_{i,h,t}: 0 = {} & \mathbb{E}^a_t[\xi_{i,t}]\left[q_t p^*_{h,t}c^*_{i,h,t} - \gamma^*\frac{q_t\pi^*_{h,t}}{p^*_{i,h,t-1}}\left(\pi^*_{h,t}\frac{p^*_{i,h,t}}{p^*_{i,h,t-1}} - \bar{\pi}^*\right)c^*_t\right] - \eta\frac{\nu^*_{i,h,t}}{p^*_{i,h,t}}c^*_{i,h,t} \\ & + \gamma^*\mathbb{E}_t\left[m_{t,t+1}\mathbb{E}^a_{t+1}[\xi_{i,t+1}]q_{t+1}\pi^*_{h,t+1}\frac{p^*_{i,h,t+1}}{p^{*2}_{i,h,t}}\left(\pi^*_{h,t+1}\frac{p^*_{i,h,t+1}}{p^*_{i,h,t}} - \bar{\pi}^*\right)c^*_{t+1}\right] \end{aligned} \tag{23}$$

Note (17) is the labor demand conditional on the level of output, which, in turn, is determined by demand given price. We combine (16) and (17), and express the efficiency condition for labor hours as

$$\kappa_{i,t} = \xi_{i,t}a_{i,t}\frac{w_t}{\alpha A_t}\left(\phi + c_{i,h,t} + c^*_{i,h,t}\right)^{\frac{1-\alpha}{\alpha}}. \tag{24}$$

2.6 Symmetric Equilibrium and International Price Wars

The last six FOCs describe the efficiency conditions for the decisions made prior to the realization of the idiosyncratic cost shock. These first-order conditions involve the expected value of internal funds $\mathbb{E}^a_t[\xi_{i,t}] \equiv \int_0^\infty \xi_{i,t}(a_{i,t})dF(a)$ where the information set of the expectations operator includes all aggregate information up to time t except the realization of the idiosyncratic shock. In contrast, the realized values $\xi_{i,h,t}$ and $a_{i,h,t}$ enter the efficiency conditions (15) and (16) without the expectation operator since equity issuance and labor hiring decisions are made after the realization of the idiosyncratic shock.

With risk-neutrality and iid idiosyncratic shocks, the timing convention adopted above implies that exante firms are identical. Hence we focus on a symmetric equilibrium whereby all monopolistically competitive firms choose identical relative price ($p_{i,h,t} = 1$ and $p^*_{i,h,t} = 1$), production scale ($c_{i,h,t} = c_{h,t}$ and $c^*_{i,h,t} = c^*_{h,t}$), habit stock ($s_{i,h,t} = s_{h,t}$ and $s^*_{i,h,t} = s^*_{h,t}$), and the shadow value of habit stock ($\lambda_{i,h,t} = \lambda_{h,t}$ and $\lambda^*_{i,h,t} = \lambda^*_{h,t}$). However, the distributions of labor hours, dividend payouts, equity issuance and the realized shadow value of internal funds ($\xi_{i,t}$) are non-degenerate and depend on the realization of idiosyncratic shock.

Note that the symmetric equilibrium condition $p_{i,h,t} = 1$ and $p^*_{i,h,t} = 1$ imply that the firms in home country choose the same price levels in home and in foreign markets vis a vis other competitors from the same country. However, this symmetric equilibrium condition does not imply that the firms in foreign country make the same pricing decision as home country firms in the same market, even when they share the same fundamentals, i.e., preference and aggregate technology level. In other words, foreign firms make the same pricing decisions among themselves both in domestic and export markets such that $p_{i,f,t} = 1$ and $p^*_{i,f,t} = 1$, but the price levels chosen by home firms and and foreign firms in a given market differ from each other, and as a result, $p_{h,t} = P_{h,t}/P_t \neq 1$, $p^*_{h,t} = P^*_{h,t}/P^*_t \neq 1$, $p_{f,t} = P_{f,t}/P_t \neq 1$, $p^*_{f,t} = P^*_{f,t}/P^*_t \neq 1$, and $p_{h,t} \neq p_{f,t}$ and $p^*_{h,t} \neq p^*_{f,t}$ in general. This is because the firms in two countries face different degrees of capital market distortions, which create different liquidity conditions for home and foreign firms and lead these two groups of firms to follow different mark-up strategies. In particular, as will be shown below, the relatively poorer financial conditions make home firms maintain higher markup and higher prices in the neighborhood of nonstochastic steady state such that $p_h > p_f$ and $p_h^* > p_f^*$.

From the vantage point of foreign firms, the same phenomenon can be thought of as them engaging in a price war, exploiting the financial vulnerability of home firms to expand their market shares both in their home and in their export markets. The stronger the long-run relationship in customer markets, the greater the incentive of foreign firms to undercut the prices of home firms. If the exchange rate is floating, however, the depreciation of home currency, assisted by easy monetary policy of home country, can greatly improve the liquidity conditions for home country firms, providing an effective defense against the foreign firms' aggressive invasion on their markets. By joining a currency union, home country essentially surrenders this armour.

2.7 Financial Friction and Phillips Curves

Imposing the symmetric equilibrium conditions to the FOCs for pricing decisions yields the following Phillips curves:

$$\begin{aligned} p_{h,t}\frac{c_{h,t}}{c_t} &= \gamma\pi_{h,t}(\pi_{h,t}-\bar{\pi}) + \eta\frac{\nu_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]}\frac{c_{h,t}}{c_t} \\ &\quad - \gamma\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}{\mathbb{E}_t^a[\xi_{i,t}]}\pi_{h,t+1}(\pi_{h,t+1}-\bar{\pi})\frac{c_{t+1}}{c_t}\right] \qquad (25) \\ \text{and } q_t p_{h,t}^*\frac{c_{h,t}^*}{c_t^*} &= \gamma q_t\pi_{h,t}^*(\pi_{h,t}^*-\bar{\pi}^*) + \eta\frac{\nu_{h,t}^*}{\mathbb{E}_t^a[\xi_{i,t}]}\frac{c_{h,t}^*}{c_t^*} \\ &\quad - \gamma^*\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}{\mathbb{E}_t^a[\xi_{i,t}]}q_{t+1}\pi_{h,t+1}^*(\pi_{h,t+1}^*-\bar{\pi})\frac{c_{t+1}^*}{c_t^*}\right] \qquad (26) \end{aligned}$$

The left side of the Phillips curve is different from 1 since $p_{h,t} \neq 1$ and $c_{h,t} \neq c_t$. This is because (25) and (26) describe 'sectoral' inflation dynamics rather than aggregate inflation. More importantly, (25) and (26) show important departure from New Keynesian Phillips curve in that the dynamic liquidity condition plays an essential role in inflation dynamics as can be seen in the fact that the ratio of shadow value of internal funds today vs tomorrow works as an additional discounting factor.

The presence of the marginal value of sales $\nu_{h,t}$ on the right side is due to the customer market feature (deep habit a la Ravn, Schmitt-Grohé, and Uribe [2006]) of the model. Given the long run relationship between the firm and the custormer base, it can be easily seen that the marginal value captures not only the value of sales today, but also future sales to the firm. What is different from deep habit model is that such marginal valuation crucially depends on the dynamic liquidity condition of the firm. Such valuation is determined by two elements: (i) future stream of profits created by sales today and long-run customer relationship; (ii) the dynamic liquidity condition represented by future shadow value of internal funds. More formally (see the appendix for derivation), it can be shown that if we define *financially adjusted* markup as

$$\tilde{\mu}_t \equiv \frac{\mathbb{E}^a_t[\xi_{i,t}]}{\mathbb{E}^a_t[\xi_{i,t}a_{i,t}]}\frac{\alpha A_t}{w_t}\left(\phi + c_{h,t} + c^*_{h,t}\right)^{\frac{\alpha-1}{\alpha}} = \frac{\mathbb{E}^a_t[\xi_{i,t}]}{\mathbb{E}^a_t[\xi_{i,t}a_{i,t}]}\mu_t,$$

the marginal value of sales normalized by the marginal value of internal funds is given by

$$\frac{\nu_{h,t}}{\mathbb{E}^a_t[\xi_{i,t}]} = p_{h,t} - \frac{1}{\tilde{\mu}_t} + \chi\mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\tilde{\beta}_{t,s}\frac{\mathbb{E}^a_s[\xi_{i,s}]}{\mathbb{E}^a_t[\xi_{i,t}]}\left(p_{h,s} - \frac{1}{\tilde{\mu}_s}\right)\right] \tag{27}$$

$$\text{and } \frac{\nu^*_{h,t}}{\mathbb{E}^a_t[\xi_{i,t}]} = q_t p^*_{h,t} - \frac{1}{\tilde{\mu}_t} + \chi\mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\tilde{\beta}^*_{t,s}\frac{\mathbb{E}^a_s[\xi_{i,s}]}{\mathbb{E}^a_t[\xi_{i,t}]}\left(q_s p^*_{h,s} - \frac{1}{\tilde{\mu}_s}\right)\right] \tag{28}$$

where $\chi \equiv (1-\rho)\theta(1-\eta)$ and the composite discounting factors are given by

$$\tilde{\beta}_{t,s} \equiv m_{s,s+1}g_{h,s+1}\cdot\prod_{j=1}^{s-t}(\rho + \chi g_{h,t+j})m_{t+j-1,t+j} \text{ with } g_{h,t} \equiv \frac{s_{h,t}/s_{h,t-1} - \rho}{1-\rho}$$

$$\tilde{\beta}^*_{t,s} \equiv m_{s,s+1}g^*_{h,s+1}\cdot\prod_{j=1}^{s-t}(\rho + \chi g^*_{h,t+j})m_{t+j-1,t+j} \text{ with } g^*_{h,t} \equiv \frac{s^*_{h,t}/s^*_{h,t-1} - \rho}{1-\rho}.$$

Hence, to analyze how the financial market friction interacts with pricing decisions we need to study how the value of internal funds $\mathbb{E}^a_t[\xi_{i,h,t}]$ is determined, and how it affects the markup decision. To that end, we impose the symmetric equilibrium condition, and define the equity issuance trigger as the idiosyncratic productivity level that satisfies the flow of funds constraint when dividend payouts are exactly zero:

$$\begin{aligned} a^E_t = \frac{A_t}{w_t(\phi + c_{h,t} + c^*_{h,t})^{1/\alpha}}\Bigg\{ & c_t\left[\frac{P_{h,t}c_{h,t}}{P_t c_t} - \frac{\gamma}{2}(\pi_{h,t} - \bar{\pi})^2\right] \\ & + q_t c^*_t\left[\frac{P^*_{h,t}c^*_{h,t}}{P^*_t c^*_t} - \frac{\gamma^*}{2}(\pi^*_{h,t} - \bar{\pi}^*)^2\right]\Bigg\} \end{aligned} \tag{29}$$

The investigation of the external financing trigger (29) provides a great opportunity to see how price war is executed in our environment and how manipulation of exchange rate provides an effective tool to counteract such price war. Using the equity issuance trigger, we can rewrite the FOC for dividends as

$$\xi(a_{i,t}) = \begin{cases} 1 & \text{if } a_{i,t} \leq a^E_{h,t} \\ 1/(1-\varphi) & \text{if } a_{i,t} > a^E_{h,t} \end{cases}. \tag{30}$$

This condition simply states that the realized shadow value of internal funds jumps to $1/(1-\varphi)$ due to the costly external financing when the idiosyncratic cost shock is greater than the threshold value. Let z^E_t denote the standardized value of a^E_t, i.e., $z^E_t = \sigma^{-1}(\log a^E_t + 0.5\sigma^2)$. From (30), the expected shadow value of internal funds is

$$\mathbb{E}_t^a[\xi_{i,t}] = \int_0^{a_t^E} 1dF(a) + \int_{a_t^E}^{\infty} \frac{1}{1-\varphi} dF(a) = 1 + \frac{\varphi}{1-\varphi}[1 - \Phi(z_t^E)] \geq 1 \tag{31}$$

where $\Phi(\cdot)$ denotes the cdf of the standard normal distribution. The expected shadow value is strictly greater than unity as long as equity issuance is costly ($\varphi > 0$) and future costs are uncertain ($\sigma > 0$). As emphasized in Gilchrist, Schoenle, Sim, and Zakrajsek [2013], this makes the firms de facto risk averse in their pricing decision: setting the price too low and taking an imprudently large number of orders by lowering the price too much exposes the firm to liquidity risk under costly external finance. Note that if $\varphi = 1$, this means that outside funding is impossible. This is the case of Brunnermeier and Sannikov [2014].

Note that the expected shadow value, which is what matters for the pricing decision, is decreasing in a_t^E. Other things being equal, the threshold value is increasing in market shares of home country firms in both markets, i.e., $P_{h,t}c_{h,t}/P_tc_t$ and $P_{h,t}^*c_{h,t}^*/P_t^*c_t^*$. If the elasticity of substitution is sufficiently large enough, which will be satisfied in any realistic calibration of ε, foreign firms increase their market shares, $1 - P_{h,t}c_{h,t}/P_tc_t$ and $1 - P_{h,t}^*c_{h,t}^*/P_t^*c_t^*$ by aggressively slashing their prices both in domestic and export markets. The drop in the market share of home country firms creates a financial strain to them as can be seen in the drop of the external financing threshold and the resulting increase of the expected shadow value of internal funds. This means that home country firms have to lower their prices in defense to maintain their expected shadow cost of financing low. However, this strategy back-fires because doing so increases production scale and wage bills and exposing them to a greater liquidity risk.

This is costly because it elevates the expected shadow cost of external financing. However, if the real exchange rate can depreciate (q_t goes up), the dilemma of home country can be solved: without changing the nominal prices of home country firms, it can lower their export prices and increase the import prices of foreign competitors, which helps them keep their market shares and maintain their expected shadow prices low at a time of price war. In contrast, under a currency union or fixed exchange rate regime, the absence of nominal exchange rate and independent monetary policy can make the situation much worse for a country with a greater degree of financial market distortion. Losing market share today, which persists in the customer market setting, aggravates tomorrow's liquidity condition, leading to a further upward pressure on the pricing of home country firms, contributing even more to foreign firms' market share.

2.8 Household Problem

To analyze how different risk sharing arrangements between the two countries affect macroeconomic allocations, we consider the problem of household in two different arrangements for international finance: (i) complete risk sharing through trading of state contingent bonds; (iii) incomplete risk sharing with only state non-contingent bonds.

2.8.1 Complete Risk Sharing Under Floating

To streamline the notations, we omit the household index j, anticipating the symmetric equilibrium. The representative household in home country works $h_{h,t}$ hours in home coutry and $h_{f,t}$ hours in foreign country. It saves by investing in the shares of home countries, state-contingent bonds that are traded internationally and state-noncontingent givernment bonds that are available in zero net supply. The household budget constraint is given by

$$\begin{aligned}0 = W_t h_t + B(s^t) + R_{t-1}B^G_t + \int_{N_h} [\max\{D_{it}, 0\} + P^S_{i,t-1,t}] s^S_{i,t} di \\ - \sum_{k=h,f} \int_{N_k} P_{i,k,t} c_{i,k,t} di - B^G_{t+1} - \int_{\mathcal{S}} M_t(s_{t+1}|s^t) B(s^{t+1}) ds - \int_{N_h} P^S_{i,t} s^S_{i,t+1} di \qquad (32)\end{aligned}$$

where $s^t \equiv s_0, \ldots, s_t$. A unit of state-contingent bond $B(s^{t+1})$ pays out one unit of home currency on the realization of state s_{t+1} and $M_t(s_{t+1}|s^t)$ is the price of the bond at time t. B^G_{t+1} is the government bond, and r_t is the interest rate on government bond. Using the accounting identity, $\int_{N_k} P_{i,k,t} c_{i,k,t} di = \tilde{P}_{k,t} x_{k,t}$ for $k = h, f$ (see the appendix), we can simplify the last term in the budget constraint as $\sum_{k=h,f} \tilde{P}_{k,t} x_{k,t}$. $s^S_{i,t}$ is the shares of outstanding stocks of home country firm i, $P^S_{i,t-1,t}$ is the time t value of shares outstanding at time $t-1$ and $P^S_{i,t}$ is the ex-dividend value of shares at time t. The last two terms are related via the accounting identity, $P^S_{i,t} = P^S_{i,t-1,t} + E^S_{i,t}$ where $E^S_{i,t}$ is the value of new shares issued at time t. The costly equity finance assumption implies that $E^S_{i,t} = -(1-\varphi)\min\{D_{i,t}, 0\}$. Using this relationship, we can express the budget constraint only in terms of $P^S_{i,t}$:

$$\begin{aligned}0 = W_t h_t + B(s^t) + R_{t-1}B^G_t + \int_{N_h} (\tilde{D}_{i,t} + P^S_{i,t}) s^S_{i,t} di \\ - \sum_{k=h,f} \tilde{P}_{k,t} x_{k,t} - B^G_{t+1} - \int_{\mathcal{S}} M(s_{t+1}|s^t) B(s^{t+1}) ds - \int_{N_h} P^S_{i,t} s^S_{i,t+1} di \qquad (33)\end{aligned}$$

where $\tilde{D}_{i,t} \equiv \max\{D_{i,t}, 0\} + (1-\varphi)\min\{D_{i,t}, 0\}$.

The above expression makes it clear that costly equity finance takes the form of sales of new shares at a discount in general equilibrium. Since the owners of old and new shares are the same entity, there is no direct wealth effect associated with costly equity financing: the losses of the old shareholders exactly offset the gains of the new shareholders. Denoting the multiplier for the budget by Λ_t and maximizing (1) subject to (33) yields

$$x_{h,t} : \Lambda_t \tilde{P}_{h,t} = \omega_h \frac{x_t}{x_{h,t}} U_{x,t} \qquad (34)$$

$$x_{f,t} : \Lambda_t \tilde{P}_{f,t} = \omega_f \frac{x_t}{x_{f,t}} U_{x,t} \qquad (35)$$

$$h_t : \Lambda_t W_t = -U_{h,t} \qquad (36)$$

$$B(s^{t+1}) : \Lambda_t M(s_{t+1}|s^t) = \beta \Pr(s_{t+1}|s^t) \Lambda(s_{t+1}|s^t) \qquad (37)$$

$$B^G_{t+1} : \Lambda_t = \beta \mathbb{E}_t[\Lambda_{t+1} R_t] \tag{38}$$

$$s^S_{i,t+1} : \Lambda_t = \beta \mathbb{E}_t\left[\Lambda_{t+1}\left(\frac{\mathbb{E}^a_{t+1}[\tilde{D}_{i,t+1}] + P^S_{t+1}}{P^S_t}\right)\right] \tag{39}$$

where we use $P^S_t = P^S_{i,t}$ in our symmetric equilibrium. Note that (34) and (35) imply $\Lambda_t \sum_{k=h,f} \tilde{P}_{k,t} x_{k,t} = x_t U_{x,t}$. Since $\sum_{k=h,f} \tilde{P}_{k,t} x_{k,t} = \tilde{P}_t x_t$ holds from an accounting identity (see the appendix), (34) and (35) are equivalent to $\tilde{P}_t \Lambda_t = U_{x,t}$. Combining this condition with (37) yields

$$M_t(s_{t+1}|s^t) = \beta \frac{U_x(s^{t+1})/\tilde{P}_{t+1}}{U_{x,t}/\tilde{P}_t} \Pr(s_{t+1}|s^t) = \beta \frac{U_x(s^{t+1})/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_t} \frac{P_t}{P_{t+1}} \Pr(s_{t+1}|s^t) \tag{40}$$

In the symmetric equilibrium, $s_{i,k,t-1} = s_{k,t-1}$, and thus it is straightforward to show that $\tilde{p}_t = [\sum_{k=h,f} \omega_k p_{k,t}^{1-\varepsilon} s_{k,t-1}^{\theta(1-\varepsilon)}]^{1/(1-\varepsilon)}$(see the appendix, (A.16)). A similar condition to (40) holds for foreign representative household, i.e.,

$$M_t(s_{t+1}|s^t) = \frac{S_t P^*_t}{S(s^{t+1}) P^*_{t+1}} \beta \frac{U^*_x(s^{t+1})/\tilde{p}^*_{t+1}}{\tilde{U}^*_{x,t}/\tilde{p}^*_t} \Pr(s_{t+1}|s^t) \tag{41}$$

(40) and (41) then jointly imply the risk sharing condition:

$$q_t = \kappa \frac{\tilde{U}^*_{x,t}}{\tilde{U}_{x,t}} \text{ where } \kappa \equiv q_0 \frac{\tilde{U}_{x,0}}{\tilde{U}^*_{x,0}} \text{ and } \tilde{U}_{x,t} \equiv U_{x,t}\left[\sum_{k=h,f} \omega_k p_{k,t}^{1-\varepsilon} s_{k,t-1}^{\theta(1-\varepsilon)}\right]^{-1/(1-\varepsilon)} \tag{42}$$

Note that summing the prices of all state-contingent bonds yields firms' discounting factor

$$\int_{\mathcal{S}} M(s_{t+1}|s^t) ds = \mathbb{E}_t[m_{t,t+1}/\pi_{t+1}] = R_t^{-1} \text{ where } m_{t,t+1} \equiv \beta \tilde{U}_{x,t+1}/\tilde{U}_{x,t}.$$

We assume that the monetary authorities of the two countries control the prices of the government bonds using an identical Taylor-type rule:

$$R_t = R^{1-\rho_R}\left[R_{t-1}\left(\frac{y_t}{y}\right)^{\rho_c}\left(\frac{\pi_t}{\pi}\right)^{\rho_\pi}\right]^{\rho_R} \tag{43}$$

where a straightforward algebra can show that the inflation rate π_t are determined as

$$\pi_t = \left[\sum_{k=h,f} \omega_k (p_{k,t-1}\pi_{k,t})^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \tag{44}$$

The foreign counterpart of (43) is symmetrically specified as

$$R^*_t = R^{1-\rho_R}\left[R^*_{t-1}\left(\frac{y^*_t}{y^*}\right)^{\rho_c}\left(\frac{\pi^*_t}{\pi^*}\right)^{\rho_\pi}\right]^{\rho_R}. \tag{45}$$

The risk sharing condition implies that the FOC of the home investor for the government bond can be rewritten

$$1=\beta\mathbb{E}_t\left[\frac{\tilde{U}_{x,t+1}}{\tilde{U}_{x,t}}\frac{R_t}{\pi_{t+1}}\right]=\beta\mathbb{E}_t\left[\frac{\tilde{U}^*_{x,t+1}}{\tilde{U}^*_{x,t}}\frac{R_t}{\pi_{t+1}}\frac{q_t}{q_{t+1}}\right] \tag{46}$$

Similarly, for the FOC of the foreign investor for the foreign government bond, the following holds.

$$1=\beta\mathbb{E}_t\left[\frac{\tilde{U}^*_{x,t+1}}{\tilde{U}^*_{x,t}}\frac{R^*_t}{\pi^*_{t+1}}\right]=\beta\mathbb{E}_t\left[\frac{\tilde{U}_{x,t+1}}{\tilde{U}_{x,t}}\frac{R^*_t}{\pi^*_{t+1}}\frac{q_{t+1}}{q_t}\right] \tag{47}$$

(46) and (47) show that the assumption of non cross-border holdings of govenrment bonds are innocuous because the risk sharing condition makes the assumption irrelevant.

2.8.2 Incomplete Risk Sharing Under Floating Exchange Rate

To analyze the effects of imcomplete risk sharing, we consider an alternative environment where the two countries trade state-noncontingent bonds. As in Ghironi and Melitz [2005], we assume that there are portfolio rebalancing costs associated with changing the level of capital accounts. These costs are specified as a short cut to real frictions that hinder efficient borrowing/lending across borders.[2] We denote home country's holdings of international bonds issued in home and foreign currency units by $B_{h,t+1}$ and $B_{f,t+1}$.[3] $B^*_{h,t+1}$ and $B^*_{f,t+1}$ denote the foreign counterparts. The interest rates are denoted by R_t and R^*_t, respectively.[4]

The portfolio rebalancing costs are specified as $(\tau/2)P_t[(B_{h,t+1}/P_t)^2+q_t(B_{f,t+1}/P^*_t)^2]$, which implies that any deviation from zero is costly, creating inelastic supply of international funds for borrowing. Under these assumptions, the marginal cost of borrowing in home currency unit is given by $R_t/(1+\tau B_{h,t+1}/P_t)$, which is strictly greater than R_t if $B_{h,t+1}<0$. The marginal return on foreign lending in home currency unit is given by $R_t(S_t/S_{t+1})/(1+\tau B^*_{h,t+1}/P^*_t)$, which is strictly less than $R_t(S_t/S_{t+1})$ if $B^*_{h,t+1}>0$. Thus, $(1+\tau B_{h,t+1}/P_t)^{-1}$ represents a welfare loss not only to borrowers but also for lenders. By varying the value of τ, one can approximate a range of international borrowing/lending relationships, including the one that is fairly close to autarky. The intertemporal budget constraint of home country household is now given by

$$\begin{aligned}0=&\,W_th_t+R_{t-1}B_{h,t}+S_tR^*_{t-1}B_{f,t}+\int_{N_h}(\tilde{D}_{i,t}+P^S_{i,t})s^S_{i,h,t}di\\&-\tilde{P}_tx_t-B_{h,t+1}-S_tB_{f,t+1}-\frac{\tau}{2}P_t\left[\left(\frac{B_{h,t+1}}{P_t}\right)^2+q_t\left(\frac{B_{f,t+1}}{P^*_t}\right)^2\right]-\int_{N_h}P^S_{i,t}s^S_{i,t+1}di\end{aligned} \tag{48}$$

The efficiency conditions of the household problem do not change from (34), (35) and (36)

---

[2] One of the fundamental barrier to efficent allocation of international funds is soverign default, and thus employing a nonlinear framework such as Eaton and Gersovitz [1981] type would be ideal. However, given the large state space, such nonlinearity is too costly for current analysis. We leave that for later study.

