

Destabilizing carry trades*

Guillaume Plantin
Toulouse School of Economics and CEPR

Hyun Song Shin
Bank for International Settlements

June 2014

Abstract

We offer a model of currency carry trades in which carry traders earn positive excess returns if they successfully coordinate on supplying excessive capital to a target economy. The interest-rate differential between their funding currency and the target currency is their coordination device. We solve for a unique equilibrium that exhibits the classic pattern of the carry-trade recipient currency appreciating for extended periods, punctuated by sharp falls.

*The views expressed here are those of the authors and not necessarily those of the Bank for International Settlements. This paper supersedes our earlier work entitled “Carry trades, monetary policy, and speculative dynamics”. We thank Patrick Bolton, Nicolas Coeurdacier, Doug Diamond, Itay Goldstein, Urban Jermann, Pete Kyle, Karen Lewis, Ady Pauzner, Lasse Pedersen, Richard Portes, Helene Rey, and Jeremy Stein for their comments on this and earlier drafts. Plantin benefitted from a European Research Council Starting Grant (N°263673 – RIFIFI).

Introduction

A currency carry trade consists in selling a low interest rate currency to fund the purchase of a high interest rate currency, or in selling forward a currency that is at a significant forward premium. The high Sharpe ratio generated by carry-trade strategies is one of the most enduring puzzles in international finance.

Carry trades have been at the forefront of the recent debate on global financial stability. The highly accommodative U.S. monetary policy that followed the 2008 financial crisis has triggered large dollar-funded credit flows towards currencies associated with a tighter monetary stance. These flows have been accused of unduly destabilizing exchange rates and local asset markets. This has led several observers to question the continuing relevance of the Mundell trilemma. Their view is that it has become unclear whether flexible exchange rates still allow independent monetary policies in the presence of international capital mobility. The argument is that there exists an important global component in local credit cycles and asset prices. This common component is highly correlated with funding conditions in U.S. dollars, and corresponds to credit flows that may be misaligned with local macroeconomic conditions, and with the objectives of the local monetary authority (Agrippino and Rey, 2014; Bruno and Shin, 2013; Rey, 2013).

This paper offers a theory that relates excess returns on carry trades to these destabilizing consequences of international capital flows. We write down a model in which carry traders may earn positive excess returns (rents) if they successfully coordinate on exploiting asynchronous monetary policies. In our setup, international investors enter into carry trades by borrowing in the world currency and investing the proceeds in assets denominated in the currency of a small open economy. Our theory rests on two central ingredients.

First, we posit that the prices of the nontradable goods in this domestic economy are much stickier than that of the tradable goods. This is consistent with evidence documented by Burstein, Eichenbaum, and Rebelo (2005).¹

Second, the domestic monetary authority anchors domestic inflation expectations by committing to a textbook interest rule that responds to realized CPI inflation. In particular, the monetary authority responds to carry-trade inflows only insofar as they affect domestic inflation. It ignores the direct effect of these inflows on domestic capital markets.

¹They argue that the slow adjustment of the prices of nontradables explains why the large devaluations that they study are associated with little inflation, and with a large decline in the real exchange rate.

Absent the first ingredient—that is, with flexible prices, this economy would feature a unique equilibrium in which carry traders invest until the rate of return on domestic assets is equal to the world interest rate. Uncovered interest parity would hold and the equilibrium return on the carry trade would be zero. This is an