[3] $B_{h,t+1}+B^*_{h,t+1}=0$, where $B_{h,t+1}$ and $B^*_{h,t+1}$ are issued in home currency as denoted by the subscripts, and are held by home and foreign countries as denoted by asterisk or lack thereof.

[4] For the imcomplete risk sharing model, we assume away government bond. However, this does not create any material difference in equlibrium because we assume zero net supply of such bond even for the complete risk sharing environment. Such bond is employed just for valuation purpose.

except for international bond holdings, which are given by

$$B_{h,t+1}: \Lambda_t(1+\tau b_{h,t+1}) = \beta\mathbb{E}_t\Big[\Lambda_{t+1}R_t\Big] \tag{49}$$

$$B_{f,t+1}: \Lambda_t(1+\tau b_{f,t+1}) = \beta\mathbb{E}_t\Big[\Lambda_{t+1}R_t^*\frac{S_{t+1}}{S_t}\Big] \tag{50}$$

where $b_{h,t+1} \equiv B_{h,t+1}/P_t$ and $b_{f,t+1} \equiv B_{f,t+1}/P_t^*$. The monetary policy is specified in an indentical way to (43). Substituting $\tilde{P}_t\Lambda_t = U_{x,t}$ in (49) and (50), one can rewrite them as

$$1+\tau b_{h,t+1} = \beta\mathbb{E}_t\left[\frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t+1}/\tilde{p}_{t+1}}\frac{R_t}{\pi_{t+1}}\right] \tag{51}$$

$$\text{and } 1+\tau b_{f,t+1} = \beta\mathbb{E}_t\left[\frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t+1}/\tilde{p}_{t+1}}\frac{q_{t+1}}{q_t}\frac{R_t^*}{\pi_{t+1}^*}\right] \tag{52}$$

The bond market clearing conditions are given by

$$0 = b_{h,t+1} + b_{h,t+1}^* \tag{53}$$

$$\text{and } 0 = b_{f,t+1} + b_{f,t+1}^* \tag{54}$$

where foreign holdings of international bonds in home and foreign currencies $b_{h,t+1}^*$ and $b_{f,t+1}^*$ satisfy

$$1+\tau b_{h,t+1}^* = \beta\mathbb{E}_t\left[\frac{U_{x,t+1}^*/\tilde{p}_{t+1}^*}{U_{x,t+1}^*/\tilde{p}_{t+1}^*}\frac{q_t}{q_{t+1}}\frac{R_t}{\pi_{t+1}}\right] \tag{55}$$

$$\text{and } 1+\tau b_{f,t+1}^* = \beta\mathbb{E}_t\left[\frac{U_{x,t+1}^*/\tilde{p}_{t+1}^*}{U_{x,t+1}^*/\tilde{p}_{t+1}^*}\frac{R_t^*}{\pi_{t+1}^*}\right]. \tag{56}$$

for foreign investors. Assuming that the portfolio rebalancing cost is transferred back to the household in lump sum, imposing the stock market equilibrium condition $s_{i,h,t}^S = s_{i,h,t+1}^S = 1$, and finally dividing the budget constraint through by P_t, one can rewrite it as a law of motion for bond holdings, i.e.,

$$b_{h,t+1} + q_t b_{f,t+1} = \frac{R_{t-1}}{\pi_t}b_{h,t} + \frac{R_{t-1}^*}{\pi_t^*}q_t b_{f,t} + w_t h_t + \tilde{d}_t - \tilde{p}_t x_t. \tag{57}$$

where $\tilde{d}_t \equiv \tilde{D}_t/P_t$ and $\tilde{d}_t^* \equiv \tilde{D}_t^*/P_t^*$. One can derive a similar law of motion for the foreign country,

$$q_t^{-1}b_{h,t+1}^* + b_{f,t+1}^* = \frac{R_{t-1}}{q_t\pi_t}b_{h,t}^* + \frac{R_{t-1}^*}{\pi_t^*}b_{f,t}^* + w_t^* h_t^* + \tilde{d}_t^* - \tilde{p}_t^* x_t^*. \tag{58}$$

After multiplying q_t to (58), subtracting (58) from (57) and imposing the bond market clearing conditions (53) and (54) yields

$$b_{h,t+1} + q_t b_{f,t+1} = \frac{R_{t-1}}{\pi_t}b_{h,t} + \frac{R_{t-1}^*}{\pi_t^*}q_t b_{f,t} + \frac{1}{2}(w_t h_t - q_t w_t^* h_t^*) + \frac{1}{2}(\tilde{d}_t - q_t\tilde{d}_t^*) - \frac{1}{2}(\tilde{p}_t x_t - q_t\tilde{p}_t^* x_t^*). \tag{59}$$

This condition replaces the risk sharing condition (42) in equilibrium.

2.8.3 Complete Risk Sharing Under Currency Union

We now consider the situation where the two countries bilaterally agree to adopt a single currency. One can think of the situation in the following way. Once the two countries adopt a single currency, all products and financial assets are nominated in the single currency unit. As a result, the nominal exchange rate is not defined. Also as a result, a single monetary authority sets the monetary policy rate, denoted by R_t^U, and all investors, regardless of their countries of origin and current locations, earn the same nominal return.

However, depending on the reference location of investors, the real return on international bond holdings differs. This is due to two factors: first, the two countries have different consumption baskets in the long-run owing to heterogeneous home biases; second, at any point in given time, the law of one price is violated as the any two consumers residing in different countries have accumulated heterogeneous degrees of habits for an identical product, and as a consequence, firms, in general, price their products to their markets, so called "pricing to habits" (see Ravn, Schmitt-Grohé, and Uribe [2007]). Hence, inflation rates are not equalized in the two countries despite the adoption of single currency and monetary policy, and the real returns on international bonds are not equalized either.

We assume that the common monetary policy is specified so as to reflect the economic fundamentals of both countries:

$$R_t^U = (R^U)^{1-\rho_R}\left[R_{t-1}^U\left(\frac{y_t^U}{y^U}\right)^{\rho_c}\left(\frac{\pi_t^U}{\pi^U}\right)^{\rho_\pi}\right]^{\rho_R} \tag{60}$$

where the union-wide variables are constructed as a weighted average with the weights given by the steady state share of output[5], i.e.,

$$y_t^U = y_t\left(\frac{y}{y+qy^*}\right) + q_t y_t^*\left(\frac{qy^*}{y+qy^*}\right) \tag{61}$$

$$\text{and } \pi_t^U = \pi_t\left(\frac{y}{y+qy^*}\right) + \pi_t^*\left(\frac{qy^*}{y+qy^*}\right). \tag{62}$$

As a complete risk sharing regime, the currency union continues to ensure the risk sharing condition (42) that prevails under the floating exchange rate regime. However, under the currency union, only one of the two consumption Euler equations (46) and (47) can be included in the system of equations that constitute the equilibrium. This is because the combination of the single monetary policy and the risk sharing condition makes the two consumption Euler equations linearly dependent. Hence, only the following efficiency condition enters the system with (42):[6]

$$1 = \beta\mathbb{E}_t\left[\frac{\tilde{U}_{x,t+1}}{\tilde{U}_{x,t}}\frac{R_t^U}{\pi_{t+1}}\right] \tag{63}$$

---

[5] We have tried different formulae, real time or lagged weights, and have found no material difference in model dynamics.

[6] Otherwise, the two consumption Euler equations held withing the system together with the common monetary policy and risk sharing condition would imply $\pi_{t+s} = \pi_{t+s}^*$ for all s, which cannot be satisfied.

2.8.4 Incomplete Risk Sharing Under Currency Union

Under the combination of incomplete risk sharing and currency union, the combined law of motion for international bond holdings (59) should be replaced with

$$b_{h,t+1} = \frac{R_{t-1}}{\pi_t} b_{h,t} + \frac{1}{2}(w_t h_t - q_t w_t^* h_t^*) + \frac{1}{2}(\tilde{d}_t - q_t \tilde{d}_t^*) - \frac{1}{2}(\tilde{p}_t x_t - q_t \tilde{p}_t^* x_t^*). \tag{64}$$

as there is no longer distinction between international bonds issued in home vs foreign currencies. In addition, the bond market clearing condition drops out of the system of equations. Furthermore, there are only two, instead of four, Euler equations characterizing the efficiency in the international bond markets:

$$b_{h,t+1} \ : \ 1 + \tau b_{h,t+1} = \beta \mathbb{E}_t \left[\frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t+1}/\tilde{p}_{t+1}} \frac{R_t^U}{\pi_{t+1}} \right] \tag{65}$$

$$\text{and } b_{h,t+1}^* \ : \ 1 + \tau b_{h,t+1}^* = \beta \mathbb{E}_t \left[\frac{U_{x,t+1}^*/\tilde{p}_{t+1}^*}{U_{x,t+1}^*/\tilde{p}_{t+1}^*} \frac{q_t}{q_{t+1}} \frac{R_t^U}{\pi_{t+1}} \right] \tag{66}$$

Note that using the definition of the real exchange rate, we have as an identity

$$\frac{q_t}{q_{t+1}} = \frac{S_t}{S_{t+1}} \cdot \frac{\pi_{t+1}}{\pi_{t+1}^*}. \tag{67}$$

However, the currency union implies $S_t/S_{t+j} = 1$ for $j \geq 1$ permanently[7], and (66) becomes equivalent to

$$1 + \tau b_{h,t+1}^* = \beta \mathbb{E}_t \left[\frac{U_{x,t+1}^*/\tilde{p}_{t+1}^*}{U_{x,t+1}^*/\tilde{p}_{t+1}^*} \frac{R_t^U}{\pi_{t+1}^*} \right] \tag{68}$$

While an indentical nominal return enters (65) and (68), their real and nominal discounting factors differ.

3 Results

3.1 Calibration

Our calibration strategy closely follows that of Gilchrist, Schoenle, Sim, and Zakrajsek [2013], expanding the set of parameters as needed for the international environment. There are three sets of parameters in the model: parameters related with preferences and technology; parameters governing the strength of nominal rigidity and monetary policy; parameters determining the strength of financial market friction, including portfolio rebalancing costs.

We set the time discounting factor equal to 0.995. We set the deep habit parmeter θ equal to -0.9 similar to the choice of Ravn, Schmitt-Grohé, and Uribe [2005]. The key tension between the market share maximization and cash flow maximization does not exist when $\theta = 0$. In this

---

[7] We assume that it is impossible to exit the union unilaterally. See Alvarez and Dixit [2014]'s real option approach for a theoretical consideration of the break up of a currency union.

Table 1: Baseline Calibration

Description	Calibration
Preferences and production	
Time discounting factor, β	0.99
Constant relative risk aversion, γ_x	2.00
Deep habit, θ	$-$ 0.80
Persistence of deep habit, ρ	0.95
Elasticity of labor supply, $1/\gamma_h$	5.00
Elasticity of substitution, η	2.00
Armington elasticity, ε	1.50
Home bias, ω_h^{ε}	0.60
Persistence of technology shock, ρ_A	0.90
Returns to scale, α	1.00
Fixed operation cost, ϕ, ϕ^*	0.30, 0.00
Nominal rigidity and monetary policy	
Price adjustment cost, γ_p	10.0
Wage adjustment cost, γ_w	30.0
Monetary policy inertia, ρ^R	0.85
Taylor rule coefficient for inflation gap, ρ^π	$0.25/(1-\rho^R)$
Taylor rule coefficient for inflation gap, ρ^π	$1.50/(1-\rho^R)$
Financial Frictions	
Equity issuance cost, φ	0.30
Idiosyncratic volatility (a.r.), σ	0.10
Persistence of financial shock, ρ_φ	0.85

environment, the financial shock we consider in this paper has no effects on real outcome. It is in this sense that the current model owes a lot to customer market settings such as "deep habit" model. We choose a fairly persistent habit formation such that only 10 percent of the habit stock is depreciated in a quarter to highlight the firms' incentive to compete on market share. The CRRA parameter is then set equal to one given that the deep habit specification provides a strong motive to smooth consumption. We set the elasticity of labor supply equal to 5. For the aggregate technology shock process, we assume $\rho_A = 0.90$, somewhat lower a value than those employed by real business cycle analysis, given that the model has a number of elements that generate persistent dynamics of the endogenous quantities.

The elasticity of substitution is a key parameter in the customer-markets model as the greater the market power the firm has, the greater incentive to invest in customer capital. We set the elasticity equal to 2 to be consistent with Broda and Weinstein [2006], who provide a set of point estimates for the elasticity of substitution for the U.S. economy. The estimates hover around 2.1∼4.8, depending on the characteristics of products (commodities vs differentiated goods) and sub-samples (before 1990 vs after 1990). Our choice is virtually indentical with the point estimate of Broda and Weinstein [2006] for the median value of the elasticity of differentiated goods of 2.1 for the differentiated products for a sub-sample period since 1990. The differentiated product is the relevant category for the deep habit model considered in this paper. The choice is also broadly consistent with Ravn, Schmitt-Grohe, Uribe, and Uuskula [2010]'s point estimate of 2.48 using their structural estimation method.

Regarding ω_h and ω_f, the weights of home and foreign goods in the untility function, we set these such that the share of imported goods in steady state consumption basket $(p_f c_f / \sum_{k=h,f} p_k c_k)$ is equal to 0.4. 0.4 is in the middle range of import/GDP ratios of European countries since 2000. For instance, Germany has 0.46 while Spain, Italy and Greece have 0.35 on average in 2012. Note that ω_f itself is not equal to the imported goods' share. ω_f should be set equal to $(p_f c_f / \sum_{k=h,f} p_k c_k)^{1/\varepsilon}$. As for the Armington elasticity, we choose 1.5 to stay close to the near unit elasticity estimated by Feenstra, Luck, Obstfeld, and Russ [2014]. However, as long as greater than 1, a value lower than our choice do not produce much difference to our main conclusion. For instance, lowering this value to 1.01 reduces the impact of financial shock on the aggregate output under currency union to 2/3 of the baseline calibration. This is because the lower elasticity of cross-border substitution implies a lower degree of price war among countries. However, even in this polar case, the qualitative features of the equilibrium remains the same.

Another important parameter is the fixed opearating cost, ϕ. This parameter is jointly determined with the returns to scale parameter α. We set α first, then choose ϕ such that dividend payout ratio (relative to income) hits the post war mean value 2.5 percent in U.S. Decreasing returns to scale enhances the link between the financial market friction and the pricing decision. For this reason, we chose $\alpha = 0.8$ in our earier study, Gilchrist, Schoenle, Sim, and Zakrajsek [2013]. However, in the current paper, we choose $\alpha = 1.0$ to be consistenet with the convention in international macroeconomics literature. In any event, the difference between 0.8 and 1.0 is marginal. With the chosen α, ϕ and η, the average mark-up is determined as 1.19. We then experimented with various values for ϕ^* to explore the implication of heterogenous financial friction for the member countries of the currency union. To emphasize this aspect, we set $\phi^* = 0$, i.e., the foreign country in the model does not face any fixed operating costs. While extreme, setting $\phi^* = \varpi\phi$ with $\varpi \in [0, 1)$ does not modify the main results of the paper in any important way. To calibrate the financial friction, we set the dilution cost $\varphi = \varphi^* = 0.30$ as 0.30 as in Cooley and Quadrini [2001]. We discuss the influence of these choices on model outcomes below. The volatility of the idiosyncratic shock is calibrated as 0.05 at a quarterly frequency, a moderate degree of idiosyncratic uncertainty.

For the parameters related to nominal rigidity, we set the adjustment costs of nominal price $\gamma_p = 10.0$. In our model presentation, we proceeded as if nominal wages were flexible. In our actual simulation, we introduce nominal rigidity for wages along the line of Bordo, Erceg, and Evans [2000] and Erceg, Henderson, and Levin [2000]. In particular, in a symmetry to the nominal price rigidity, we assume a market power for households that supply labor to production firms, and a quadratic cost of adjusting nominal wage. In this case, and under the assumption of a separable, constant elasticity of labor supply, $U_{h,t} = -h_t^{1/\zeta}$, the efficiency condition (36) is modified into

$$\eta_w \frac{h_t^{1/\zeta}/U_{x,t}}{w_t/\tilde{p}_t} = \eta_w - 1 + \gamma_w(\pi_{w,t} - \pi_w)\pi_{w,t} - \beta\mathbb{E}_t \left[\frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_t} \gamma_w(\pi_{w,t+1} - \pi_w)\pi_{w,t+1} \frac{\pi_{w,t+1}}{\pi_{t+1}} \frac{h_{t+1}}{h_t} \right] \tag{69}$$

where $\pi_{w,t} \equiv W_{t+1}/W_t$, γ_w is the coefficient of nominal wage adjusment cost, and η_w is the elasticity of substitution of labor. We choose $\eta_w = 3$ and $\gamma_w = 30$. Our choice of norminal rigidity for both price and wage are very close to the point estimates of $\gamma_p = 14.5$ and $\gamma_w = 41.0$ by Ravn, Schmitt-Grohe, Uribe, and Uuskula [2010], who show that deep habit model substantially enhances the persistence of inflation dynamics without the help of implausibly large amount of adjustment friction in nominal prices. While nominal wage rigidity does not modify the main conclusions of the paper in any important way, it does help create a greater volatility of real exchange rate. This is because the countercyclical markup of the country under a financial crisis, which is essential to the main conclusions of the paper, is achieved more by an immediate fall in nominal wage than by an increase in product price in a flexible wage environment. The relatively more stable final product prices then lead to a less volatile real exchange rate, which runs counter to our intention, in addition to being unrealistic.

Finally, we set the inertial Taylor coefficient at a conventional level of 0.85 and the long coefficient of inflation gap as 1.5, following Taylor [1993]. The long run coefficient for output gap is less obvious. In traditional New Keynesian literature, this coefficient does not play an important role due to so called “divine coincidence”, which describes the strong comovement between demand pressure measured by output gap and inflation. As a consequence, a strong reaction to inflation often makes the response to output gap redundant or even inefficient. However, this is not the case in our current paper. As shown by Gilchrist, Schoenle, Sim, and Zakrajsek [2013], the specific combination of customer market setting and financial friction breaks the divine coincidence in the sense that a strong negative demand shock under a severe financial strain may lead to a higher inflation pressure as firms under financial distress may find it optimal to raise prices in order to secure short-term liquidity, giving up on long-run market share. For this reason, we take a 50:50 prior by choosing a middle value between 0 and 0.5 suggested by Taylor [1993].

3.2 Currency Regime and Impacts of External Disturbances

In this section, we study the macroeconomic consequences of adopting a common currency, and hence a single monetary policy in an environment where member countries face heterogeneous degrees of financial frictions, and member countries neither have complete risk sharing arrangement through cross-border transfer of funds nor cross-border labor mobility. We compare the international macroeconomic dynamics under two environments: floating exchange rate regime and currency union.

3.2.1 Impact of Financial Shock

To study the effects of financail instability under various currency regimes, we a financial shock that elevates the cost of outside equity capital for member countries. More specifically, we subject the cost of issuing new equities to random shocks, and we call it the cost of capital shock:

$$\varphi_t = \bar{\varphi} f_t, \quad \log f_t = \rho_f \log f_{t-1} + \epsilon_{f,t}, \quad \epsilon_{f,t} \sim N(-0.5\sigma_f^2, \sigma_f^2) \tag{70}$$

To create a financial crisis situation initiated by a shock hitting the financially vulnerable region, which we take as 'home' country, we calibrate $\epsilon_{t,f}$ such that the cost of capital is elevated to $2\bar{\varphi}$ on impact, and gradually comes down to the normal level $\bar{\varphi}$ thereafter. In contrast, we shut down the shock to the cost of capital for the country with a substantial financial slack, which we take as 'foreign' country, i.e., $\varphi_t^* = \bar{\varphi}$ for all t.[8] The shock is designed to elevate the expected shadow value of internal funds for the firms in home country. Without the shock, the exected shadow value of internal funds in the model is calibrated to 1.16. The shock immediately increases the expected shadow value to 1.50.

As we mentioned earlier, when the shadow cost of capital is elevated, firms in the model rebalance the benefit of expanding custormer base by lowering their prices and the benefit of increasing current cashflow by temporarily raising their prices. The former strategy is profitable in the long run, but in the short run, exposes the firms to risk that firms take greater orders from customers before the resolution of indiosyncratic uncertainty about their marginal costs, and hence the risk of having to rely on costly external financing. The latter strategy only makes sense when firms have 'loyal' and 'sticky' customer base.[9] When the expected cost of outside capital is substantially elevated, the firms are likely to lean toward the latter option, i.e., raising their prices in order to limit the liquidity risk exposure.

Figure 3 displays the impact of the shock under a floating exchange rate regime. Blue, solid line shows the case of home country and red, dash-dotted line the case of foreign country. Panel (f) of figure 3 shows that the firms in the two countries indeed increase their prices in response to the financial strain. This results are consistent with the pattern we have shown in our earlier paper (Gilchrist, Schoenle, Sim, and Zakrajsek [2013]), but in a closed economy setting. Since the shock affects home country directly and only indirectly foreign country, the response in inflation is disproportionately larger for home country. In our empirical section, we provide some evidence for this pattern in the data in the context of European financial crisis during the period of 2008-2012.

If nominal exchange rate does not respond to the shock, the differential responses of inflation rates of the two country would imply a substantial appreciation of real exchange rate for home country. However this is not the case under a floating exchange rate regime. As shown in panel (3) of the figure, the nominal exchange rate, shown by red, dash-dotted line, strongly depreciates. In fact, the depreciation is strong enough that the real exchange rate also depreciates despite the price differential moving in the opposite direction. As in the data, the real exchange rate dynamics in the short run is dominated by the nominal exchange rate rather than price changes. Note that in panel (e), the deviation of nominal exchange rate looks like disappearing in the long run. However, this is simply a coincidence. The New Keyinesian framework adopted in the current paper does not have a prediction for the level of nominal exchange rate just as it does not have one for price level.[10] There is no reason why the levels of price index and nominal exchange rate should converge

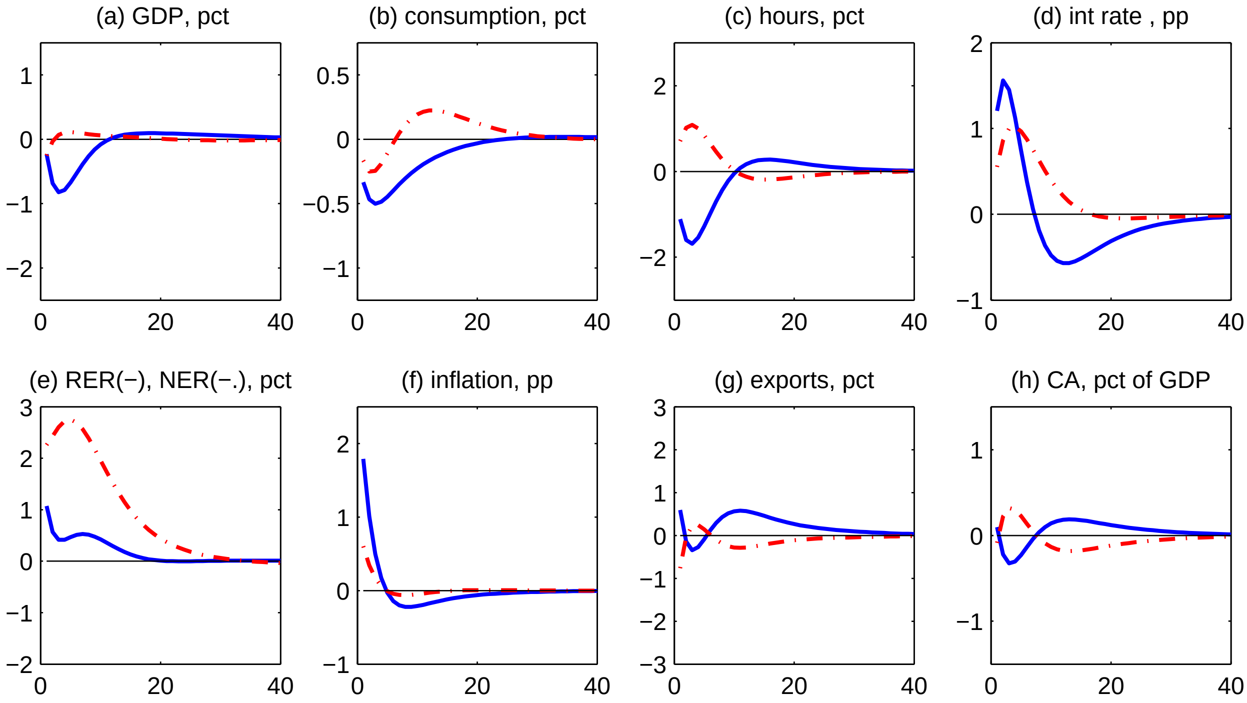
---

[8] However, in evaluating the consequences of different currency regimes, we assume that the foreign country is also subject to its own financial shocks given by $\varphi_t^* = \bar{\varphi} f_t^*, \quad \log f_t^* = \rho_f \log f_{t-1}^* + \epsilon_{f,t}^*, \quad \epsilon_{f,t}^* \sim N(-0.5\sigma_f^2, \sigma_f^2)$.

[9] In turn, that is the very reason why they want to invest in customer base.

[10] The figure assumes that the initial value of nominal exchange rate is one, which is an arbitrary, but innocuous

Figure 3: Financial Shock to Peripheral Country Under Floating



Note: Blue, solid line is the peripheral country and red, dash-dotted line is the core country. The shock assumes that the dilution cost for the peripheral countries go up by 100

to specific levels.

The short-run depreciation of nominal exchange rate explains why GDP of home country is affected only mildly despite the large size of the financial shock.[11] Panel (a) shows that GDP of home country drops about one percent in the trough. The drop in aggregate consumption is even milder: only about a half percent. This is owing to relatively strong, initial gains in export, shown in panel (g), which is driven by the depreciation of real exchange rate. While short lived, the depreciation of nominal exchange rate helps firms avoid having to increase the relative prices of export products in foreign market too much with a view to increasing short-run liquidity precisely because the nominal depreciation does the job partially. This keeps them from losing their export market shares too much and thus limits the overall downside risk of the economy.

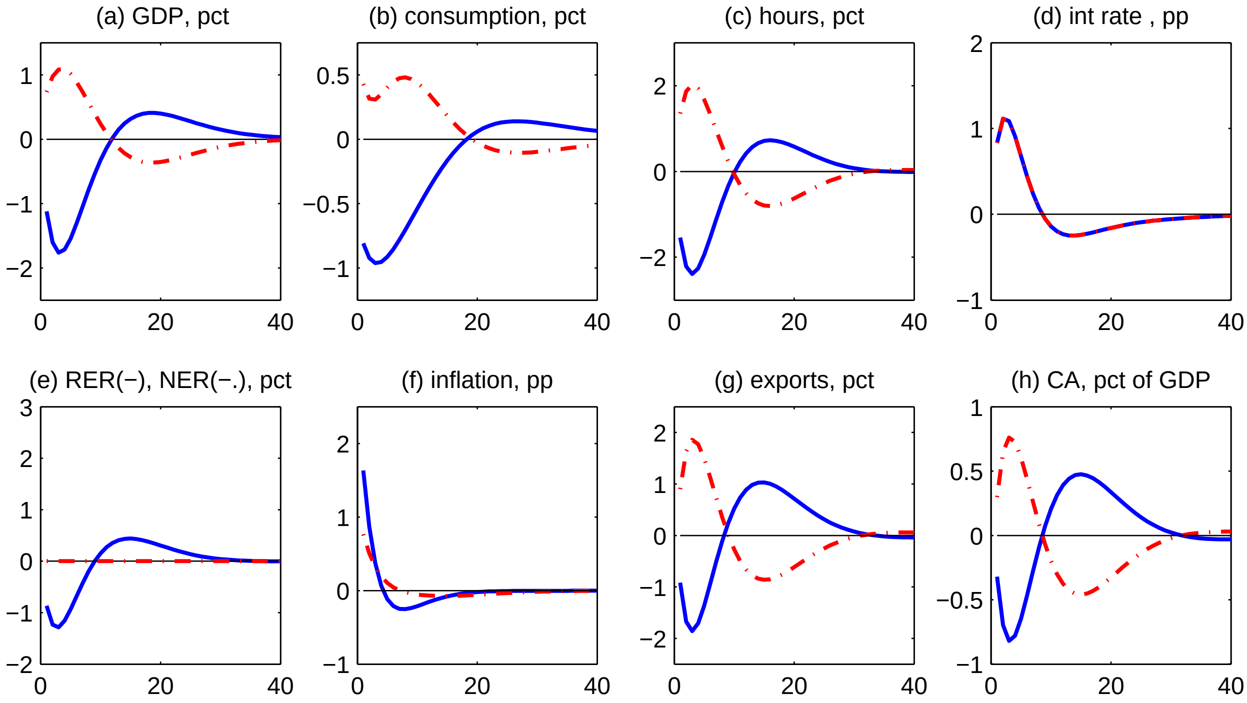
Figure 4 shows a dramatically different pattern of international macroeconomic adjustment ensuing the financial shock under the currency union. In panel (a) through (c), one can see that the drops in GDP, consumption and hours of home country are almost two times greater under the currency union than under the floating exchange rate regime. In panel (g), the troughs of export and current account deficit (relative to GDP) of home country is almost 7 times and three times greater with the currency union. More strikingly, the financial crisis of home country is coupled with a modest boom in foreign country: GDP, consumption and hours go up by 1, 0.5 and 2 percent

---

assumption. However, the inflation rate of nominal exchange rate is well defined and is one of the model variables.

[11] GDP in the model is defined as domestic consumption plus export minus import, i.e., $p_t c_t + q_t p^*_{h,t} c^*_{h,t} - p_{f,t} c_{f,t}$. This is not equal to the volume index of aggregate output, y_t.

Figure 4: Financial Shock to Peripheral Country Under Monetary Union



Note: Blue, solid line is the peripheral country and red, dash-dotted line is the core country. The shock assumes that the dilution cost for the peripheral countries go up by 100

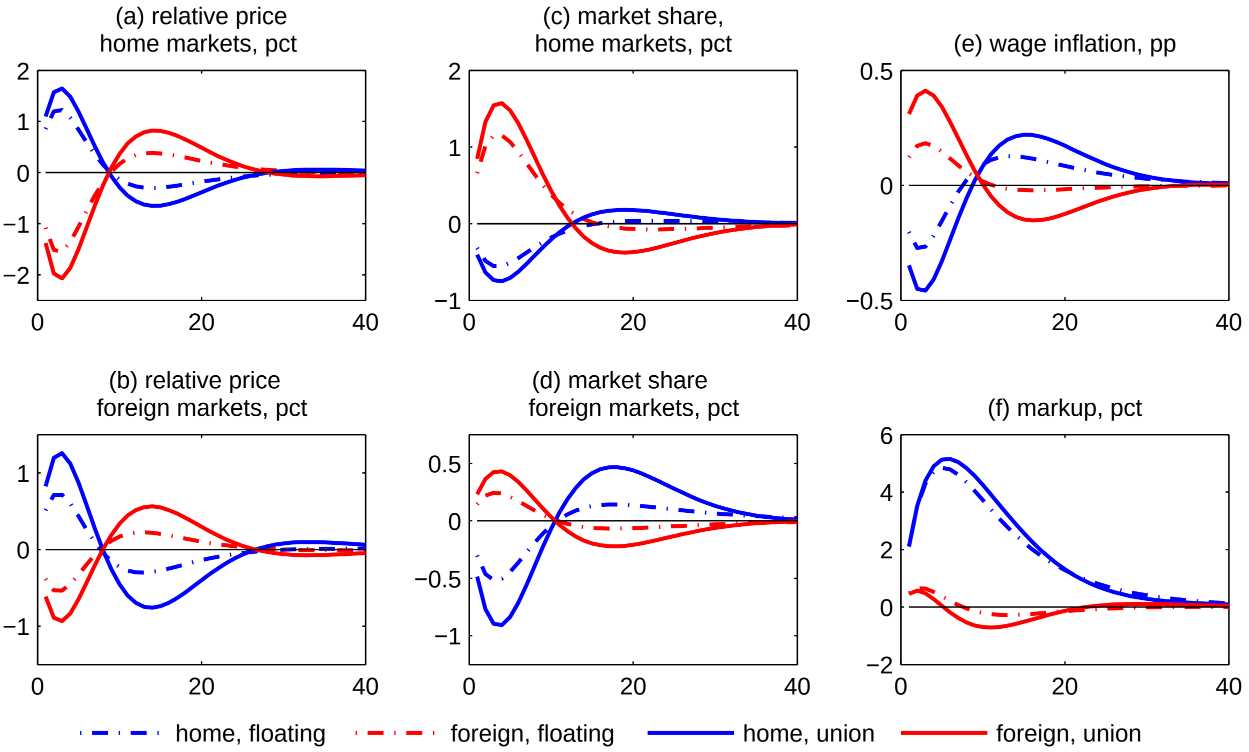
each. The export and current account surplus (relative to GDP) of foreign country to home country go up by 2 and 3/4 percent, respectively.

The mechanism behind the stark difference in international macroeconomic adjustment patterns can be seen in the panel (d) through (f) of figure 4. Owing to the financial friction, the firms in the country hit by the financial shock has a greater incentive to raise their prices more than its trading partner. This pattern in the inflation differential during the financial strain is similar across the two currency regimes as shown by panel (f)s of figure 3 and 4. What is different under the currency union is the real exchange rate dynamics. Under a floating exchange rate regime, the international bond holding conditions (51) and (52) impliy the following no-arbitrage condition:

$$\tau(b_{h,t+1} - b_{f,t+1}) = \mathbb{E}_t\left[m_{t,t+1}\left(\frac{R_t}{\pi_{t+1}} - \frac{q_{t+1}}{q_t}\frac{R_t^*}{\pi_{t+1}^*}\right)\right]. \tag{71}$$

In equilibrum with a relatively small cost of portfolio rebalancing, the left side is close to zero to a first order approximation. This means $R_t/\pi_{t+1} - (q_{t+1}/q_t)(R_t^*/\pi_{t+1}^*) \approx 0$ in expectation. As shown in panel (d) and (f) of figure (3), nominal interest differential between home and foreign countries is smaller than inflation differential, i.e., the real interest rate is lower in home country than in foreign country, which is intuitive given the recession in home country. The *absence of capital control* then implies that the real exchange rate should appreciate over time ($q_{t+1}/q_t < 1$) to avoid the exodus of capital from home to foreign country. This requires that nominal exchange rate should *depreciate*

Figure 5: Price Wars and Market Shares During A Financial Crisis



Note: Solid lines are the cases of the currency union with blue and red indicating the periphery and the core, respectively. Dash-dotted lines are the case of the floating exchange rate regime with the same color convention.

today such that $q_{t+1}/q_t < 1$. This restriction from free capital account is exactly what is missing in the currency union. The bond marke efficinecy conditions (65) and (68) impose no restriction on the dynamics of real exchange rate. While the real interest differential still exists as in the case of the floating exchange regime, the differential does not have to be compensated by expected changes in nominal exchange rate: one can never exit from one currency to the same currency. (65) and (68) jointly requires[12]

$$1 = \mathbb{E}_t \left[\frac{1}{2} \left(\frac{m_{t,t+1}}{\pi_{t+1}} + \frac{m^*_{t,t+1}}{\pi^*_{t+1}} \right) R^U_t \right], \tag{72}$$

which implies that the single policy rate should be set according to the *average* fundamenatal of the two economies regardless of the coefficients of the monetary policy reaction function (60).

As a consequence, any differential in inflation rates is directly translated into the movement in real exchange rate. Since the model's financial friction implies that the financially more vulnerable firms optimally choose higher relative prices, the real exchange rate appreciates substantially. This causes the export of home country to contract severely and so does GDP. The consumption does not contract as much as GDP as international borrowing, while subject to the costly rebalancing friction, allows consumers in home country to smooth out the effects of financial crisis to a certain

---

[12] One can derive this condition by simply adding the efficiency conditions and imposing the bond market clearing condition.

degree. The boom in economic activity of foreign country is simply a mirror image of home country's economic plight. The contrast between the two countries is quite reminiscent of the European dichotomy bewteen central and pheripheral countries during the recent financial crisis.

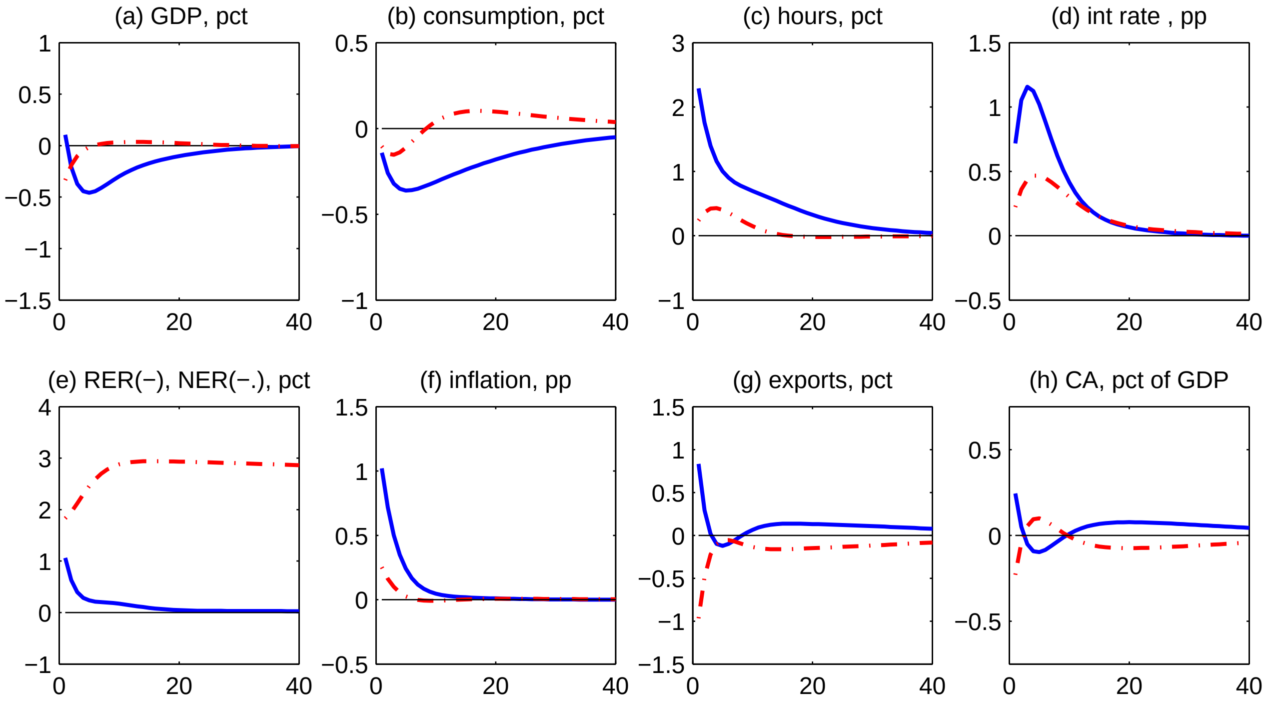
One seemingly illogical result is that despite the harsher financial environment under the currency union, neither the level of home country inflation rate nor the difference from that of foreign country are very different from those under the floating exchange rate regime. This is because certain factors of internaiotnal price wars are creating offsetting dynamics for overall inflation rates. Figure 5 provides a bit more detailed inside look of international price war during the financial crisis. The solid lines, both blue and red, show the case of the currency union with blue and red lines representing home and foreign countries, respectively. The dash-dotted lines show the case of the floating exchange rate regime with blue and red lines representing home and foreign countries, respectively.

Panel (a) and (b) show the endogenous dispersion of relative prices. Regardless of the exchange rate arrangement, the firms in the country hit by the financial shock increase their relative prices both in their domestic (panel (a)) and export markets (panel (b)). In contrast, the firms in the country with relatively ample financial resources substantially lower their relative prices. The firms in home country raises their relative prices to secure short-term liquidity, sacrificing their market shares, both in home and foreign markets (panel (c) and (d)). The firms in foreign country follows the opposite strategy to lower their relative prices to expand their market shares. It is an interesting observation that these firms slash their prices more in home country (export prices) than in foreign country (domestic prices). Note that the intensity of price war, in terms of the dispersion of prices, is much greater with the currency union. This is because the firms in home country cannot rely on the depreciation of their currency to improve their cashflow. Since the higher prices of home firms (domestic prices in home country) are offset by the lower prices of foreign firms (imported prices in home country), overall inflation rate of home country is not greatly affected by the currency regime, which explains the seemingly illogical result. Finally, note that the markup is strongly countercyclical in home country regardless of currency regime during the financial crisis (see panel (e) and (f)).

3.2.2 Impact of Technology Shock

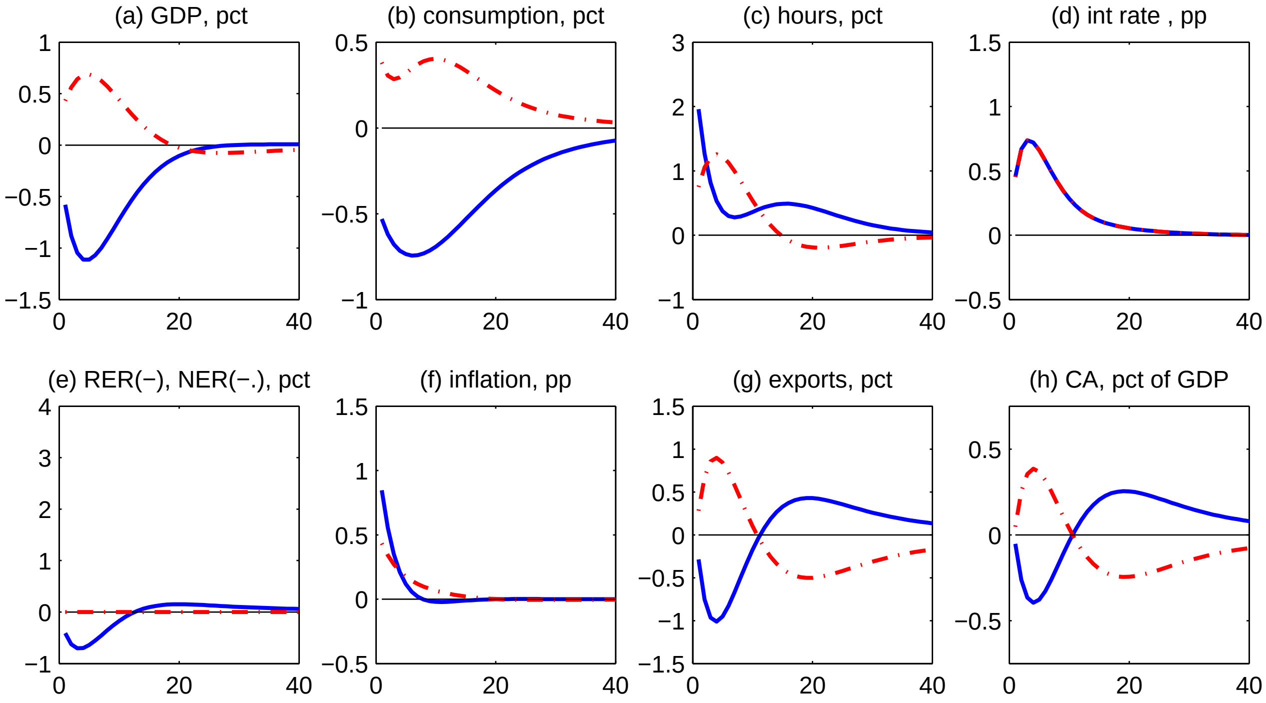
One might wonder if the results shown above are specific to the nature of the shock–the shock that raises the cost of outside capital for firms. We now show that this is not the case, and in fact, the stylized patterns shown above hold true for more conventional shocks. To that end, we use the most popular driving force of international macroeconomics: a technology shock. Figure 6 and 7 show the impacts of a technology shock to home country under the floating and currency union, respectively. One can easily recognize the same pattern of intenational macroeconomic adjustment following the technology shock: the impact of the technology shock on home country is much greater under the currency union. The contraction in GDP is more than two times greater under the currency union. The impact on consumption is almost three times greater. Perhaps, the most striking difference can

Figure 6: Technology Shock to Peripheral Country Under Floating



Note: Blue, solid line is the peripheral country and red, dash-dotted line is the core country.

Figure 7: Technology Shock to Peripheral Country Under Monetary Union



Note: Blue, solid line is the peripheral country and red, dash-dotted line is the core country.

be found in the reponse of export and current account. In response to the technology shock, home country increases its exports about 1 percent and runs a small current acount surplus under the floating exchange rate regime. In a stark contrast, home country exports decline about 1 percent at the peak of the cycle and the current account deficits amounts 0.5 percent of GDP under the currency union. As in the case of the financial shock, with the help of strong gains in exports and current account surplus, foreign country is experiencing an economic boom at a time of recession in home country.

The key mechanism behind this stark contrast in economic performances of home country under the two different currency regimes can be found in the real exchange rate dynamics. As shown in panel (e) of figure 6, the nominal exchange rate depreciates as much as 3 percent in response to the technology shock. As mentioned earlier, the New Keynesian theory does not have a prediction on the level of the nominal exchange rate, which seems to settle down on a permanently higher level. What matters is the inflation rate of the nominal exchange rate as that is the only relevant for staisfying the no-arbitrage condition (71). Without the nominal exchange rate adjustment, the differential in inflation rates shown in panel (f) would have implied a strong appreciation of the real exchange rate. However, the strong depreciation of the nominal exchange rate actually makes the real exchange rate depreciate. This limits the downside of the technology driven recession for home country. Home country essentially exports its way out with the help of real depreciation. In contrast, under the currency union, the absence of bond market arbitrage condition implies that the real exchange rate appreciates for country that is inflating its price more than its trading partner. While the focus in our study is not on the analysis of the impact of technology shock, this experiment shows well that our argument that the currency union can make situations a lot worse for financially vulnerable countries at a time of economic plight, and this conclusion is robust for the source of business cycle fluctuation.[13]

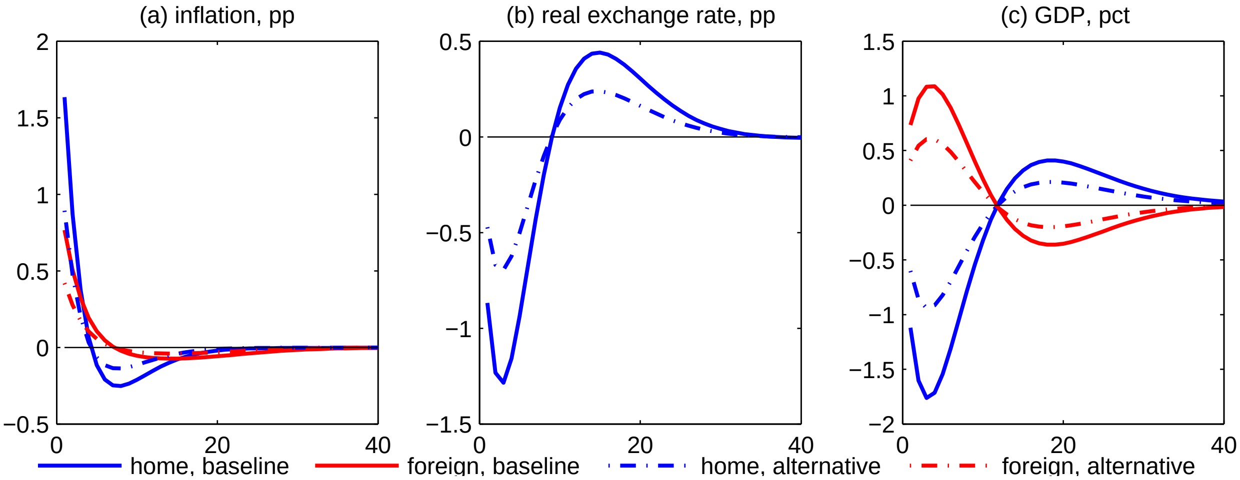
3.2.3 Financial Vulnerability and Real Exchange Rate Misalignment

We have shown that currency union may bring about undesirable business cycle properties when countries facing nontrivial *financial frictions*, which generate differential inflation dynamics between two countries in response to financial and real shocks to the country that is more vulnerable to external disturbances owing to the lack of financial buffer. Such inflation differentials, when not

---

[13] A somewhat unusual aspect of the technology shock can be seen in panel (c)s of figure 6 and 7, which show that hours increase substantially in response to a negative technology shock. There are two mechanism at work to produce this result: wealth effect on labor and the countercyclicality of labor in response to technology shock in the absense of monetary policy strongly reacting to technology shock (Gali [1999]). In the model, the real wage is weakly procyclical and the marginal utility of consumption is strongly countercyclical. This increases labor supply substantially. Meanwhile, the monopolistic competition implies that output is demand-driven in the short run, and thus, without interest rate policy negatively reacting to technology shock, labor demand is negatively related with output, producing a counterintuitive outcome for labor hours (we have verified this using an alternative monetary policy rule). Given that the initial response of hours is about positive 2 to 2.5 percent depending ot the currency regime, the aggregate output (volume) should contract about 0.5-1 percent on impact. Figure 16 in the appendix shows this is indeed the case. However, as hours go back to normal level after the first period, the dynamics of the aggregate outputs of the two economies undergo substantially different routes.

Figure 8: Financial Friction and Misalignment of Real Exchange Rate



Note: Solid lines are the baseline case with blue and red indicating the periphery and the core, respectively. Dash-dotted lines are the alternative case with zero fixed cost. The same color convention is used for the alternative case.

corrected by nominal exchange rate adjustments, may distort international trade flows and magnify the impact of initial shocks substantially.

We emphasize the importance of the financial vulnerability of member countries in establishing these results since the model does not have ability to generate a significant degree of real exchange rate misalignment without such financial friction. To illustrate this point, we use an alternative calibration that represents economies that are more resilient in their financial condition. In particular, we assume that the fixed operation cost is zero identically across the two countries. The firms in the two countries are still subject to the costly financing friction with the same amount of dilution cost.[14] Under this alternative calibration, the expected shadow value of internal funds of the firms ($\mathbb{E}_t^a[\xi_{i,t}] = 1 + \varphi/(1-\varphi)[1 - \Phi(z_t^E)]$) under the currency union is only 1.07 as compared with 1.16 in the baseline economy, while the probability of issuing outside equity $(1 - \Phi(z_t^E))$ is 0.21, which is substantially lower than 0.48 in the baseline calibration.

Figure 8 compares the dynamics of inflation rates, real exchange rate and GDPs of the two countries in response to the financial shock under the currency union for the baseline and alternative calibrations. Solid lines (blue for home and red for foreing) show the baseline case while dash-dotted lines show the alternative. In panel (a), inflation responses are almost two time greater for the baseline case, which features more vulnerable financial condition for home country. This is because the harsher financial condition creates a greater incentive for financially more vulnerable firms to deviate from long run market share maximization. The combination of the greater difference in inflation rate and the absence of nominal exchange rate adjustment implies a much stronger

---

[14] Note that the firms can realize negative profits even with zero fixed operation costs. This is due to the timing convention we have adopted: Since the firms take orders from customers and commit to ship the products before the resolution of idiosyncratic uncertainty, it is possible that the firms produce suboptimally large amount of output in ex post sense. In other words, the option to adjust labor freely after the realization of idiosyncratic shock (so called *Oi-Hartman effect*) does not exist for this economy.

Table 2: Welfare Consequence of Currency Union

	Welfare		Consumption Equivalent
	Currency Union (A)	Floating Ex. Rate (B)	Percent
Home country	-274.86	-274.37	0.22
Foreign country	-217.86	-217.37	0.38
Joint welfare	-492.82	-491.48	-

Note: The consumption equivalent is the required minimum increase in average consumption per period holding labor hours constant to make the representative agent living in the economy under the floating exchange rate regime no worse off by transitioning to the currency union.

appreciation in the real exchange rate for home country in the baseline case, as shown in panel (b). As a consequence, home country undergoes considerably stronger contraction in economic activity under this case.

3.3 Heterogeneous Financial Condition and Policy Dilemma

3.3.1 Welfare Consequence of Currency Union

Table 2 summarizes the welfare consequences of adopting a policy union when member countries face heterogeneous financial market friction. To evaluate the effect on welfare, we adopt the following, simple and stylized calibration strategy: we assume that the two countries are subject to aggregate technology shocks ($\epsilon_{A,t}$ and $\epsilon^*_{A,t}$) and financial shocks ($\epsilon_{f,t}$ and $\epsilon^*_{f,t}$) only; we calibrate the standard deviation of aggregate technology shocks as 1 percent each, and set the standard deviations of financial shocks such that they account for 50 percent of variance decomposition of home country output.[15] To evaluate the welfare, we define the value functions of the representative agents of the two countries as

$$W(\mathbf{s}) = U(x, h) + \beta\mathbb{E}[W(\mathbf{s}')|\mathbf{s}]$$
$$\text{and } W^*(\mathbf{s}) = U(x^*, h^*) + \beta\mathbb{E}[W^*(\mathbf{s}')|\mathbf{s}],$$

and approximate them using a second order approxiamtion and report the analytical first moment in table 2.

The first and second rows of table 2 show that the welfares of both home and foreign countries deteriorate by adopting a common currency. To put this result in perspective, we also report the consumption equivalent in the third column of the table, which is formally defined as the required increase in average consumption per period to make the agent living in an economy with the common currency indifferent with transitioning to an economy with the floating exchange rate.

---

[15] Obviously, more elaborate strategies can be adopted to provide more realistic representation of the macroeconomy. However, our main conclusions hold true even when a radically different strategy of calibrating the structure of shocks is employed as we have shown that the main problem associated with the currency union stays the same when the driving force of business cycle is entirely dominated by the aggregate technology shocks.

Table 3: Output and Consumption Volatility Under Alternative Environment

	Output (GDP) volatility			Consumption volatility		
	Union (A)	Floating (B)	B/A	Union (A)	Floating (B)	B/A
Home country	.0151	.0108	.72	.0219	.0099	.45
Foreign country	.0149	.0087	.58	.0204	.0093	.46

Note: The consumption equivalent is the required minimum increase in average consumption per period holding labor hours constant to make the representative agent living in the economy under the floating exchange rate regime no worse off by transitioning to the currency union.

While the sign of the certainty equivalent change in consumption is intuitive, the degree of welfare deterioration caused by adopting a single currency appears to be small at least in terms of the certainty equivalent changes in consumption.

However, as is standard of welfare cost analysis of business cycle, this may be misleading because the representative agent is clearly a theoretical artifact we use for the sake of conveniency, and the aggregate uncertainty clearly and substantially understates the uncertainty facing individuals without perfect insurance. In this regard, somewhat more useful statistics can be found in standard deviations of output and consumption, which are reported in table 3.[16] The results shown in the table are striking: by agreeing to abolish the currency union and return to the floating exchange rate, home and foreign can reduce the output volatility as much as 28 and 42 percent, respectively. Even more strikingly, such a transition would reduce the consumption volatility of both countries more than 50 percent in the baseline calibration.

3.3.2 A Benchmark Allocation: Complete Risk Sharing

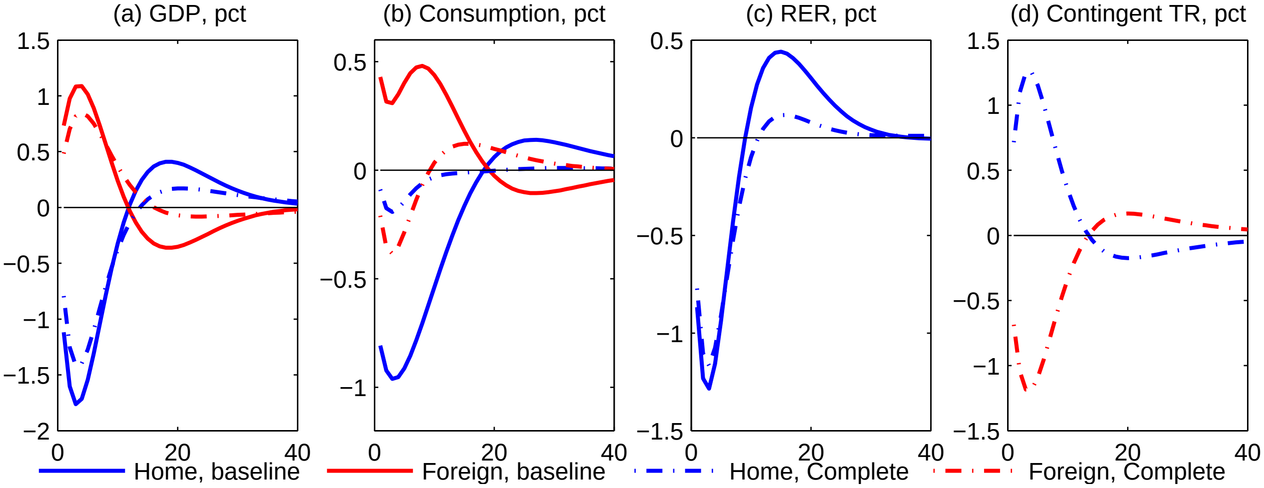
Assuming that a return to the floating exchange rate is either infeasible or undesirable for whatever non-economic reasons, what could remedy the distortions without breaking the single currency regime? To study this issue, we start by analyzing the nature of real allocation under a currency union with the two countries trading a complete set of state contingent bonds. This provides a natural benchmark against which the efficiency of other policy proposals can be guaged. For the sake of space, we mainly focus on the four aspects of the economy in this section: dynamics of GDP, consumption, real exchange and state-contingent cross-border transfer.

In the international macroeconomics literature, researchers often make the assumption of complete risk sharing. This is because it is often the case that the presence and absence of such risk sharing arrangement do not make substantial differences in the dynamics of endogenous quantities, including real exchange rate (see Steinsson [2008], for example). However, as shown by figure 9, this is not the case in the current environment. In the figure, solid lines represent the case of the baseline, i.e., currency union without the complete risk sharing arrangement, with blue and red

---

[16] Since about a half of output variation is due to the financial shock, one can easily map the changes in standard deviation of output into variation in unemployment rate using a variant of so called *Okun's law*.

Figure 9: Financial shock and Currency Union with Complete Risk Sharing



Note: Solid lines are the baseline case of the currency union with incomplete risk sharing arrangement with blue and red indicating the periphery and the core countries, respectively. Dashed-dotted lines are the case of the currency union with complete risk sharing arrangement with the the same color convention.

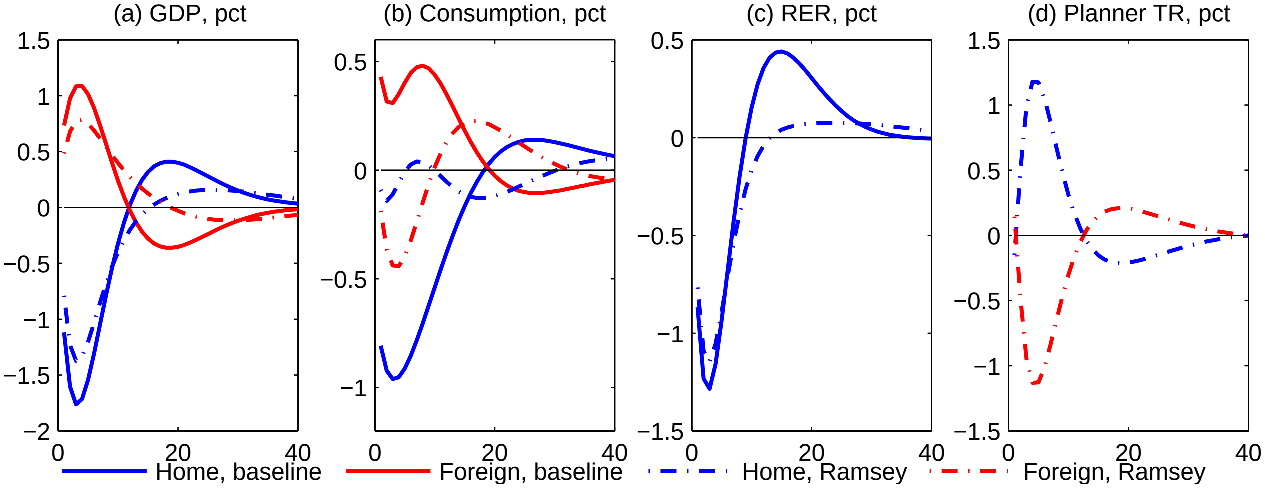
being home and foreign cases. Dash-dotted lines show the case of the currency union with the complete risk sharing with the same color convention.

A few conclusions can be easily made from the figure. First, the state contingent bond trading evenly spreads out the cost of financial shock to the two countries, as shown by panel (b) of the figure. In terms of the vertical distance from the baseline consumption paths, the two countries undergo the same degree of improvement/sacrifice depending on which country is hit by the shock. Second, the complete risk sharing arrangement works mainly through cross-border wealth transfer, rather than the changes in production share. Panel (a) shows that the production scales of the two allocations are not very different from each other, although the complete risk sharing does reduce the volatility of GDP for both countries. This is owing to the relative inefficiency of home country. To the contrary, the complete risk sharing arrangement organize the production such that the marginal costs are equalized across the countries and then redistribute the proceeds to achieve the risk sharing. Third, as shown by panel (d), this requires a substantial amount of international transfer of wealth, which, in this particular example, amounts to 1.2 percent of GDP. Finally, the complete risk sharing arrangement does not abolish the fluctuations in the real exchange rate, although the volatility is somewhat subdued in this environment. This is because, despite the cross-border transfer of wealth, the ratio of marginal utilities of consumption (more accurately, marginal utilties of consumption habit aggregator, x) declines.

3.3.3 An Alternative Benchmark: Ramsey Tax

However great the improvement may be made by the state contingent bonds trading, such a risk sharing arrangement for countries can rarely be found in reality. With a view to searching for more realistic policy options, we consider an alternative benchmark, which can guide policymakers in

Figure 10: Financial shock and Ramsey Allocation for Currency Union



Note: Solid lines are the baseline case of the currency union with incomplete risk sharing arrangement with blue and red indicating the periphery and the core countries, respectively. Dashed-dotted lines are the case of the currency union with incomplete risk sharing arrangement, but with Ramsey planner determining the level of cross-border wealth transfer. The same color convention is used for this case.

constructing a realistic policy proposal: Ramsey tax/subsidy. This provides us with a constraint optimum allocation, which takes as given the distortions brought about by the single currency and potentially suboptimal monetary policy (60). In this allocation, a Ramsey planner chooses a set of lump sum taxes, τ and τ^* to maximize the joint welfare defined as

$$W^{SP}(\mathbf{s}) = U(x(\mathbf{s}), h(\mathbf{s})) + \kappa U(x^*(\mathbf{s}), h^*(\mathbf{s})) + \beta \mathbb{E}[W^{SP}(\mathbf{s}')|\mathbf{s}] \tag{73}$$

subject to all private equilibrium conditions and a budget constraint, $0 = \tau(\mathbf{s}) + q(\mathbf{s})\tau^*(\mathbf{s})$.

Figure 10 shows the result of the Ramsey allocation problem in a format symmetric to figure 9. From the figure 9and 10, it is straightforward to conclude that the Ramsey planner achieves an allocation more or less identical to that under the complete risk sharing using state dependent lump sum tax policies. There is a subtle difference however. The Ramsey planner does not take as given the aggregate consumption. In other words, the planner can exploit the externality created by the “Catching Up with Jonneses” preferences. As a consequence, the consumption dynamics exhibit somewhat more oscillatory dynamics in this environment than in the complete risk sharing environment. It is an interesting question if Ramsey allocation can actually improve upon the outcome of the complete risk sharing arrangement through this channel (we will get to this discussion shortly).

Obviously, such a planner does not exist in reality. A natural question is then if a rule based tax policy can achieve such allocations (whether or not the countries agree to adopt such a policy is a different problem). We consider the following tax rule:

$$\tau_t = \alpha_\tau \times 100 \times \log(q_t/\bar{q}), \quad \alpha_\tau > 0 \tag{74}$$

Table 4: **Effects on Welfare of Alternative Environments**

	Baseline	Complete market	Ramsey	Optimal taxation
Joint Welfare	-492.82	-490.17	-491.50 (-491.86)	-492.35
Welfare, Home	-274.86	-253.21	-	-274.77
Welfare, Foreign	-217.96	-236.96	-	-217.58
Cons. Equiv., Home	-	10.28	-	0.04
Cons. Equiv., Foreign	-	-9.13	-	0.19

Note: The consumption equivalent is the required minimum increase in average consumption per period holding labor hours constant to make the representative agent living in the economy under the floating exchange rate regime no worse off by transitioning to the currency union.

When the real exchange rate appreciates, the rule prescribes a negative tax, i.e., a lump sum subsidy.[17] Rather than arbitrarily specifying the parameter α_τ, we optimize it such that the joint welfare (73) is maximized.

Table 4 summarizes the effects on welfare of various policy environments under the currency union. The first row of the table shows the impact on the joint welfare. A remarkable property of these experiment is that the complete risk sharing arrangement achieves the highest joint welfare. It is notable that the Ramsey allocation[18], even though it exploits the externality of habit, fails to improve upon the outcome of the complete risk sharing arrangement. The optimized tax rule in the form of (74) does improve upon the baseline case, although the degree of improvement is relatively small, and falls short of both the complete risk sharing and Ramsey allocation.

A policy dilemma of any risk sharing arrangement between the two countries can be seen in the rest of the rows of the table. The complete risk sharing is a polar case. While it substantially improves the join welfare, it involves nontrival, cross-border transfer of wealth.Home country improves its welfare substantially, but the welfare of foreign country deteriorates dramatically. The improvement in the joint welfare implies that the gains for home dominates the loss for foreign country. The last two rows show the certainty equivalent changes in consumptions. Strikingly enough, the complete risk sharing arrangement increases the steady state consumption level 10 percent for home country, but decreases 9 percent for foreign country. The reason why the currency union cannot become a true union is that there simply is no reason for the residents of foreign country to agree with such transfers.

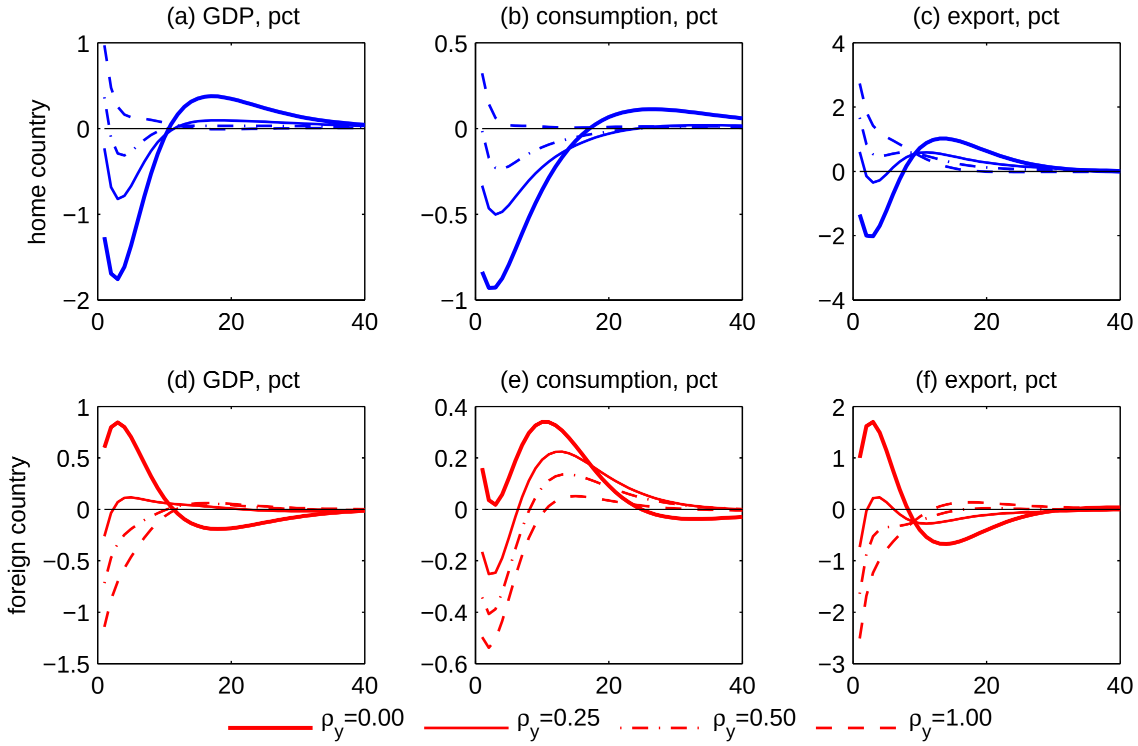
In this regard, the results for the optimized tax rule are interesting on several aspects. First, it shows that a cross-border taxation that is mutually beneficial to both country can exist, although it falls far short of the complete risk sharing in terms of improving the joint welfare. Second, the magnitudes of welfare improvement for member countries are miniscule: it improves home country welfare only 4 basis points. Finally, surprisingly enough, such an optimized rule brings more benefits

---

[17] An alternative would be to specify the tax as a function of current account surplus.

[18] We report two values for Ramsey allocation: the first one is when the planner can set the initial Lagrangian multipliers equal to zero; the second one is the when the planner sets the initial Lagrangian multipliers equal to their steady state values.

Figure 11: Monetary Policy Activism and Benefits of Floating Exchange Rate



Note: Blue lines are the case of the peripheral county and red lines are the case of the core country.

to foreign country than to home country under the baseline calibration.

3.3.4 Back to Floating?

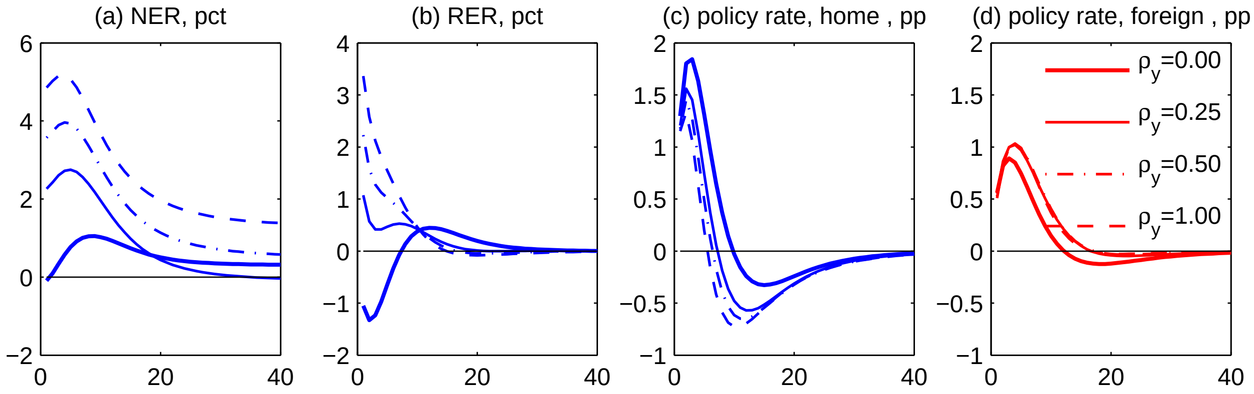
We have shown that the distortions caused by the currency union are not easily overcome by alternative risk sharing arrangements. A main problem is that such a solution may involve potentially large tranfer of wealth among member countries when they have heterogenous financial conditions. Does that mean that the countries can get better off by breaking up the union and going back to the floating exchange rate. The answer is, it depends: depending on the nature of monetary policy taken by the member countries after the break-up, it may make matters worse. Under the baseline calibration, this is not the case. However, if the countries insist upon non-acitivist monetary policy regimes, they can jointly deteriorate their welfares. Furthermore, it is possible for the independent monetary authorities of the two countries to replicate the allocation under the currency union.

Figure 11 show the responses of GDP, consumption and export of the two countries to the financial shock to home country under the foating exchange rate regime with the monetary policy in each country determined by the inertial Taylor rule, which only reacts to national fundamentals. Holding the inflation coefficient at 1.5, we consider 4 cases: $\rho_y = 0.00$, 0.25 (the baseline), 0.50 (Taylor [1993]), and 1.00 (**?**).[19] The results suggest that home country gets substantially better

---

[19] We assume that the coefficients of the monetary policy reaction functions of the two countries are symmetrically

Figure 12: Monetary Activism, Nominal and Real Exchange Rate



Note: Blue lines are the case of the peripheral county and red lines are the case of the core country.

off by adopting an activist monetary policy. In fact, if the monetary policies are prescribed not to respond to output gap, the two countries actually achieve the allocation that is very close to that under the currency union. As shown by figure 12, such monetary policy raises the real interest rate in the middle of the recession to stabilize the inflation and thus does not generate enough real interest differential that has to be offset by a large depreciation of nominal exchange rate to safisfy the international bond market arbitrage free condition (71). As a result, while the nominal exchange rate does depreciate somewhat as shown in the first panel of figure 12, it fails to avoid the real exchange rate appreciation.

In contrast, the most activist monetary policy produces the best outcome for home country by allowing the country hit by the financial shock to export its way out. The better performance of activist monetary policy is owing to the lack of the *divine coincidence* that is typical of New Keynesian models–the perfect comovement between output gap and inflation gap. The liquidity problem caused by the financial shock leads the firms in home country to raise their product prices during a recession. An expansionary monetary policy helps the situation by boosting domestic aggregate demand. The real exchange rate depreciation under such a policy also improves the cashflow of the firms, thereby reducing the pressure to raise their product prices to secure short-term liquidity, which then holds down the negative demand pressure.

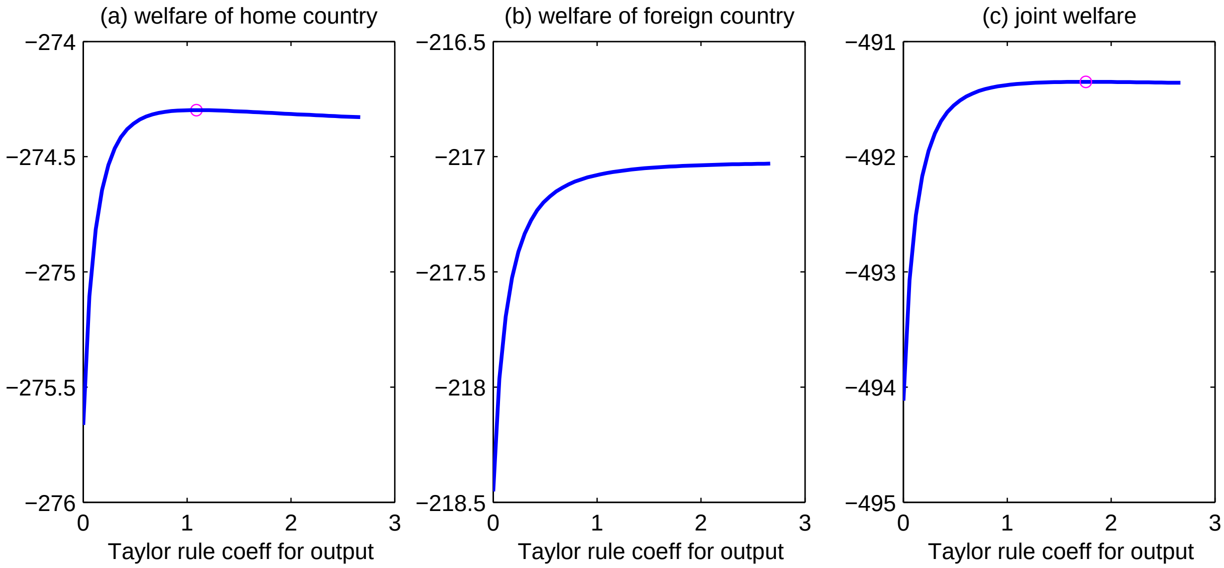
Can such a monetary policy regime be mutually benefical to both countries? The popular press would dub such a regime "beggar-thy-neighbor" policy. Assuming that the countries follow a symmetric choice on the output gap coefficient, figure 13 shows the impact of the changes in the monetary policy on individual and joint welfares of the two countries.[20] In this figure, one can see that the output coefficients that maximize the welfares of the countries are far from zero. Home country welfare is maximized around $\rho_y = 1$. Foreign country's welfare is actually maximized around $\rho_y = 30$ (not shown for scale). While foreign country's welfare continues to improve albeit marginally, too high a value for the output coefficient starts deteriorting the quality of the allocation

---

given.

[20] To perform the experiment, we use the same calibration strategy adopted in the earlier tables.

Figure 13: Monetary Activism and Welfare



Note: The welfare is approximated with a second order perturbation method. The welfare figures shown in the figure are analytical moments of the second order approximation.

for home country, resulting in the maxium joint welfare around $\rho_y = 1.7$.

It is a both surprising and important result that foreign country with relatively stronger financial position may gain more than home country with its more vulnerable financial structure. Without the activist monetary policies of the both countries, the firms in foreign country try to exploit the vulnerable financial position of home country firms, and aggresively cut their relative prices to expand their long term market shares. While such a move is consistent with individual rationality, it fails to incorporate the negative externality of such policy on the union-wide aggregate demand. The real exchange rate depreciation for home country under the mix of floating exchange rate and activist monetary policy helps these firms internalize the negative externality caused by their aggresive pricing strategy.

4 Inflation and Financial Frictions in the Euro Zone

In this section, we present evidence on the relationship between price-setting and financial frictions in the Euro Zone.

What motivates our analysis are the findings of Gilchrist et al. (2013) for U.S. firms during the Great Recession: During the height of the crisis in late 2008, firms with “weak” balance sheets increased prices significantly relative to industry averages, whereas firms with “strong” balance sheets lowered prices, a response consistent with an adverse demand shock. These stark differences in price-setting behavior clearly contradict the standard price-adjustment mechanism emphasized by the New Keynesian literature, where firms financial conditions play no role in determining their price-setting behavior.

Inflation dynamics and financial turmoil in the Euro Crisis mirror this pattern exactly when we compare financially strong and financial weak crisis countries: Anecdotally, it is well known that prices continued to rise in financially weak countries during the Euro Crisis, whereas financially strong, Northern European countries saw only very little inflation or deflation. Here, we systematically quantify this relationship between price-setting and financial frictions in the Euro Zone. We demonstrate that these two are indeed both economically and statistically significantly related.

4.1 Data

To do so, we construct a dataset on inflation and financial frictions in the Euro Zone from two sources. First, we use data from Eurostat on inflation, real marginal cost and unemployment. In particular, we use the all-item harmonized inflation price index (HICP) data to measure inflation, which we seasonally adjust, starting in 1996Q1 and running through 2014Q1. Our measure of real marginal cost is given by seasonally adjusted real unit labor costs from Eurostat. These are defined as total compensation of employees to GDP in market prices. In the case of Greece, the seasonally adjusted series end in the first quarter of 2011, so we link a four-quarter moving average of the available non-adjusted series to the existing series. We complement the real marginal cost data with a series for the unemployment rate, seasonally adjusted, from Eurostat for the same period.

Second, we measure financial frictions using CDS data from Markit. Our main measure of financial frictions is the quarterly average CDS spread of EUR-denominated contracts. To check robustness, we also use the annualized realized volatility of the daily CDS spreads, and the annualized realized volatility of the daily difference in CDS spreads.

We collect data for the following countries: Austria, Finland, France, Germany, Netherlands to which we refer to as “Core Europe”; and Greece, Ireland, Italy, Portugal, Spain, to which we refer to as “PIIGS” countries. All countries taken together account for approximately 94.33% of the Euro Zone’s GDP (World Bank, 2009).

4.2 Macroeconomic Imbalances of the Euro Countries

(a) GDP/unemployment rate (b) price levels (or inflation rates) (c) real exchange rate (misalignment) (d) current account.

4.3 Measuring the Impact of Financial Frictions on Inflation in the Euro Zone

We establish a relationship between price-setting and financial frictions in the Euro zone in two steps: First, we estimate country-specific Phillips curves through the end of 2008.[21] Second, we predict inflation during the Euro crisis based on these estimates, and relate residuals of actual less predicted inflation to financial frictions. We expect larger actual inflation than inflation predicted based on pre-crisis estimates of a Phillips curve if financial frictions in fact – as in our model – are associated with revenue stabilization through higher inflation.

---

[21] Our choice of date is motivated by the fact that the crisis started with a delay only in 2009Q1 in the Euro zone

First, we estimate country-specific Phillips curves through 2008Q4:

$$\pi_{i,t} = \beta_i \mathbb{E}_t[\pi_{i,t+1}] + \lambda_i mc_{i,t} \tag{75}$$

where $E_t[\pi_{i,t+1}]$ denotes expectations of future inflation and $mc_{i,t}$ is our measure of marginal cost. We employ GMM as described in Gali et al. (2001), with the same moment condition:

$$\mathbb{E}_t[(\pi_{i,t} - \beta_i \mathbb{E}_t[\pi_{i,t+1}] - \lambda_i mc_{i,t})\mathbf{z}_{i,t}] = 0 \tag{76}$$

where $\mathbf{z}_{i,t}$ is a country-specific set of instruments. We include five lags of inflation, and two lags of marginal cost into the set of instruments. Alternatively, we have estimated a backward-looking Phillips curve specification, as well as one with deviations of unemployment from the natural rate of unemployment. We prefer the marginal cost specification because it yields – as in Gali et al. (2001) – coefficients of the right signs. However, both alternative specifications leave results qualitatively unchanged.

Second, we use these pre-crisis estimates to predict country-specific Phillips-curve residuals for the Euro crisis from 2009Q1 through 2014Q1. These residuals will systematically vary with financial frictions if financial variables play a role for inflation, but are not included in the pre-crisis estimates of the Phillips curve: Since these Phillips curve estimates are from a "normal" economic environment where financial frictions do not influence inflation dynamics, any predicted inflation movements based on these estimates will only capture dynamics that we would expect if financial frictions are not important. At the same time, the residuals of predicted less actual inflation movements should reflect the effect of financial frictions.

We define residuals as the difference between observed and predicted inflation rates: $\epsilon^{\pi}_{i,t} = \pi_{i,t} - \hat{\pi}_{i,t}$. We follow Gali et al. (2001) and Sbordone (1999) in constructing predicted, "fundamental" inflation $\hat{\pi}_{i,t}$ as a function of the discounted stream of expected future real marginal costs. Using $\hat{\beta}_i$ and $\hat{\lambda}_i$, predicted inflation is given as follows:

$$\hat{\pi}_{i,t} = \lambda_i \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t[\widehat{mc}_{t+k}] \tag{77}$$

To obtain estimates of expected future real marginal costs, we estimate a VAR(1) specification for $\mathbf{z_{i,t}} = [mc_{i,t}, mc_{i,t-1}, mc_{i,t-2}, ..., mc_{i,t-q}]$ with $q = 5$ lags. This implies that $E_t[\widehat{mc}_{t+k}|\mathbf{z_t}] = e_1' A_i^k \mathbf{z_{i,t}}$ and $\hat{\pi}_{i,t} = \lambda_i e_1'(I - \beta_i A_i)^{-1}\mathbf{z_{i,t}}$. Note that our VAR(1) is based solely on marginal costs and excludes observed inflation as Gali et al. (2001). Unlike Gali et al. (2001), we thus do not use any information in the VAR(1) on the effect of financial frictions that may be directly contained in observed inflation rates.

We can then directly relate these residuals to observed financial frictions, and test whether they are indeed related. We do so by estimating the following regression specification:

$$\pi_{i,t} - \hat{\pi}_{i,t} = \beta_0 + \beta_1 CDS_{i,t} + \nu_{i,t} \tag{78}$$

where $CDS_{i,t}$ denotes country-specific CDS data. Motivated by the empirical evidence in Gilchrist et al. (2013), we estimate the above regression separately for financially strong "core" countries, and the set of financially weak, PIIGS countries. If financial frictions play a role for inflation during the crisis, then we would expect a positive relationship between residuals and CDS spreads. We also present our findings for each set of countries graphically. As robustness check, we estimate eqn.(77) using our alternative measures of country-level financial frictions in the Euro zone.

4.4 Results

To highlight the role of financial distortions in determining inflation dynamics during the Euro crisis, we first plot the predicted inflation residuals obtained from estimating eqn.(77). Figure 14 shows our main result: the financially weak PIIGS countries have a very strong, positive relationship with average CDS spreads during the Euro crisis. Panel (a) summarizes this relationship graphically. When we only consider Italy and Spain, the two largest amongst the PIIGS countries, the relationship continues to hold and is even more pronounced and downwards-sloping. Panel (b) summarizes this result. By contrast, there is no evidence for a significantly upwards- or downwards-sloping relationship between average CDS spreads and predicted Phillips curve residuals for core European countries. Panel (c) summarizes this finding.

These results from Figure 14 are difficult to reconcile with the standard price-adjustment mechanism emphasized by the New Keynesian literature, a paradigm where financial conditions play no role in determining price-setting behavior. In general, we would expect that adverse demand shocks – the kind that hit the PIIGS countries between 2009 and 2012 – should induce firms to lower prices. Moreover, if our proxies used to measure financial distortions are also indicative of the weakness in demand, we would expect financially vulnerable firms in the PIIGS countries to lower prices even more relative to financially strong firms in the Northern countries. However, we observe exactly the opposite reaction in the data.

Interestingly, most of the residuals have a positive sign for both the PIIGS countries, and Italy and Spain taken separately. This is what one would expect if financial frictions worsened during the Euro crisis, and induced firms in the PIIGS countries to stabilize their cash flow by setting higher prices than they would have had financial frictions not affected them. Even more strikingly, we find that the majority of inflation residuals during the Euro crisis becomes almost perfectly positive during the crisis, in 2010 through 2012. However, in the pre-crisis period, there is no discernible pattern, and again in 2013, some residuals become positive again. This latter reversal coincides with the "whatever it takes" announcement of Mario Draghi in the summer of 2013 that reduced CDS spreads and calmed markets. Table **??** summarizes the trends by year.

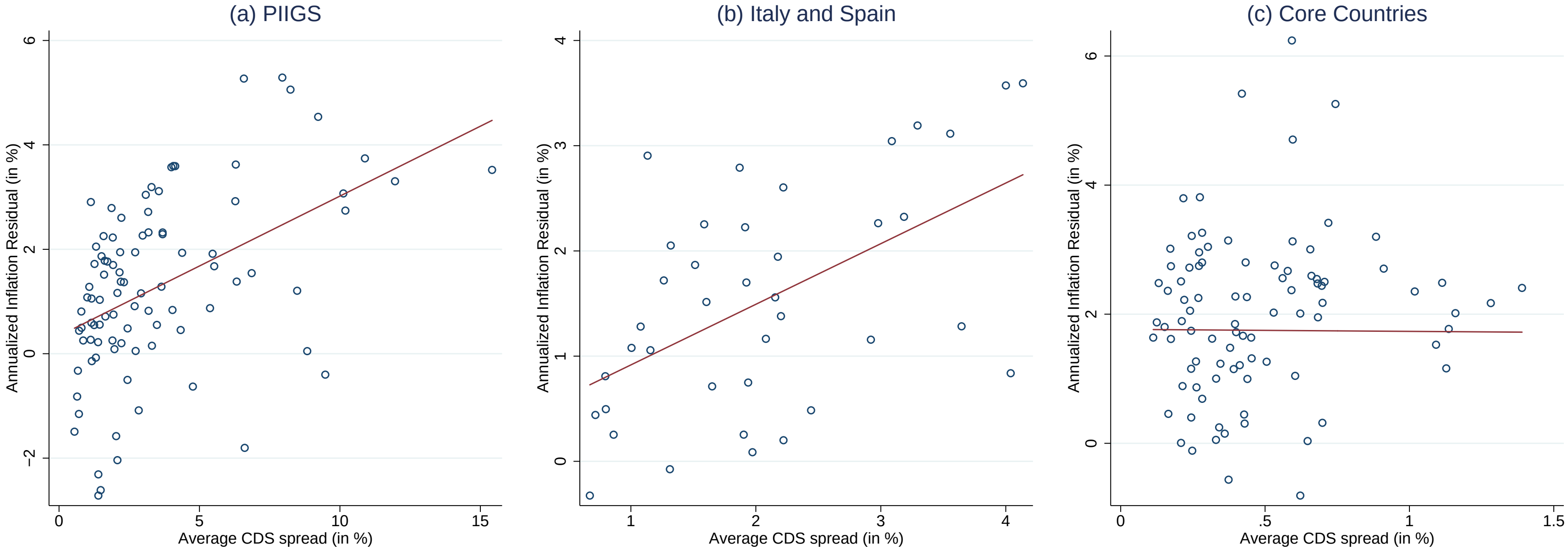
What is the magnitude of the effect of financial distortions on inflationary movements? We present estimates of eqn.(77) in Tables 6 - 8. For the set of PIIGS countries, we find that a one percentage point increase in the average quarterly CDS spread – ceteris paribus – is associated with a 0.10 percentage point increase in the inflation residual. This relationship is statistically significant different from 0 at the 5% level. In economic terms, this estimate implies a positive 1.20%

quarterly inflation residual associated with a one standard deviation increase in CDS spreads. This is summarized in specification (I) of Table 6. For Italy and Spain, we find a statistically significant coefficient of 0.44. At the height of the Euro crisis, this is equivalent to a striking 1.8% predicted quarterly inflation residual. The corresponding regression findings are shown in specification (I) of Table 7. By contrast, there is no statistically significant relationship for the Northern European countries at all. Table 8 presents this finding.

We find that our result are robust in two ways. First, we estimate the regression specifications using our alternative measures of financial distortions in the Euro crisis, namely the annualized realized volatility of the daily CDS spreads, and the annualized realized volatility of the daily difference in CDS spreads. We find that using either measure leaves our results unchanged. Columns (II) and (III) of Tables 6-8 summarize these findings. Second, we take log transformation of CDS spreads to mitigate the effect of the very large CDS spreads of Ireland and Greece during the Euro crisis. Columns (IV) and (VI) of Tables 6-8 show that this also leaves our results economically and statistically unchanged.

Finally, while we do not have sufficient evidence to statistically quantify the recent impact of financial frictions for deflationary tendencies in the Euro zone, the evidence that we have presented for a positive relationship between financial distortions and predicted inflation residuals is consistent with these recent trends: as financial conditions have started to improve in the PIIGS countries, this allows firms to finally lower prices, generating deflationary trends in the aggregate.

Figure 14: Inflation Forecast Errors and Financial Friction in Euro Zone: Level

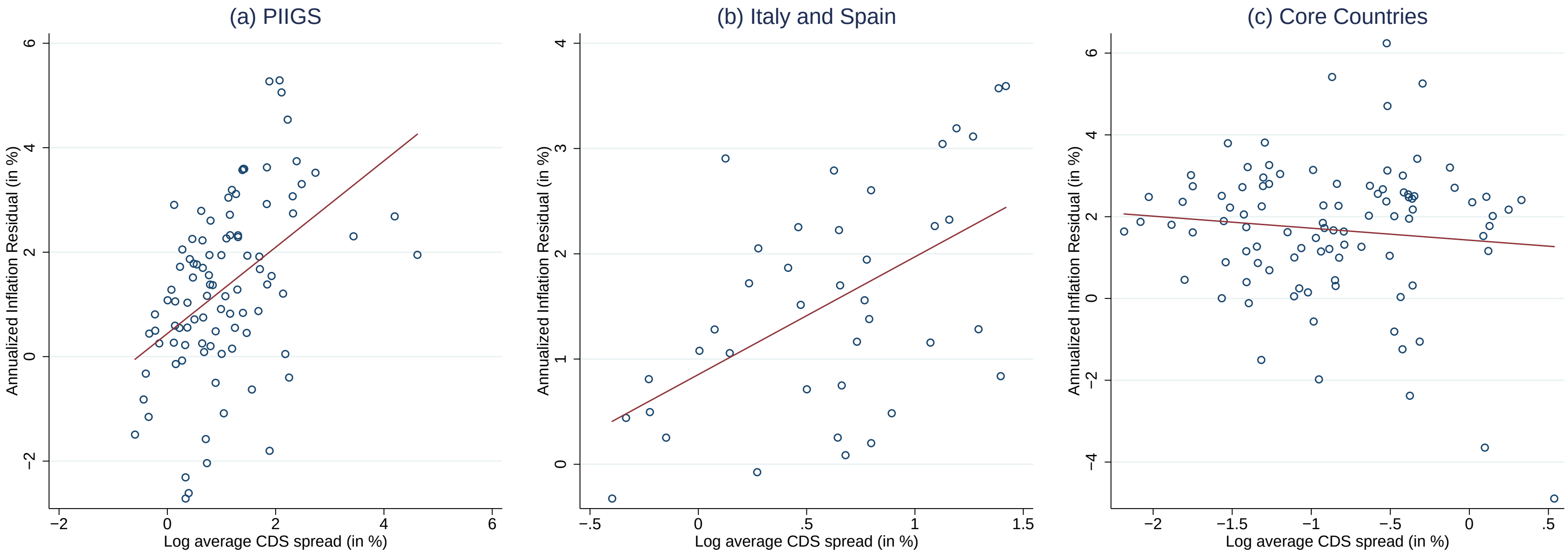

(a) PIIGS

(b) Italy and Spain

(c) Core Countries

Note: The panels show the relationship of predicted inflation residuals and CDS spreads for the respective countries during the Euro Crisis at a quarterly frequency. Residuals are obtained from estimating a Phillips curve, (75), from 1996Q1 through 2008Q4, and predicting residuals for 2009Q1 through 2013Q4. The solid line represents the estimate of (**??**) relating residuals and CDS spreads.

Figure 15: Inflation Forecast Errors and Financial Friction in Euro Zone: Log



Note: The panels show the relationship of predicted inflation residuals and the log of CDS spreads for the respective countries during the Euro Crisis at a quarterly frequency. Residuals are obtained from estimating a Phillips curve, (75), from 1996Q1 through 2008Q4, and predicting residuals for 2009Q1 through 2013Q4. The solid line represents the estimate of (**??**) relating residuals and the log of CDS spreads.

Table 5: Sign of Predicted Residuals in PIIGS Countries

Sign of Residuals	Year 02	03	04	05	06	07	08	09	10	11	12	13	14
> 0	10	4	13	14	7	13	7	12	16	20	20	15	-
< 0	10	16	7	6	13	7	13	8	4	0	0	0	5

Table 6: Inflation and Financial Frictions: PIIGS Countries

	(I)	(II)	(III)	(IV)	(V)	(VI)
average CDS spread	0.094**					
	(0.04)					
annualized realized volatility of the daily CDS spreads		0.097**				
		(0.04)				
annualized realized volatility of the daily difference in CDS spreads			0.362**			
			(0.14)			
Log average CDS spread				0.365***		
				(0.11)		
Log annualized realized volatility of the daily CDS spreads					0.297**	
					(0.09)	
Log annualized realized volatility of the daily difference in CDS spreads						0.311**
						(0.1)
Intercept	0.002	0.028	0.001	-0.025	0.088	0.442***
	(0.16)	(0.15)	(0.16)	(0.15)	(0.13)	(0.1)

The table shows results obtained in two steps: First, we estimate a Phillips Curve $\pi_{i,t} = \beta_{0,i} + \beta_{1,i}\pi_{i,t-1} + \beta_{2,i}(u_{i,t} - u_i^n) + \beta_3 TIME + \epsilon_{i,t}$ for Portugal, Ireland, Italy, Greece and Spain through the end of 2008Q4. Second, we use predicted residuals $\hat{\epsilon}_{i,t}$ for $2009Q1 \leq t \leq 2014Q1$ to estimate $\hat{\epsilon}_{i,t} = \gamma_0 + \gamma_1 CDS_{i,t-1} + \nu_{i,t}$ where $CDS_{i,t}$ is either the average CDS spread (pps.), the annualized realized volatility of the daily CDS spreads (pps.), the annualized realized volatility of the daily difference in CDS spreads (pps.) or the natural log of either of them. We report standard errors in brackets.

Table 7: Inflation and Financial Frictions: Italy and Spain

	(I)	(II)	(III)	(IV)	(V)	(VI)
average CDS spread	0.437**					
	(0.16)					
annualized realized volatility of the daily CDS spreads		0.218*				
		(0.12)				
annualized realized volatility of the daily difference in CDS spreads			0.819*			
			(0.41)			
Log average CDS spread				0.828**		
				(0.32)		
Log annualized realized volatility of the daily CDS spreads					0.611**	
					(0.26)	
Log annualized realized volatility of the daily difference in CDS spreads						0.557**
						(0.26)
Intercept	-0.392	0.049	-0.034	0.007	0.176	0.842***
	(0.37)	(0.29)	(0.32)	(0.25)	(0.21)	(0.22)

The table shows results obtained in two steps: First, we estimate a Phillips Curve $\pi_{i,t} = \beta_{0,i} + \beta_{1,i}\pi_{i,t-1} + \beta_{2,i}(u_{i,t} - u_i^n) + \beta_3 TIME + \epsilon_{i,t}$ for Italy and Spain through the end of 2008Q4. Second, we use predicted residuals $\hat{\epsilon}_{i,t}$ for $2009Q1 \leq t \leq 2014Q1$ to estimate $\hat{\epsilon}_{i,t} = \gamma_0 + \gamma_1 CDS_{i,t-1} + \nu_{i,t}$ where $CDS_{i,t}$ is either the average CDS spread (pps.), the annualized realized volatility of the daily CDS spreads (pps.), the annualized realized volatility of the daily difference in CDS spreads (pps.) or the natural log of either of them. We report standard errors in brackets.

Table 8: Inflation and Financial Frictions: Northern Europe

	(I)	(II)	(III)	(IV)	(V)	(VI)
average CDS spread	0.2061					
	(0.28)					
annualized realized volatility of the daily CDS spreads		-0.122				
		(0.15)				
annualized realized volatility of the daily difference in CDS spreads			0.325			
			(0.36)			
Log average CDS spread				0.020		
				(0.11)		
Log annualized realized volatility of the daily CDS spreads					-0.037	
					(0.08)	
Log annualized realized volatility of the daily difference in CDS spreads						0.073
						(0.06)
Intercept	0.181	0.239**	0.114	0.195	0.141	0.326**
	(0.151)	(0.10)	(0.09)	(0.13)	(0.10)	(0.14)

The table shows results obtained in two steps: First, we estimate a Phillips Curve $\pi_{i,t} = \beta_{0,i} + \beta_{1,i}\pi_{i,t-1} + \beta_2(u_{i,t} - u_i^n) + \beta_3 TIME + \epsilon_{i,t}$ for Austria, Finland, France, Germany, and the Netherlands through the end of 2008Q4. Second, we use predicted residuals $\hat{\epsilon}_{i,t}$ for $2009Q1 \leq t \leq 2014Q1$ to estimate $\hat{\epsilon}_{i,t} = \gamma_0 + \gamma_1 CDS_{i,t-1} + \nu_{i,t}$ where $CDS_{i,t}$ is either the average CDS spread (pps.), the annualized realized volatility of the daily CDS spreads (pps.), the annualized realized volatility of the daily difference in CDS spreads (pps.) or the natural log of either of them. We report standard errors in brackets.

References

Alvarez, F., and A. Dixit (2014): "A real options perspective on the future of the Euro," Journal of Monetary Economics, 61(C), 78–109.

Bernanke, B. S., M. Gertler, and S. Gilchrist (1999): "The financial accelerator in a quantitative business cycle framework," in Handbook of Macroeconomics, ed. by J. B. Taylor, and M. Woodford, vol. 1 of Handbook of Macroeconomics, chap. 21, pp. 1341–1393. Elsevier.

Bordo, M. D., C. J. Erceg, and C. L. Evans (2000): "Money, Sticky Wages, and the Great Depression," American Economic Review, 90(5), 1447–1463.

Broda, C., and D. E. Weinstein (2006): "Globalization and the Gains from Variety," The Quarterly Journal of Economics, 121(2), 541–585.

Brunnermeier, M. K., and Y. Sannikov (2014): "A Macroeconomic Model with a Financial Sector," American Economic Review, 104(2), 379–421.

Cooley, T. F., and V. Quadrini (2001): "Financial Markets and Firm Dynamics," The American Economic Review, 91(5), pp. 1286–1310.

Eaton, J., and M. Gersovitz (1981): "Debt with Potential Repudiation: Theoretical and Empirical Analysis," Review of Economic Studies, 48(2), 289–309.

Erceg, C. J., D. W. Henderson, and A. T. Levin (2000): "Optimal monetary policy with staggered wage and price contracts," Journal of Monetary Economics, 46(2), 281–313.

Feenstra, R. C., P. Luck, M. Obstfeld, and K. N. Russ (2014): "In Search of the Armington Elasticity," NBER Working Papers 20063, National Bureau of Economic Research, Inc.

French, K. R., and J. M. Poterba (1991): "Investor Diversification and International Equity Markets," The American Economic Review, 81(2), pp. 222–226.

Gali, J. (1999): "Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?," American Economic Review, 89(1), 249–271.

Ghironi, F., and M. J. Melitz (2005): "International Trade and Macroeconomic Dynamics with Heterogeneous Firms," The Quarterly Journal of Economics, 120(3), 865–915.

Gilchrist, S., R. Schoenle, J. Sim, and E. Zakrajsek (2013): "Inflation Dynamics During the Financial Crisis," Discussion paper.

Gomes, J. F. (2001): "Financing Investment," The American Economic Review, 91(5), pp. 1263–1285.

Obstfeld, M., and K. Rogoff (2000): "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?," NBER Macroeconomics Annual, 15, pp. 339–390.

Ravn, M., S. Schmitt-Grohé, and M. Uribe (2005): "Relative Deep Habits: A," Discussion paper.

——— (2006): "Deep Habits," The Review of Economic Studies, 73(1), pp. 195–218.

Ravn, M. O., S. Schmitt-Grohé, and M. Uribe (2007): “Pricing to Habits and the Law of One Price,” American Economic Review, 97(2), 232–238.

Ravn, M. O., S. Schmitt-Grohe, M. Uribe, and L. Uuskula (2010): “Deep habits and the dynamic effects of monetary policy shocks,” Journal of the Japanese and International Economies, 24(2), 236–258.

Rotemberg, J. J. (1982): “Monopolistic Price Adjustment and Aggregate Output,” The Review of Economic Studies, 49(4), pp. 517–531.

Stein, J. C. (2003): “Agency, information and corporate investment,” in Handbook of the Economics of Finance, ed. by G. Constantinides, M. Harris, and R. M. Stulz, vol. 1 of Handbook of the Economics of Finance, chap. 2, pp. 111–165. Elsevier.

Steinsson, J. (2008): “The Dynamic Behavior of the Real Exchange Rate in Sticky Price Models,” The American Economic Review, 98(1), pp. 519–533.

Taylor, J. B. (1993): “Discretion versus policy rules in practice,” Carnegie-Rochester Conference Series on Public Policy, 39(0), 195 – 214.

Tesar, L. L., and I. M. Werner (1995): “Home bias and high turnover,” Journal of International Money and Finance, 14(4), 467 – 492.

Appendices

A Product Demand and Identities

A.1 Derivation of Product Demands

In the symmetric equilibrium, all households choose the same levels of consumptions. Henceforth, we omit the household sperscript. The cost minimization problem is then given by

$$\mathcal{L}_c = \sum_{k=h,f} \int_{N_k} P_{i,k,t} c_{i,k,t} di - \lambda_{c,t} \left[\left\{ \sum_{k=h,f} \omega_k \left[\int_{N_k} (c_{i,k,t}/s^{\theta}_{i,k,t-1})^{1-1/\eta} dk \right]^{\frac{1-1/\varepsilon}{1-1/\eta}} \right\}^{1/(1-1/\varepsilon)} - x_t \right]$$

The efficiency condition for $c_{i,h,t}$ is given by

$$P_{i,h,t} = \omega_h \lambda_{c,t} \frac{(c_{i,h,t}/s^{\theta}_{i,h,t-1})^{1-1/\eta}}{c_{i,h,t}} \left[\int_{N_h} (c_{i,h,t}/s^{\theta}_{i,h,t-1})^{1-1/\eta} dk \right]^{\frac{1-1/\varepsilon}{1-1/\eta}-1} x_t^{1/\varepsilon} \tag{A.1}$$

Similarly, the efficiency condition $c_{j,h,t}$ is given by

$$P_{j,h,t} = \omega_h \lambda_{c,t} \frac{(c_{j,h,t}/s^{\theta}_{j,h,t-1})^{1-1/\eta}}{c_{j,h,t}} \left[\int_{N_k} (c_{j,h,t}/s^{\theta}_{j,h,t-1})^{1-1/\eta} dk \right]^{\frac{1-1/\varepsilon}{1-1/\eta}-1} x_t^{1/\varepsilon} \tag{A.2}$$

Taking the ratio of (A.1) and (A.2) yields

$$\frac{P_{i,h,t}}{P_{j,h,t}} = \frac{c_{j,h,t}}{c_{i,h,t}} \frac{(c_{i,h,t}/s^{\theta}_{i,h,t-1})^{1-1/\eta}}{(c_{j,h,t}/s^{\theta}_{j,h,t-1})^{1-1/\eta}}$$

or equivalently,

$$(c_{j,h,t}/s^{\theta}_{j,h,t-1})^{-1/\eta} = \frac{P_{j,h,t} s^{\theta}_{j,h,t-1}}{P_{i,h,t}} \frac{(c_{i,h,t}/s^{\theta}_{i,h,t-1})^{1-1/\eta}}{c_{i,h,t}}$$

Rasing this expression to the power $1 - 1/\eta$, integrating the resulting expression with respect to j, and finally raising the resulting expression to the power $1/(1 - 1/\eta)$ yields

$$\left[\int_{N_h} (c_{j,h,t}/s^{\theta}_{j,h,t-1})^{1-1/\eta} dj \right]^{1/(1-1/\eta)} = \left[\int_{N_h} (P_{j,h,t} s^{\theta}_{j,h,t-1})^{1-\eta} dj \right]^{1/(1-1/\eta)} c_{i,h,t} (s^{\theta}_{i,h,t-1})^{\eta-1} P^{\eta}_{i,h,t} \tag{A.3}$$

We define two aggregates

$$x_{h,t} \equiv \left[\int_{N_h} (c_{j,h,t}/s^{\theta}_{j,h,t-1})^{1-1/\eta} dj \right]^{1/(1-1/\eta)} \quad \text{and} \tag{A.4}$$

$$\tilde{P}_{h,t} \equiv \left[\int_{N_h} (P_{j,h,t} s^{\theta}_{j,h,t-1})^{1-\eta} dj \right]^{1/(1-\eta)} \tag{A.5}$$

We can then rewrite (A.3) in terms of the two aggregates (A.4) and (A.5) as

$$\begin{aligned} c_{i,h,t} &= \left(\frac{P_{i,h,t}}{\tilde{P}_{h,t}} \right)^{-\eta} s^{\theta(1-\eta)}_{i,h,t-1} x_{h,t} \qquad \text{(A.6)} \\ &= \left(\frac{P_{i,h,t}}{P_{h,t}} \right)^{-\eta} \left(\frac{\tilde{P}_{h,t}}{P_{h,t}} \right)^{\eta} s^{\theta(1-\eta)}_{i,h,t-1} x_{h,t}. \end{aligned}$$

Following the same steps, one can derive the home demand for foreign product as

$$\begin{aligned} c_{i,f,t} &= \left(\frac{P_{i,f,t}}{\tilde{P}_{f,t}}\right)^{-\eta} s_{i,f,t-1}^{\theta(1-\eta)} x_{f,t} \qquad \text{(A.7)} \\ &= \left(\frac{P_{i,f,t}}{P_{f,t}}\right)^{-\eta} \left(\frac{\tilde{P}_{f,t}}{P_{f,t}}\right)^{\eta} s_{i,f,t-1}^{\theta(1-\eta)} x_{f,t}, \end{aligned}$$

where

$$x_{f,t} \equiv \left[\int_{N_f} (c_{j,f,t}/s_{j,f,t-1}^{\theta})^{1-1/\eta} dj\right]^{1/(1-1/\eta)} \qquad \text{(A.8)}$$

$$\text{and } \tilde{P}_{f,t} \equiv \left[\int_{N_f} (P_{j,f,t} s_{j,f,t-1}^{\theta})^{1-\eta} dj\right]^{1/(1-\eta)} \qquad \text{(A.9)}$$

Note that using (A.4) and (A.8), the consumption/habit aggregator x_t can be written as

$$x_t = \left[\sum_{k=h,f} \omega_k x_{k,t}^{1-1/\varepsilon}\right]^{1/(1-1/\varepsilon)} \qquad \text{(A.10)}$$

We can then think of another cost minimization problem: minimizing the cost of obtaining x_t by choosing $x_{k,t}$ when the unit price of $x_{k,t}$ is given by $P_{k,t}$, i.e.,

$$\mathcal{L}_x = \sum_{k=h,f} \tilde{P}_{k,t} x_{k,t} - \tilde{P}_t \left[\left(\sum_{k=h,f} \omega_k x_{k,t}^{1-1/\varepsilon}\right)^{1/(1-1/\varepsilon)} - x_t\right]$$

where $\tilde{P}_t$ is the Lagrangian multiplier. The efficiency conditions for this program are given by

$$x_{h,t} = \omega_h^{\varepsilon} \left(\frac{\tilde{P}_{h,t}}{\tilde{P}_t}\right)^{-\varepsilon} x_t \qquad \text{(A.11)}$$

$$\text{and } x_{f,t} = \omega_f^{\varepsilon} \left(\frac{\tilde{P}_{f,t}}{\tilde{P}_t}\right)^{-\varepsilon} x_t \qquad \text{(A.12)}$$

Substituting these conditions in (A.10) yields the following condition.

$$1 = \left\{\sum_{k=h,f} \omega_k \left[\omega_k^{\varepsilon} \left(\frac{\tilde{P}_{k,t}}{\tilde{P}_t}\right)^{-\varepsilon}\right]^{1-1/\varepsilon}\right\}^{1/(1-1/\varepsilon)}$$

Solving this expression for $\tilde{P}_t$ results in an expression for a welfare based aggregate price index:

$$\tilde{P}_t = \left[\sum_{k=h,f} \omega_k \tilde{P}_{k,t}^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \qquad \text{(A.13)}$$

A.2 Accounting Identities

The following accounting identities are used in the main text:

$$\begin{aligned}\int_{N_k} P_{i,k,t} c_{i,k,t} di &= \int_{N_k} P_{i,k,t} \left(\frac{P_{i,k,t}}{\tilde{P}_{k,t}}\right)^{-\eta} s_{i,k,t-1}^{\theta(1-\eta)} x_{k,t} di \qquad \text{(A.14)}\\ &= \tilde{P}_{k,t}^{\eta} x_{k,t} \int_{N_k} (P_{i,k,t} s^{\theta}_{i,k,t-1})^{1-\eta} di = \tilde{P}_{k,t} x_{k,t} \text{ for } k = h, f;\end{aligned}$$

$$\sum_{k=h,f} \tilde{P}_{k,t} x_{k,t} = \sum_{k=h,f} \tilde{P}_{k,t} \omega_k \left(\frac{\tilde{P}_{k,t}}{\tilde{P}_t}\right)^{-\varepsilon} x_t = \tilde{P}_t^{\varepsilon} x_t \sum_{k=h,f} \omega_k \tilde{P}_{k,t}^{1-\varepsilon} = \tilde{P}_t x_t; \qquad \text{(A.15)}$$

We define $p_{i,k,t} \equiv P_{i,k,t}/P_{k,t}$, $\tilde{p}_{k,t} \equiv \tilde{P}_{k,t}/P_{k,t}$ and $p_{k,t} \equiv P_{k,t}/P_t$ for $k = h, f$. Similarly, we define $p^*_{i,k,t} \equiv P^*_{i,k,t}/P^*_{k,t}$, $\tilde{p}^*_{k,t} \equiv \tilde{P}^*_{k,t}/P^*_{k,t}$ and $p^*_{k,t} \equiv P^*_{k,t}/P^*_t$ for $k = h, f$. Using these relative prices together with (5) and (8) under symmetric equilibrium, we can derive

$$\begin{aligned}\tilde{p}_t \equiv \frac{\tilde{P}_t}{P_t} &= \left[\sum_{k=h,f} \omega_k \tilde{p}_{k,t}^{1-\varepsilon} p_{k,t}^{1-\varepsilon}\right]^{1/(1-\varepsilon)}\\ &= \left[\sum_{k=h,f} \omega_k p_{k,t}^{1-\varepsilon} s_{k,t-1}^{\theta(1-\varepsilon)}\right]^{1/(1-\varepsilon)}. \qquad \text{(A.16)}\end{aligned}$$

B Phillips Curve

Using the symmetric equilibrium condition and dividing the FOC for $c_{i,h,t}$ of the firm problem by $\mathbb{E}^a_t[\xi_{i,t}]$, one can express the ratio of the marginal sales to the marginal value of internal funds as

$$\begin{aligned}\frac{\nu_{h,t}}{\mathbb{E}^a_t[\xi_{i,t}]} &= p_{h,t} - \frac{\mathbb{E}^a_t[\kappa_{i,t}]}{\mathbb{E}^a_t[\xi_{i,t}]} + (1-\rho)\frac{\lambda_{h,t}}{\mathbb{E}^a_t[\xi_{i,t}]} \qquad \text{(B.1)}\\ &= p_{h,t} - \frac{\mathbb{E}^a_t[\xi_{i,t} a_{i,t}]}{\mathbb{E}^a_t[\xi_{i,t}]} \frac{w_t}{\alpha A_t} \left(\phi + c_{h,t} + c^*_{h,t}\right)^{\frac{1-\alpha}{\alpha}} + (1-\rho)\frac{\lambda_{h,t}}{\mathbb{E}^a_t[\xi_{i,t}]}\end{aligned}$$

Define aggregate (marginal) gross mark-up $\mu(s_t)$ as $\mu_t \equiv \dfrac{\alpha A_t}{w_t}\left(\phi + c_{h,t} + c^*_{h,t}\right)^{\frac{\alpha-1}{\alpha}}$. Define also financially adjusted markup $\tilde{\mu}_{h,t}$ as

$$\tilde{\mu}_t \equiv \frac{\mathbb{E}^a_t[\xi_{i,t}]}{\mathbb{E}^a_t[\xi_{i,t} a_{i,t}]}\mu_t = \frac{\mathbb{E}^a_t[\xi_{i,t}]}{\mathbb{E}^a_t[\xi_{i,t} a_{i,t}]} \frac{\alpha A_t}{w_t} \left(\phi + c_{h,t} + c^*_{h,t}\right)^{\frac{\alpha-1}{\alpha}}$$

We can then express (B.1) as

$$\frac{\nu_{h,t}}{\mathbb{E}^a_t[\xi_{i,t}]} = p_{h,t} - \frac{1}{\tilde{\mu}_{h,t}} + (1-\rho)\frac{\lambda_{h,t}}{\mathbb{E}^a_t[\xi_{i,h,t}]} \qquad \text{(B.2)}$$

Dividing the FOC for $s_{i,h,t}$ through by $\mathbb{E}^a_t[\xi_{i,t}]$ and rearranging terms yields

$$\begin{aligned}\frac{\lambda_{h,t}}{\mathbb{E}^a_t[\xi_{i,t}]} &= \rho \mathbb{E}_t\left[m_{t,t+1} \frac{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}{\mathbb{E}^a_t[\xi_{i,t}]} \frac{\lambda_{h,t+1}}{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}\right] \qquad \text{(B.3)}\\ &+ \theta(1-\eta)\mathbb{E}_t\left[m_{t,t+1} \frac{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}{\mathbb{E}^a_t[\xi_{i,t}]} \frac{\nu_{h,t+1}}{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]} \frac{c_{h,t+1}}{s_{h,t}}\right]\end{aligned}$$

After substituting (B.2) in (B.3) and solving the expression forwardly, one can verify that

$$\frac{\lambda_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]} = \theta(1-\eta)\mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\tilde{\beta}_{t,s}\frac{\mathbb{E}_s^a[\xi_{i,s}]}{\mathbb{E}_t^a[\xi_{i,t}]}\left(p_{h,s}-\frac{1}{\tilde{\mu}_s}\right)\right] \tag{B.4}$$

where $\tilde{\beta}_{t,s} \equiv m_{s,s+1}g_{h,s+1} \cdot \prod_{j=1}^{s-t}[\rho + \theta(1-\eta)(1-\rho)g_{h,t+j}]m_{t+j-1,t+j}$ with $g_{h,t} \equiv c_{h,t}/s_{h,t-1} = (s_{h,t}/s_{h,t-1}-\rho)/(1-\rho)$ denotes a growth-adjusted discount factor. Hence,

$$\frac{\nu_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]} = p_{h,t} - \frac{1}{\tilde{\mu}_t} + (1-\rho)\theta(1-\eta)\mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\tilde{\beta}_{t,s}\frac{\mathbb{E}_s^a[\xi_{i,s}]}{\mathbb{E}_t^a[\xi_{i,t}]}\left(p_{h,s}-\frac{1}{\tilde{\mu}_s}\right)\right] \tag{B.5}$$

C Nonstochastic Steady State

To derive the steady state relationship, it is useful to state the problem of foreign firms and derive FOCs first. The firm problem can be expressed as the following Lagrangian:

$$\begin{aligned}
\mathcal{L}^* = \mathbb{E}_0\sum_{t=0}^{\infty} m_{0,t}^*\Bigg\{ & d_{i,t}^* + \kappa_{i,t}^*\left[\left(\frac{A_t^*}{a_{i,t}^*}h_{i,t}^*\right)^{\alpha} - \phi^* - (c_{i,f,t}^* + c_{i,f,t})\right] \\
& + \xi_{i,t}^*\Bigg[p_{i,f,t}^* p_{f,t}^* c_{i,f,t}^* + q_t^{-1} p_{i,f,t} p_{f,t} c_{i,f,t} - w_t^* h_{i,t}^* - d_{i,t}^* + \varphi^* \min\{0, d_{i,t}^*\} \\
& \qquad - \frac{\gamma}{2}\left(\frac{p_{i,f,t}^*}{p_{i,f,t-1}^*}\pi_{f,t}^* - \bar{\pi}^*\right)^2 c_t^* - \frac{\gamma^*}{2} q_t^{-1}\left(\frac{p_{i,f,t}}{p_{i,f,t-1}}\pi_{f,t} - \bar{\pi}\right)^2 c_t\Bigg] \\
& + \nu_{i,f,t}^*\left[(p_{i,f,t}^*)^{-\eta} s_{i,f,t-1}^{*\theta(1-\eta)} x_{f,t}^* - c_{i,f,t}^*\right] \\
& + \nu_{i,f,t}\left[(p_{i,f,t})^{-\eta} s_{i,f,t-1}^{\theta(1-\eta)} x_{f,t} - c_{i,f,t}\right] \\
& + \lambda_{i,f,t}^*\left[\rho s_{i,f,t-1}^* + (1-\rho)c_{i,f,t}^* - s_{i,f,t}^*\right] \\
& + \lambda_{i,f,t}\left[\rho s_{i,f,t-1} + (1-\rho)c_{i,f,t} - s_{i,f,t}\right]\Bigg\}
\end{aligned}$$

C.1 Efficiency Conditions of Foreign Firms

The efficiency conditions for the firm problem in foreign country are given by the followings:

$$d^*_{i,t} : \xi^*_{i,t} = \begin{cases} 1 & \text{if } d^*_{i,t} \geq 0 \\ 1/(1-\varphi^*) & \text{if } d^*_{i,t} < 0 \end{cases} \tag{C.1}$$

$$h^*_{i,t} : \xi^*_{i,t} w^*_t = \alpha \kappa^*_{i,t} \left(\frac{A^*_t}{a^*_{i,t}} h^*_{i,t} \right)^{\alpha - 1} \tag{C.2}$$

$$\text{where } \; h^*_{i,t} \; = \; \frac{a^*_{i,t}}{A^*_t} (\phi^* + c^*_{i,f,t} + c_{i,f,t})^{1/\alpha}$$

$$c^*_{i,f,t} : \nu^*_{i,f,t} = \mathbb{E}^a_t[\xi^*_{i,t}] \; p^*_{i,f,t} p^*_{f,t} - \mathbb{E}^a_t[\kappa^*_{i,t}] + (1-\rho)\lambda^*_{i,f,t} \tag{C.3}$$

$$c_{i,f,t} : \nu_{i,f,t} = \mathbb{E}^a_t[\xi^*_{i,t}] \; q_t^{-1} p_{i,f,t} p_{f,t} - \mathbb{E}^a_t[\kappa^*_{i,t}] + (1-\rho)\lambda_{i,f,t} \tag{C.4}$$

$$\begin{aligned} s^*_{i,f,t} : \lambda^*_{i,f,t} = {} & \rho \mathbb{E}_t[m^*_{t,t+1} \lambda^*_{i,f,t+1}] \\ & + \theta(1-\eta)\mathbb{E}_t \left\{ m^*_{t,t+1} \mathbb{E}^a_{t+1} \left[\nu^*_{i,f,t+1} \frac{c^*_{i,f,t+1}}{s^*_{i,f,t}} \right] \right\} \end{aligned} \tag{C.5}$$

$$\begin{aligned} s_{i,f,t} : \lambda_{i,f,t} = {} & \rho \mathbb{E}_t[m^*_{t,t+1} \lambda_{i,f,t+1}] \\ & + \theta(1-\eta)\mathbb{E}_t \left\{ m^*_{t,t+1} \mathbb{E}^a_{t+1} \left[\nu_{i,f,t+1} \frac{c_{i,f,t+1}}{s_{i,f,t}} \right] \right\} \end{aligned} \tag{C.6}$$

$$\begin{aligned} p^*_{i,f,t} : 0 = {} & \mathbb{E}^a_t[\xi^*_{i,t}] \left[p^*_{f,t} c^*_{i,f,t} - \gamma \frac{\pi^*_{f,t}}{p^*_{i,f,t-1}} \left(\pi^*_{f,t} \frac{p^*_{i,f,t}}{p^*_{i,f,t-1}} - \bar{\pi}^* \right) c^*_t \right] - \eta \frac{\nu^*_{i,f,t}}{p^*_{i,f,t}} c^*_{i,f,t} \\ & + \gamma \mathbb{E}_t \left[m^*_{t,t+1} \mathbb{E}^a_{t+1}[\xi^*_{i,t+1}] \pi^*_{f,t+1} \frac{p^*_{i,f,t+1}}{p^{*2}_{i,f,t}} \left(\pi^*_{f,t+1} \frac{p^*_{i,f,t+1}}{p^*_{i,f,t}} - \bar{\pi}^* \right) c^*_{t+1} \right] \end{aligned} \tag{C.7}$$

$$\begin{aligned} p_{i,f,t} : 0 = {} & \mathbb{E}^a_t[\xi^*_{i,t}] \left[q_t^{-1} p_{f,t} c_{i,f,t} - \gamma \frac{q_t^{-1} \pi_{f,t}}{p_{i,f,t-1}} \left(\pi_{f,t} \frac{p_{i,f,t}}{p_{i,f,t-1}} - \bar{\pi} \right) c_t \right] - \eta \frac{\nu_{i,f,t}}{p_{i,f,t}} c_{i,f,t} \\ & + \gamma \mathbb{E}_t \left[m^*_{t,t+1} \mathbb{E}^a_{t+1}[\xi^*_{i,t+1}] q_{t+1}^{-1} \pi_{f,t+1} \frac{p_{i,f,t+1}}{p^2_{i,f,t}} \left(\pi_{f,t+1} \frac{p_{i,f,t+1}}{p_{i,f,t}} - \bar{\pi} \right) c_{t+1} \right] \end{aligned} \tag{C.8}$$

C.2 Symmetric Equilibrium and Relative Prices

Before we move onto the model dynamics, it is useful to discuss how relative prices are determined and how they are related with each other in symmetric equilibrium. The risk neutrality, i.i.d. idiosyncratic shock and the timing convention aforementioned imply that all home country firms choose an identical price level for a given market, i.e., $P_{i,h,t} = P_{h,t}$ and $P^*_{i,h,t} = P^*_{h,t}$. Similarly, $P_{i,f,t} = P_{f,t}$ and $P^*_{i,f,t} = P^*_{f,t}$. Due to the pricing to market mechanism, $P_{i,h,t} \neq S_t P^*_{i,h,t}$ and $P_{i,f,t} \neq S_t^{-1} P^*_{i,f,t}$ in general. However, the symmetric equilibrium implies $p_{i,h,t}(= P_{i,h,t}/P_{h,t}) = p^*_{i,h,t}(= P^*_{i,h,t}/P^*_{h,t}) = p_{i,f,t}(= P_{i,f,t}/P_{f,t}) = p^*_{i,f,t}(= P^*_{i,f,t}/P^*_{f,t}) = 1$ always.

In any path of symmetric equilibrium, the relative ratio of tytpe specific, habit adjusted price index ($\tilde{P}_{k,t}$) and CPI index ($P_{k,t}$) satisfy the followings:

$$\begin{aligned} \tilde{p}_{h,t} = \tilde{P}_{h,t}/P_{h,t} &= s^{\theta}_{h,t-1} \\ \tilde{p}^*_{h,t} = \tilde{P}^*_{h,t}/P^*_{h,t} &= s^{*\theta}_{h,t-1} \\ \tilde{p}_{f,t} = \tilde{P}_{f,t}/P_{f,t} &= s^{\theta}_{f,t-1} \\ \text{and } \tilde{p}^*_{f,t} = \tilde{P}^*_{f,t}/P^*_{f,t} &= s^{*\theta}_{f,t-1}. \end{aligned}$$

These relative prices can then be used to derive the demands for habit adjusted consumption baskets in the symmetric equilbrium: With a symmetric equilibrium condition $x^j_{h,t} = x_{h,t}$,

$$\begin{aligned} x_{h,t} &= \omega_h^{\varepsilon}\left(\frac{\tilde{P}_{h,t}}{\tilde{P}_t}\right)^{-\varepsilon} x_t \\ &= \omega_h^{\varepsilon}\left(\frac{\tilde{P}_{h,t}}{P_{h,t}}\cdot\frac{P_{h,t}}{P_t}\cdot\frac{P_t}{\tilde{P}_t}\right)^{-\varepsilon} x_t \\ &= \omega_h^{\varepsilon} p_{h,t}^{-\varepsilon}\left(\frac{\tilde{p}_{h,t}}{\tilde{p}_t}\right)^{-\varepsilon} x_t \end{aligned}$$

where

$$\begin{aligned} \tilde{p}_t &= \left[\sum_{k=h,f}\omega_k\left(\frac{\tilde{P}_{k,t}}{P_t}\right)^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \\ &= \left[\sum_{k=h,f}\omega_k\left(\frac{\tilde{P}_{k,t}}{P_{k,t}}\cdot\frac{P_{k,t}}{P_t}\right)^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \\ &= \left[\sum_{k=h,f}\omega_k s_{k,t-1}^{\theta(1-\varepsilon)} p_{k,t}^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \end{aligned}$$

Similary, it can be shown that

$$\begin{aligned} x_{f,t} &= \omega_f^{\varepsilon} p_{f,t}^{-\varepsilon}\left(\frac{\tilde{p}_{f,t}}{\tilde{p}_t}\right)^{-\varepsilon} x_t, \\ x^*_{h,t} &= \omega_h^{*\varepsilon}(p^*_{h,t})^{-\varepsilon}\left(\frac{\tilde{p}^*_{h,t}}{\tilde{p}^*_t}\right)^{-\varepsilon} x^*_t, \\ x^*_{f,t} &= \omega_f^{*\varepsilon}(p^*_{f,t})^{-\varepsilon}\left(\frac{\tilde{p}^*_{f,t}}{\tilde{p}^*_t}\right)^{-\varepsilon} x^*_t, \end{aligned}$$

and

$$\tilde{p}^*_t = \left[\sum_{k=h,f}\omega^*_k s_{k,t-1}^{*\theta(1-\varepsilon)} p_{k,t}^{*(1-\varepsilon)}\right]^{1/(1-\varepsilon)}.$$

As is usual, only relative prices are determined in equilibrium: $p_{h,t}$, $p^*_{h,t}$, $p_{f,t}$, $p^*_{f,t}$, $\tilde{p}_{h,t}$, $\tilde{p}^*_{h,t}$, $\tilde{p}_{f,t}$, $\tilde{p}^*_{f,t}$, $\tilde{p}_t$, $\tilde{p}^*_t$ and q_t.

C.3 Equilibrium Relative Prices and Quantities

The Phillips curves in the steady state is given by

$$p_h = \eta\frac{\nu_h}{\mathbb{E}^a_t[\xi_i]} \tag{C.9}$$

$$qp^*_h = \eta\frac{\nu^*_h}{\mathbb{E}^a_t[\xi_i]} \tag{C.10}$$

$$p^*_f = \eta\frac{\nu^*_f}{\mathbb{E}^a_t[\xi^*_i]} \tag{C.11}$$

$$\text{and } p_f q^{-1} = \eta\frac{\nu_f}{\mathbb{E}^a_t[\xi^*_i]}. \tag{C.12}$$

(C.9)~(C.12) are the steady state Phillips curves of home good in home country, home good in foreign country, foreign good in home country and foreign good in foreign country, respectively. The notational convention is that h and f indicate the origin of the good, and asterisks and the absence thereof indicate the destination of the good with asterisks indicating foreign country. For instance, p_f^* is the (relative) price of good produced by and sold in foreign country, whereas p_f is the price of good produced by foreign country, but sold in home country in home currency unit, and hence $1/q$ attached to it to convert it to foreign currency unit.

The symmetric equilibrium and the law of motion for habit stock imply $c_h = s_h$, $c_h^* = s_h^*$, $c_f = s_f$ and $c_f^* = s_f^*$. Using these conditions together with the FOCs for habit stocks, one can derive

$$\frac{\lambda_h}{\mathbb{E}^a[\xi_i]} = \frac{\theta(1-\eta)\beta}{1-\rho\beta}\frac{\nu_h}{\mathbb{E}^a[\xi_i]}, \tag{C.13}$$

$$\frac{\lambda_h^*}{\mathbb{E}^a[\xi_i]} = \frac{\theta(1-\eta)\beta}{1-\rho\beta}\frac{\nu_h^*}{\mathbb{E}^a[\xi_i]}, \tag{C.14}$$

$$\frac{\lambda_f^*}{\mathbb{E}^a[\xi_i^*]} = \frac{\theta(1-\eta)\beta}{1-\rho\beta}\frac{\nu_f^*}{\mathbb{E}^a[\xi_i^*]}, \tag{C.15}$$

$$\text{and } \frac{\lambda_f}{\mathbb{E}^a[\xi_i^*]} = \frac{\theta(1-\eta)\beta}{1-\rho\beta}\frac{\nu_f}{\mathbb{E}^a[\xi_i^*]}. \tag{C.16}$$

Combining (C.9)~(C.12) and (C.13)~(C.16) yields

$$\frac{\lambda_h}{\mathbb{E}^a[\xi_i]} = p_h\frac{\theta(1-\eta)\beta}{\eta(1-\rho\beta)}, \tag{C.17}$$

$$\frac{\lambda_h^*}{\mathbb{E}^a[\xi_i]} = qp_h^*\frac{\theta(1-\eta)\beta}{\eta(1-\rho\beta)}, \tag{C.18}$$

$$\frac{\lambda_f^*}{\mathbb{E}^a[\xi_i^*]} = p_f^*\frac{\theta(1-\eta)\beta}{\eta(1-\rho\beta)}, \tag{C.19}$$

$$\text{and } \frac{\lambda_f}{\mathbb{E}^a[\xi_i^*]} = \frac{p_f}{q}\frac{\theta(1-\eta)\beta}{\eta(1-\rho\beta)}, \tag{C.20}$$

which imply $qp_h^*/p_h = \lambda_h^*/\lambda_h$ and $qp_f^*/p_f = \lambda_f^*/\lambda_f$. Combining the FOCs (16), (18), (19), (C.2), (C.3) and (C.4), we have

$$\frac{\nu_h}{\mathbb{E}^a[\xi_i]} = p_h - \frac{\mathbb{E}^a[\xi_i a_i]}{\mathbb{E}^a[\xi_i]}\frac{w}{\alpha A}\left(\phi + c_h + c_h^*\right)^{\frac{1-\alpha}{\alpha}} + (1-\rho)\frac{\lambda_h}{\mathbb{E}^a[\xi_i]}, \tag{C.21}$$

$$\frac{\nu_h^*}{\mathbb{E}^a[\xi_i]} = qp_h^* - \frac{\mathbb{E}^a[\xi_i a_i]}{\mathbb{E}^a[\xi_i]}\frac{w}{\alpha A}\left(\phi + c_h + c_h^*\right)^{\frac{1-\alpha}{\alpha}} + (1-\rho)\frac{\lambda_h^*}{\mathbb{E}^a[\xi_i]}, \tag{C.22}$$

$$\frac{\nu_f^*}{\mathbb{E}^a[\xi_i^*]} = p_f^* - \frac{\mathbb{E}^a[\xi_i^* a_i^*]}{\mathbb{E}^a[\xi_i^*]}\frac{w^*}{\alpha A^*}\left(\phi^* + c_f^* + c_f\right)^{\frac{1-\alpha}{\alpha}} + (1-\rho)\frac{\lambda_f^*}{\mathbb{E}^a[\xi_i^*]}, \tag{C.23}$$

$$\text{and } \frac{\nu_f}{\mathbb{E}^a[\xi_i^*]} = \frac{p_f}{q} - \frac{\mathbb{E}^a[\xi_i^* a_i^*]}{\mathbb{E}^a[\xi_i^*]}\frac{w^*}{\alpha A^*}\left(\phi^* + c_f^* + c_f\right)^{\frac{1-\alpha}{\alpha}} + (1-\rho)\frac{\lambda_f}{\mathbb{E}^a[\xi_i^*]}. \tag{C.24}$$

Substituting (C.9)∼(C.12) and(C.17)∼(C.20) in (C.21)∼(C.24) and solving for p_k and p_k^* yields

$$p_h = \frac{\eta(1-\rho\beta)}{(\eta-1)[(1-\rho\beta)-\theta\beta(1-\rho)]}\frac{\mathbb{E}^a[\xi_i a_i]}{\mathbb{E}^a[\xi_i]}\frac{w}{\alpha A}(\phi + c_h + c_h^*)^{\frac{1-\alpha}{\alpha}} \tag{C.25}$$

$$p_h^* = \frac{\eta(1-\rho\beta)}{(\eta-1)[(1-\rho\beta)-\theta\beta(1-\rho)]}q^{-1}\frac{\mathbb{E}^a[\xi_i a_i]}{\mathbb{E}^a[\xi_i]}\frac{w}{\alpha A}(\phi + c_h + c_h^*)^{\frac{1-\alpha}{\alpha}} \tag{C.26}$$

$$p_f^* = \frac{\eta(1-\rho\beta)}{(\eta-1)[(1-\rho\beta)-\theta\beta(1-\rho)]}\frac{\mathbb{E}^a[\xi_i^* a_i^*]}{\mathbb{E}^a[\xi_i^*]}\frac{w^*}{\alpha A^*}(\phi^* + c_f^* + c_f)^{\frac{1-\alpha}{\alpha}} \tag{C.27}$$

$$\text{and } p_f = \frac{\eta(1-\rho\beta)}{(\eta-1)[(1-\rho\beta)-\theta\beta(1-\rho)]}q\frac{\mathbb{E}^a[\xi_i^* a_i^*]}{\mathbb{E}^a[\xi_i^*]}\frac{w^*}{\alpha A^*}(\phi^* + c_f^* + c_f)^{\frac{1-\alpha}{\alpha}} \tag{C.28}$$

Note that the law of one price holds in the non-stochastic steady state: $p_h = qp_h^*$ and $p_f^* = p_f/q$, which also impliy $\lambda_h^*/\lambda_h = 1$ and $\lambda_f^*/\lambda_f = 1$. This is simply because we assume the symmetry of the two markets in terms of elasticity of substitution, strength of customer relationship, etc. However, the law of one price is generally violated in stochastic simulation as two countries undergo different histories of asymmetric shocks, which affect the intensity of customer relationships, and hence the demand elasticities in the two countries, and different financing conditions. Firms in general exploit any discrepancies in customer relationship and discriminate prices across the border.

The external financing triggers in the steady state are given by

$$a^E = \frac{A}{w(\phi + c_h + c_h^*)^{1/\alpha}}(p_h c_h + qp_h^* c_h^*) \tag{C.29}$$

$$a^{E*} = \frac{A^*}{w^*(\phi^* + c_f^* + c_f)^{1/\alpha}}(p_f^* c_f^* + q^{-1}p_f c_f), \tag{C.30}$$

which can be used to compute $\mathbb{E}^a[\xi_i]$, $\mathbb{E}^a[\xi_i a_i]$, $\mathbb{E}^a[\xi_i^*]$ and $\mathbb{E}^a[\xi_i^* a_i^*]$:

$$\mathbb{E}^a[\xi_i] = 1 + \frac{\varphi}{1-\varphi}[1 - \Phi(z^E)] \tag{C.31}$$

$$\mathbb{E}^a[\xi_i a_i] = 1 + \frac{\varphi}{1-\varphi}[1 - \Phi(z^E - \sigma)] \tag{C.32}$$

$$\mathbb{E}^a[\xi_i^*] = 1 + \frac{\varphi^*}{1-\varphi^*}[1 - \Phi(z^{*E})] \tag{C.33}$$

$$\mathbb{E}^a[\xi_i^* a_i^*] = 1 + \frac{\varphi^*}{1-\varphi^*}[1 - \Phi(z^{*E} - \sigma)] \tag{C.34}$$

where

$$z^E \equiv \sigma^{-1}(\log a^E + 0.5\sigma^2) \tag{C.35}$$

$$\text{and } z^{*E} \equiv \sigma^{-1}(\log a^{*E} + 0.5\sigma^2). \tag{C.36}$$

(4) and (7) and their foreign counterparts imply that the following ratios should be satisfied in the steady state

$$\frac{c_{i,h}}{c_{i,f}} = \frac{p_{i,h}^{-\eta}\tilde{p}_h^{\eta}s_{i,h}^{\theta(1-\eta)}x_h}{p_{i,f}^{-\eta}\tilde{p}_f^{\eta}s_{i,f}^{\theta(1-\eta)}x_f} = \frac{p_{i,h}^{-\eta}\tilde{p}_h^{\eta}s_{i,h}^{\theta(1-\eta)}\omega_h^{\varepsilon}\tilde{p}_h^{-\varepsilon}p_h^{-\varepsilon}\tilde{p}^{\varepsilon}x}{p_{i,f}^{-\eta}\tilde{p}_f^{\eta}s_{i,f}^{\theta(1-\eta)}\omega_f^{\varepsilon}\tilde{p}_f^{-\varepsilon}p_f^{-\varepsilon}\tilde{p}^{\varepsilon}x}$$

$$\frac{c_{i,h}^*}{c_{i,f}^*} = \frac{p_{i,h}^{*-\eta}\tilde{p}_h^{*\eta}s_{i,h}^{*\theta(1-\eta)}x_h^*}{p_{i,f}^{*-\eta}\tilde{p}_f^{*\eta}s_{i,f}^{*\theta(1-\eta)}x_f^*} = \frac{p_{i,h}^{*-\eta}\tilde{p}_h^{*\eta}s_{i,h}^{*\theta(1-\eta)}\omega_h^{\varepsilon}\tilde{p}_h^{*-\varepsilon}p_h^{*-\varepsilon}\tilde{p}^{*\varepsilon}x^*}{p_{i,f}^{*-\eta}\tilde{p}_f^{*\eta}s_{i,f}^{*\theta(1-\eta)}\omega_f^{\varepsilon}\tilde{p}_f^{*-\varepsilon}p_f^{*-\varepsilon}\tilde{p}^{*\varepsilon}x^*}$$

Imposing the symmetric equilibrium conditions and using $\tilde{p}_k = s_k^{\theta}$, we have

$$\frac{c_h}{c_f} = \left(\frac{\omega_h}{\omega_f}\right)^{\varepsilon}\left(\frac{p_h}{p_f}\right)^{-\varepsilon}\left(\frac{s_h^{\theta}}{s_f^{\theta}}\right)^{1-\varepsilon} \tag{C.37}$$

$$\text{and } \frac{c_h^*}{c_f^*} = \left(\frac{\omega_h}{\omega_f}\right)^{\varepsilon}\left(\frac{p_h^*}{p_f^*}\right)^{-\varepsilon}\left(\frac{s_h^{*\theta}}{s_f^{*\theta}}\right)^{1-\varepsilon}. \tag{C.38}$$

Since $c_{i,k} = c_k = s_k = s_{i,k}$ and $c_{i,k} = c_k^* = s_k^* = s_{i,k}^*$, (6) and its foreign counterpart imply

$$x = \left[\sum_{k=h,f} \omega_k (c_k^{1-\theta})^{1-1/\varepsilon}\right]^{1/(1-1/\varepsilon)} \tag{C.39}$$

$$x^* = \left[\sum_{k=h,f} \omega_k (c_k^{*1-\theta})^{1-1/\varepsilon}\right]^{1/(1-1/\varepsilon)}. \tag{C.40}$$

Aggregate (conditional) labor demand in home and foreign markets satisfy

$$h = \left[\frac{\phi + c_h + c_h^*}{A^{\alpha}\exp[0.5\alpha(1+\alpha)\sigma^2]}\right]^{1/\alpha}, \tag{C.41}$$

$$\text{and } h^* = \left[\frac{\phi^* + c_f + c_f^*}{A^{*\alpha}\exp[0.5\alpha(1+\alpha)\sigma^2]}\right]^{1/\alpha}, \tag{C.42}$$

which also implies goods market clearing conditions. The FOCs of households for labor hours can be expressed as

$$h = U_h^{-1}\left[-\frac{w}{\tilde{p}}\frac{\eta_w - 1}{\eta_w}U_x\right], \tag{C.43}$$

$$\text{and } h^* = U_h^{-1}\left[-\frac{w^*}{\tilde{p}^*}\frac{\eta_w - 1}{\eta_w}U_x^*\right], \tag{C.44}$$

The labor market clearing conditions in home and abroad can then be given by

$$U_h^{-1}\left[-\frac{w}{\tilde{p}}\frac{\eta_w - 1}{\eta_w}U_x\right] = \left[\frac{\phi + c_h + c_h^*}{A^{\alpha}\exp[0.5\alpha(1+\alpha)\sigma^2]}\right]^{1/\alpha} \tag{C.45}$$

$$\text{and } U_h^{-1}\left[-\frac{w^*}{\tilde{p}^*}\frac{\eta_w - 1}{\eta_w}U_x^*\right] = \left[\frac{\phi^* + c_f + c_f^*}{A^{*\alpha}\exp[0.5\alpha(1+\alpha)\sigma^2]}\right]^{1/\alpha}, \tag{C.46}$$

which charaterize the labor market clearing conditions in home and abroad, and can be used to equilibrium wages in both markets. Finally, equilibrium consistency requires

$$1 = \left[\sum_{k=h,f} \omega_k p_k^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \tag{C.47}$$

$$\text{and } 1 = \left[\sum_{k=h,f} \omega_k p_k^{*1-\varepsilon}\right]^{1/(1-\varepsilon)} \tag{C.48}$$

C.4 Real Exchange Rate

In the case of complete risk sharing between the two countries, the real exchange rate at any point in time should satisfy (42).

$$q = \kappa \frac{U_x^*}{U_x} \left[\frac{\sum_{k=h,f} \omega_k (p_k^* s_k^{*\theta})^{(1-\varepsilon)}}{\sum_{k=h,f} \omega_k (p_k s_k^{\theta})^{(1-\varepsilon)}} \right]^{-1/(1-\varepsilon)} \tag{C.49}$$

We assume that the equilibrium interest rates are determined by time preferences: $r = r^* = \beta^{-1} - 1$. This condition, in the case of incomplete risk sharing, pins down the equilibrium holdings of international bonds: $B_h = B_f = 0$, which, via the bond market clearing conditions, $B_h + B_h^* = 0$ and $B_f + B_f^* = 0$, pins down $B_h^* = B_f^* = 0$. In the case of incomplete risk sharing, the real exchange rate is determined such that $b_h = b_f = 0$, which, together with (59) implies

$$0 = wh - qw^*h^* + \tilde{d} - q\tilde{d}^* - (\tilde{p}x - q\tilde{p}^*x^*)$$

or equivalently,

$$q = \frac{wh + \tilde{d} - \tilde{p}x}{w^*h^* + \tilde{d}^* - \tilde{p}^*x^*}. \tag{C.50}$$

D System of Equations

There are total 71 equations for 71 endogenous variables in the system in the case with a floating exchange rate under the complete risk sharing arrangement. We provide these equations in the symmetric equilibrium forms:

$$\begin{aligned} 0 = & -\frac{h_t^{1/\zeta}/U_{x,t}}{w_t/\tilde{p}_t} + \frac{\eta_w - 1}{\eta_w} + \frac{\gamma_w}{\eta_w}(\pi_{w,t} - \pi_w)\pi_{w,t} \\ & -\beta\frac{\gamma_w}{\eta_w}\mathbb{E}_t\left[\frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_t}(\pi_{w,t+1} - \pi_w)\pi_{w,t+1}\frac{\pi_{w,t+1}}{\pi_{t+1}}\frac{h_{t+1}}{h_t}\right] \end{aligned} \tag{D.1}$$

$$\begin{aligned} 0 = & -\frac{h_t^{*1/\zeta}/U_{x,t}}{w_t^*/\tilde{p}_t^*} + \frac{\eta_w - 1}{\eta_w} + \frac{\gamma_w}{\eta_w}(\pi_{w,t}^* - \pi_w)\pi_{w,t}^* \\ & -\beta\frac{\gamma_w}{\eta_w}\mathbb{E}_t\left[\frac{U_{x,t+1}^*/\tilde{p}_{t+1}^*}{U_{x,t}^*/\tilde{p}_t^*}(\pi_{w,t+1}^* - \pi_w)\pi_{w,t+1}^*\frac{\pi_{w,t+1}^*}{\pi_{t+1}^*}\frac{h_{t+1}^*}{h_t^*}\right] \end{aligned} \tag{D.2}$$

$$0 = -\frac{c_{h,t}}{c_{f,t}} + \left(\frac{\omega_h}{\omega_f}\right)^{\varepsilon}\left(\frac{p_{h,t}}{p_{f,t}}\right)^{-\varepsilon}\left(\frac{s_{h,t-1}^{\theta}}{s_{f,t-1}^{\theta}}\right)^{1-\varepsilon} \tag{D.3}$$

$$0 = -\frac{c_{h,t}^*}{c_{f,t}^*} + \left(\frac{\omega_h}{\omega_f}\right)^{\varepsilon}\left(\frac{p_{h,t}^*}{p_{f,t}^*}\right)^{-\varepsilon}\left(\frac{s_{h,t-1}^{*\theta}}{s_{f,t-1}^{*\theta}}\right)^{1-\varepsilon} \tag{D.4}$$

$$0 = -\tilde{p}_{h,t} + s_{h,t-1}^{\theta} \tag{D.5}$$

$$0 = -\tilde{p}_{f,t} + s_{f,t-1}^{\theta} \tag{D.6}$$

$$0 = -\tilde{p}_{h,t}^* + s_{h,t-1}^{*\theta} \tag{D.7}$$

$$0 = -\tilde{p}_{f,t}^* + s_{f,t-1}^{*\theta} \tag{D.8}$$

$$0 = -x_{h,t} + \omega_h^{\varepsilon}p_{h,t}^{-\varepsilon}\left(\frac{\tilde{p}_{h,t}}{\tilde{p}_t}\right)^{-\varepsilon}x_t \tag{D.9}$$

$$0 = -x_{f,t} + \omega_f^{\varepsilon}p_{f,t}^{-\varepsilon}\left(\frac{\tilde{p}_{f,t}}{\tilde{p}_t}\right)^{-\varepsilon}x_t \tag{D.10}$$

$$0 = -x_{h,t}^* + \omega_h^{*\varepsilon}(p_{h,t}^*)^{-\varepsilon}\left(\frac{\tilde{p}_{h,t}^*}{\tilde{p}_t^*}\right)^{-\varepsilon}x_t^* \tag{D.11}$$

$$0 = -x_{f,t}^* + \omega_f^{*\varepsilon}(p_{f,t}^*)^{-\varepsilon}\left(\frac{\tilde{p}_{f,t}^*}{\tilde{p}_t^*}\right)^{-\varepsilon}x_t^* \tag{D.12}$$

$$0 = -\tilde{p}_t + \left[\sum_{k=h,f}\omega_k s_{k,t-1}^{\theta(1-\varepsilon)}p_{k,t}^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \tag{D.13}$$

$$0 = -\tilde{p}_t^* + \left[\sum_{k=h,f} \omega_k^* s_{k,t-1}^{*\theta(1-\varepsilon)} p_{k,t}^{*(1-\varepsilon)} \right]^{1/(1-\varepsilon)} \tag{D.14}$$

$$0 = -\pi_{h,t} + \frac{p_{h,t}}{p_{h,t-1}} \pi_t \tag{D.15}$$

$$0 = -\pi_{h,t}^* + \frac{p_{h,t}^*}{p_{h,t-1}^*} \pi_t^* \tag{D.16}$$

$$0 = -\pi_{f,t} + \frac{p_{f,t}}{p_{f,t-1}} \pi_t \tag{D.17}$$

$$0 = -\pi_{f,t}^* + \frac{p_{f,t}^*}{p_{f,t-1}^*} \pi_t^* \tag{D.18}$$

$$0 = -h_t^S + h_t^D \tag{D.19}$$

$$0 = -h_t^{*S} + h_t^{*D} \tag{D.20}$$

$$0 = -\mathbb{E}_t^a[\kappa_{i,t}] + \mathbb{E}_t^a[\xi_{i,t} a_{i,t}] \frac{w_t}{\alpha A_t} (\phi + c_{h,t} + c_{h,t}^*)^{\frac{1-\alpha}{\alpha}} \tag{D.21}$$

$$0 = -\mathbb{E}_t^a[\kappa_{i,t}^*] + \mathbb{E}_t^a[\xi_{i,t}^* a_{i,t}^*] \frac{w_t^*}{\alpha A_t^*} (\phi + c_{f,t} + c_{f,t}^*)^{\frac{1-\alpha}{\alpha}} \tag{D.22}$$

$$0 = -\nu_{h,t} + \mathbb{E}_t^a[\xi_{i,t}]\ p_{h,t} - \mathbb{E}_t^a[\kappa_{i,t}] + (1-\rho)\lambda_{h,t} \tag{D.23}$$

$$0 = -\nu_{h,t}^* + \mathbb{E}_t^a[\xi_{i,t}]\ q_t p_{h,t}^* - \mathbb{E}_t^a[\kappa_{i,t}] + (1-\rho)\lambda_{h,t}^* \tag{D.24}$$

$$0 = -\lambda_{h,t} + \rho \mathbb{E}_t[m_{t,t+1} \lambda_{h,t+1}] + \theta(1-\eta) \mathbb{E}_t \left\{ m_{t,t+1} \mathbb{E}_{t+1}^a \left[\nu_{h,t+1} \frac{c_{h,t+1}}{s_{h,t}} \right] \right\} \tag{D.25}$$

$$0 = -\lambda_{h,t}^* + \rho \mathbb{E}_t[m_{t,t+1} \lambda_{h,t+1}^*] + \theta(1-\eta) \mathbb{E}_t \left\{ m_{t,t+1} \mathbb{E}_{t+1}^a \left[\nu_{h,t+1}^* \frac{c_{h,t+1}^*}{s_{h,t}^*} \right] \right\} \tag{D.26}$$

$$0 = -\nu_{f,t}^* + \mathbb{E}_t^a[\xi_{i,t}^*]\ p_{f,t}^* - \mathbb{E}_t^a[\kappa_{i,t}^*] + (1-\rho)\lambda_{f,t}^* \tag{D.27}$$

$$0 = -\nu_{f,t} + \mathbb{E}_t^a[\xi_{i,t}^*]\ q_t^{-1} p_{f,t} - \mathbb{E}_t^a[\kappa_{i,t}^*] + (1-\rho)\lambda_{f,t} \tag{D.28}$$

$$0 = -\lambda_{f,t}^* + \rho \mathbb{E}_t[m_{t,t+1}^* \lambda_{f,t+1}^*] + \theta(1-\eta) \mathbb{E}_t \left\{ m_{t,t+1}^* \mathbb{E}_{t+1}^a \left[\nu_{f,t+1}^* \frac{c_{f,t+1}^*}{s_{f,t}^*} \right] \right\} \tag{D.29}$$

$$0 = -\lambda_{f,t} + \rho \mathbb{E}_t[m_{t,t+1}^* \lambda_{f,t+1}] + \theta(1-\eta) \mathbb{E}_t \left\{ m_{t,t+1}^* \mathbb{E}_{t+1}^a \left[\nu_{f,t+1} \frac{c_{f,t+1}}{s_{f,t}} \right] \right\} \tag{D.30}$$

$$\begin{aligned}0 = {}& -p_{h,t}\frac{c_{h,t}}{c_t} + \gamma\pi_{h,t}(\pi_{h,t}-\bar{\pi}) + \eta\frac{\nu_{h,t}}{\mathbb{E}^a_t[\xi_{i,t}]}\frac{c_{h,t}}{c_t} \\ & -\gamma\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}{\mathbb{E}^a_t[\xi_{i,t}]}\pi_{h,t+1}(\pi_{h,t+1}-\bar{\pi})\frac{c_{t+1}}{c_t}\right]\end{aligned} \tag{D.31}$$

$$\begin{aligned}0 = {}& -q_t p^*_{h,t}\frac{c^*_{h,t}}{c^*_t} + \gamma q_t\pi^*_{h,t}(\pi^*_{h,t}-\bar{\pi}^*) + \eta\frac{\nu^*_{h,t}}{\mathbb{E}^a_t[\xi_{i,t}]}\frac{c^*_{h,t}}{c^*_t} \\ & -\gamma^*\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}^a_{t+1}[\xi_{i,t+1}]}{\mathbb{E}^a_t[\xi_{i,t}]}q_{t+1}\pi^*_{h,t+1}(\pi^*_{h,t+1}-\bar{\pi})\frac{c^*_{t+1}}{c^*_t}\right]\end{aligned} \tag{D.32}$$

$$\begin{aligned}0 = {}& -p^*_{f,t}\frac{c^*_{f,t}}{c^*_t} + \gamma\pi^*_{f,t}(\pi^*_{f,t}-\bar{\pi}^*) + \eta\frac{\nu^*_{i,f,t}}{\mathbb{E}^a_t[\xi^*_{i,t}]}\frac{c^*_{f,t}}{c^*_t} \\ & -\gamma\mathbb{E}_t\left[m^*_{t,t+1}\frac{\mathbb{E}^a_{t+1}[\xi^*_{i,t+1}]}{\mathbb{E}^a_t[\xi^*_{i,t}]}\pi^*_{f,t+1}\left(\pi^*_{f,t+1}-\bar{\pi}^*\right)\frac{c^*_{t+1}}{c^*_t}\right]\end{aligned} \tag{D.33}$$

$$\begin{aligned}0 = {}& -q_t^{-1}p_{f,t}\frac{c_{f,t}}{c_t} + \gamma q_t^{-1}\pi_{f,t}(\pi_{f,t}-\bar{\pi}) + \eta\frac{\nu_{i,f,t}}{\mathbb{E}^a_t[\xi^*_{i,t}]}\frac{c_{f,t}}{c_t} \\ & -\gamma\mathbb{E}_t\left[m^*_{t,t+1}\frac{\mathbb{E}^a_{t+1}[\xi^*_{i,t+1}]}{\mathbb{E}^a_t[\xi^*_{i,t}]}q^{-1}_{t+1}\pi_{f,t+1}\left(\pi_{f,t+1}-\bar{\pi}\right)\frac{c_{t+1}}{c_t}\right]\end{aligned} \tag{D.34}$$

$$0 = -\mu_t + \frac{\alpha A_t}{w_t}\left(\phi + c_{h,t} + c^*_{h,t}\right)^{\frac{\alpha-1}{\alpha}} \tag{D.35}$$

$$0 = -\mu^*_t + \frac{\alpha A^*_t}{w^*_t}(\phi^* + c_{f,t} + c^*_{f,t})^{\frac{\alpha-1}{\alpha}} \tag{D.36}$$

$$0 = -\tilde{\mu}_t + \frac{\mathbb{E}^a_t[\xi_{i,t}]}{\mathbb{E}^a_t[\xi_{i,t}a_{i,t}]}\mu_t \tag{D.37}$$

$$0 = -\tilde{\mu}^*_t + \frac{\mathbb{E}^a_t[\xi^*_{i,t}]}{\mathbb{E}^a_t[\xi^*_{i,t}a^*_{i,t}]}\mu^*_t \tag{D.38}$$

$$\begin{aligned}0 = {}& -a^E_t + \frac{A_t}{w_t(\phi + c_{h,t} + c^*_{h,t})^{1/\alpha}} \\ & \times\left\{c_t\left[\frac{p_{h,t}c_{h,t}}{c_t} - \frac{\gamma}{2}(\pi_{h,t}-\bar{\pi})^2\right] + q_t c^*_t\left[\frac{p^*_{h,t}c^*_{h,t}}{c^*_t} - \frac{\gamma^*}{2}(\pi^*_{h,t}-\bar{\pi}^*)^2\right]\right\}\end{aligned} \tag{D.39}$$

$$\begin{aligned}0 = {}& -a^{*E}_t + \frac{A^*_t}{w^*_t(\phi^* + c_{f,t} + c^*_{f,t})^{1/\alpha}} \\ & \times\left\{c^*_t\left[\frac{p^*_{f,t}c^*_{f,t}}{c^*_t} - \frac{\gamma^*}{2}(\pi^*_{f,t}-\bar{\pi})^2\right] + q_t^{-1}c_t\left[\frac{p_{f,t}c_{f,t}}{c_t} - \frac{\gamma}{2}(\pi_{f,t}-\bar{\pi})^2\right]\right\}\end{aligned} \tag{D.40}$$

$$0 = -z^E_t + \sigma^{-1}(\log a^E_t + 0.5\sigma^2) \tag{D.41}$$

$$0 = -z_t^{*E} + \sigma^{-1}(\log a_t^{*E} + 0.5\sigma^2) \tag{D.42}$$

$$0 = -\mathbb{E}_t^a[\xi_{i,t}] + 1 + \frac{\varphi_t}{1-\varphi_t}[1 - \Phi(z_t^E)] \tag{D.43}$$

$$0 = -\mathbb{E}_t^a[\xi_{i,t} a_{i,t}] + 1 + \frac{\varphi_t}{1-\varphi_t}[1 - \Phi(z_t^E - \sigma)] \tag{D.44}$$

$$0 = -\mathbb{E}_t^a[\xi_{i,t}^*] + 1 + \frac{\varphi_t^*}{1-\varphi_t^*}[1 - \Phi(z_t^{*E})] \tag{D.45}$$

$$0 = -\mathbb{E}_t^a[\xi_{i,t}^* a_{i,t}^*] + 1 + \frac{\varphi_t^*}{1-\varphi_t^*}[1 - \Phi(z_t^{*E} - \sigma)] \tag{D.46}$$

$$0 = -h_t^D + \left[\frac{\phi + c_{h,t} + c_{h,t}^*}{A_t^\alpha \exp[0.5\alpha(1+\alpha)\sigma^2]}\right]^{1/\alpha} \tag{D.47}$$

$$0 = -h_t^{*S} + \left[\frac{\phi^* + c_{f,t} + c_{f,t}^*}{A_t^{*\alpha} \exp[0.5\alpha(1+\alpha)\sigma^2]}\right]^{1/\alpha} \tag{D.48}$$

$$0 = -U_{x,t} + (x_t - \delta_t)^{-\gamma_x} \tag{D.49}$$

$$0 = -U_{x,t}^* + (x_t^* - \delta_t^*)^{-\gamma_x} \tag{D.50}$$

$$0 = -y_t + \exp[0.5\alpha(1+\alpha)\sigma^2](A_t h_t)^\alpha - \phi \tag{D.51}$$

$$0 = -y_t^* + \exp[0.5\alpha(1+\alpha)\sigma^2](A_t^* h_t^*)^\alpha - \phi^* \tag{D.52}$$

$$0 = -1 + \mathbb{E}_t\left[\beta \frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_t} \frac{R_t}{\pi_{t+1}}\right] \tag{D.53}$$

$$0 = -1 + \mathbb{E}_t\left[\beta \frac{U_{x,t+1}^*/\tilde{p}_{t+1}^*}{U_{x,t}^*/\tilde{p}_t^*} \frac{R_t^*}{\pi_{t+1}^*}\right] \tag{D.54}$$

$$0 = -R_t + R^{1-\rho_R}\left[R_{t-1}\left(\frac{y_t}{y}\right)^{\rho_c}\left(\frac{\pi_t}{\bar{\pi}}\right)^{\rho_\pi}\right]^{\rho_R} \tag{D.55}$$

$$0 = -R_t^* + R^{*1-\rho_R}\left[R_{t-1}^*\left(\frac{y_t^*}{y^*}\right)^{\rho_c}\left(\frac{\pi_t^*}{\bar{\pi}^*}\right)^{\rho_\pi}\right]^{\rho_R} \tag{D.56}$$

$$0 = -c_t + p_{h,t} c_{h,t} + p_{f,t} c_{f,t} \tag{D.57}$$

$$0 = -c_t^* + p_{h,t}^* c_{h,t}^* + p_{f,t}^* c_{f,t}^* \tag{D.58}$$

$$0 = -\pi_t + \left[\sum_{k=h,f} \omega_k (p_{k,t-1}\pi_{k,t})^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \tag{D.59}$$

$$0 = -\pi_t^* + \left[\sum_{k=h,f} \omega_k (p_{k,t-1}^* \pi_{k,t}^*)^{1-\varepsilon} \right]^{1/(1-\varepsilon)} \tag{D.60}$$

$$0 = -x_t + \left[\sum_{k=h,f} \omega_k (c_{k,t}^{1-\theta})^{1-1/\varepsilon} \right]^{1/(1-1/\varepsilon)} \tag{D.61}$$

$$0 = -x^* + \left[\sum_{k=h,f} \omega_k (c_{k,t}^{*1-\theta})^{1-1/\varepsilon} \right]^{1/(1-1/\varepsilon)} \tag{D.62}$$

$$0 = -s_{h,t} + \rho s_{h,t-1} + (1-\rho) c_{h,t} \tag{D.63}$$

$$0 = -s_{f,t} + \rho s_{f,t-1} + (1-\rho) c_{f,t} \tag{D.64}$$

$$0 = -s_{h,t}^* + \rho s_{h,t-1}^* + (1-\rho) c_{h,t}^* \tag{D.65}$$

$$0 = -s_{f,t}^* + \rho s_{f,t-1}^* + (1-\rho) c_{f,t}^* \tag{D.66}$$

D.1 Complete Risk Sharing With Floating Exchange Rate

Under the complete risk sharing arrangement, the real exchange rate is determined by the risk-sharing condition:

$$0 = -q_t + \kappa \frac{U_{x,t}^* / \tilde{p}_t^*}{U_{x,t} / \tilde{p}_t} \tag{D.67}$$

D.2 Incomplete Risk Sharing With Floating Exchange Rate

Under the incomplete risk sharing arrangement, the risk sharing condition (D.71), and the FOCs for risk-free bonds (D.57) and (D.58) should be replaced by the following equations:

$$0 = -(1 + \tau b_{h,t+1}) + \beta \mathbb{E}_t \left[\frac{U_{x,t+1} / \tilde{p}_{t+1}}{U_{x,t} / \tilde{p}_t} \frac{R_t}{\pi_{t+1}} \right] \tag{D.68}$$

$$0 = -(1 + \tau b_{f,t+1}) + \beta \mathbb{E}_t \left[\frac{U_{x,t+1} / \tilde{p}_{t+1}}{U_{x,t} / \tilde{p}_t} \frac{R_t^*}{\pi_{t+1}^*} \frac{q_{t+1}}{q_t} \right] \tag{D.69}$$

$$0 = -(1 + \tau b_{h,t+1}^*) + \beta \mathbb{E}_t \left[\frac{U_{x,t+1}^* / \tilde{p}_{t+1}^*}{U_{x,t}^* / \tilde{p}_t^*} \frac{R_t}{\pi_{t+1}} \frac{q_t}{q_{t+1}} \right] \tag{D.70}$$

$$0 = -(1 + \tau b_{f,t+1}^*) + \beta \mathbb{E}_t \left[\frac{U_{x,t+1}^* / \tilde{p}_{t+1}^*}{U_{x,t}^* / \tilde{p}_t^*} \frac{R_t^*}{\pi_{t+1}^*} \right] \tag{D.71}$$

$$0 = b_{h,t+1} + b_{h,t+1}^* \tag{D.72}$$

$$0 = b_{f,t+1} + b_{f,t+1}^* \tag{D.73}$$

$$0 = -(b_{h,t+1} + q_t b_{f,t+1}) + \frac{R_{t-1}}{\pi_t} b_{h,t} + \frac{R^*_{t-1}}{\pi^*_t} q_t b_{f,t} + \frac{1}{2}(w_t h_t - q_t w^*_t h^*_t) + \frac{1}{2}(\tilde{d}_t - q_t \tilde{d}^*_t) - \frac{1}{2}(\tilde{p}_t x_t - q_t \tilde{p}^*_t x^*_t) \tag{D.74}$$

where

$$0 = -\tilde{d}_t + \tilde{d}^+_t + (1 - \varphi_t)\tilde{d}^-_t, \tag{D.75}$$

$$0 = -\tilde{d}^*_t + \tilde{d}^{*+}_t + (1 - \varphi^*_t)\tilde{d}^{*-}_t, \tag{D.76}$$

$$\begin{aligned} 0 = -\tilde{d}^+_t + \Phi(z^E_t)\Bigg[& p_{h,t}c_{h,t} + q_t p^*_{h,t} c^*_{h,t} - \frac{w_t}{A_t}\frac{\Phi(z^E_t - \sigma)}{\Phi(z^E_t)}(\phi + c_{h,t} + c^*_{h,t})^{1/\alpha} \\ & -\frac{\gamma}{2}(\pi_{h,t} - \bar{\pi})^2 c_t - \frac{\gamma^*}{2} q_t(\pi^*_{h,t} - \bar{\pi}^*)^2 c^*_t\Bigg], \end{aligned} \tag{D.77}$$

$$\begin{aligned} 0 = -\tilde{d}^-_t + \frac{1 - \Phi(z^E_t)}{1 - \varphi_t}\Bigg[& p_{h,t}c_{h,t} + q_t p^*_{h,t} c^*_{h,t} - \frac{w_t}{A_t}\frac{1 - \Phi(z^E_t - \sigma)}{1 - \Phi(z^E_t)}(\phi + c_{h,t} + c^*_{h,t})^{1/\alpha} \\ & -\frac{\gamma}{2}(\pi_{h,t} - \bar{\pi})^2 c_t - \frac{\gamma^*}{2} q_t(\pi^*_{h,t} - \bar{\pi}^*)^2 c^*_t\Bigg], \end{aligned} \tag{D.78}$$

$$\begin{aligned} 0 = -\tilde{d}^{*+}_t + \Phi(z^{*E}_t)\Bigg[& q_t^{-1} p_{f,t}c_{f,t} + p^*_{f,t} c^*_{f,t} - \frac{w^*_t}{A^*_t}\frac{\Phi(z^{*E}_t - \sigma)}{\Phi(z^{*E}_t)}(\phi^* + c_{f,t} + c^*_{f,t})^{1/\alpha} \\ & -\frac{\gamma}{2} q_t^{-1}(\pi_{f,t} - \bar{\pi})^2 c_t - \frac{\gamma^*}{2}(\pi^*_{f,t} - \bar{\pi}^*)^2 c^*_t\Bigg], \end{aligned} \tag{D.79}$$

and

$$\begin{aligned} 0 = -\tilde{d}^{*-}_t + \frac{1 - \Phi(z^{*E}_t)}{1 - \varphi^*_t}\Bigg[& q_t^{-1} p_{f,t}c_{f,t} + p^*_{f,t} c^*_{f,t} - \frac{w^*_t}{A^*_t}\frac{1 - \Phi(z^{*E}_t - \sigma)}{1 - \Phi(z^{*E}_t)}(\phi^* + c_{f,t} + c^*_{f,t})^{1/\alpha} \\ & -\frac{\gamma}{2} q_t^{-1}(\pi_{f,t} - \bar{\pi})^2 c_t - \frac{\gamma^*}{2}(\pi^*_{f,t} - \bar{\pi}^*)^2 c^*_t\Bigg]. \end{aligned} \tag{D.80}$$

D.3 Incomplete Risk Sharing With Monetary Union

Under the incomplete risk sharing with monetary union, (D.68)∼(D.71) should be replaced with

$$0 = -(1 + \tau b_{h,t+1}) + \beta \mathbb{E}_t\left[\frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t+1}/\tilde{p}_{t+1}}\frac{R^U_t}{\pi_{t+1}}\right] \tag{D.81}$$

$$0 = -(1 + \tau b^*_{h,t+1}) + \beta \mathbb{E}_t\left[\frac{U^*_{x,t+1}/\tilde{p}^*_{t+1}}{U^*_{x,t+1}/\tilde{p}^*_{t+1}}\frac{R^U_t}{\pi^*_{t+1}}\right] \tag{D.82}$$

and the bond market clearing condition $0 = b_{f,t+1} + b^*_{f,t+1}$ is deleted. The following identity is added to the system:

$$\frac{\mathbb{E}_t[q_{t+1}]}{q_t} = \frac{\mathbb{E}_t[S_{t+1}]}{S_t} \cdot \frac{\mathbb{E}_t[\pi^*_{t+1}]}{\mathbb{E}_t[\pi_{t+1}]}. \tag{D.83}$$

Note that S_t is not a model variable as the level of nominal exchange rate cannot be determined in the steady state. However, $\pi^S_{t+1} \equiv S_{t+1}/S_t$ is well-defined as a model variable.

D.4 Exogenous Variables

There are 6 exogenous variables:

$$0 = -\log A_t + \rho_A \log A_{t-1} + \epsilon_{A,t} \tag{D.84}$$

$$0 = -\log A_t^* + \rho_A \log A_{t-1}^* + \epsilon_{A,t}^* \tag{D.85}$$

$$0 = -\delta_t + \rho_\delta \delta_{t-1} + \epsilon_{\delta,t} \tag{D.86}$$

$$0 = -\delta_t^* + \rho_\delta \delta_{t-1}^* + \epsilon_{\delta,t}^* \tag{D.87}$$

$$0 = -\log \varphi_t + (1-\rho_\varphi) \log \bar{\varphi} + \rho_\varphi \log \varphi_{t-1} + \epsilon_{\varphi,t} \tag{D.88}$$

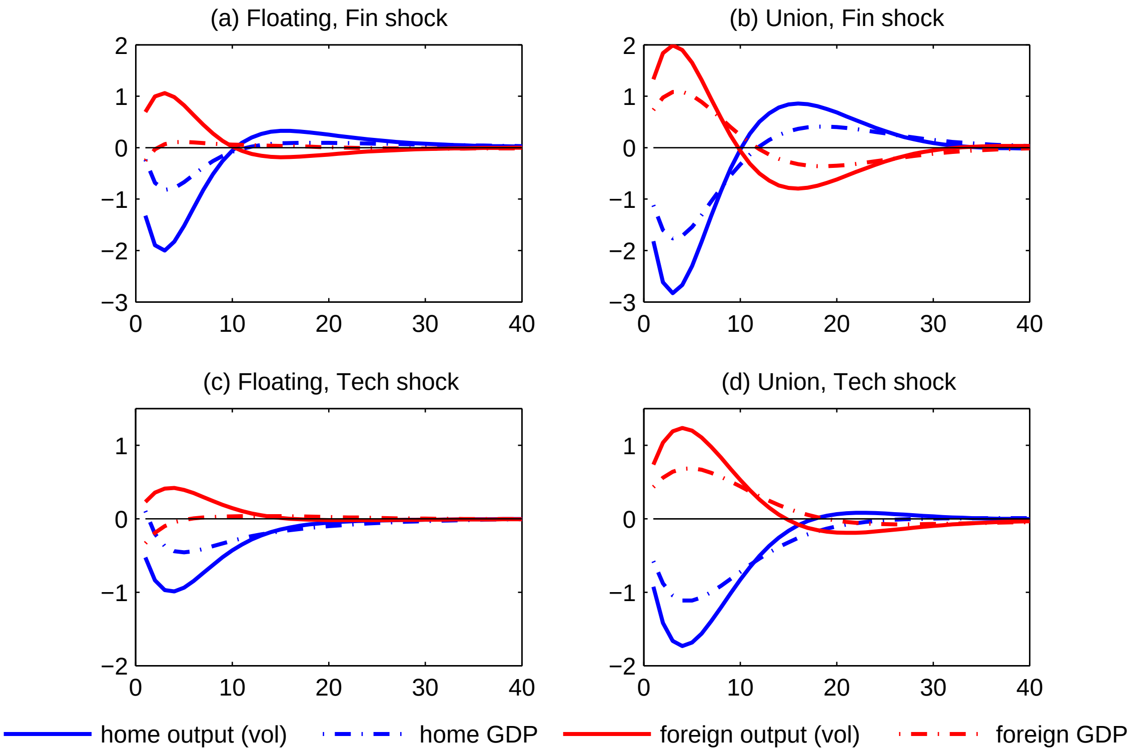
$$0 = -\log \varphi_t^* + (1-\rho_\varphi) \log \bar{\varphi}^* + \rho_\varphi \log \varphi_{t-1}^* + \epsilon_{\varphi,t}^* \tag{D.89}$$

E Volume and Value of GDP

Figure 16 shows the responses of volume index of output and the value of aggregate output (GDP) of the two countries in response to the financial and technology shocks under the two currency arrangements. It is evident that the volume index tends to react to shock more than the value of products, and this applies to the both currency regimes. The reason for the relative stability of the value of products as compared with that of volume can be found the changes in the relative prices. GDP in the steady state can be expressed as $c+x-im = p_h c_h + p_f c_f + q p_h^* c_h^* - p_f c_f = p_h c_h + q p_h^* c_h^*$ whereas the volume index is given by $y = c_h + c_h^*$.

As we have shown in the main text, regardless of the direction in the real exchange rate, the relative prices in both market p_h and p_h^* go up substantially when c_h and c_h^* go down for home country. Exactly opposite is true for foreign country. Such offsetting movements in quantity and prices are responsible for the relative stability of value index such as GDP. This tendency is even more stronger under the floating exchange rate regime as q moves in the same direction as p_h and p_h^*.

Figure 16: Impacts of Shocks on Output Volume and Value (GDP)



Note: Blue, solid line is the peripheral country and red, dash-dotted line is the central country. The shock assumes that the dilution cost for the peripheral countries go up by 100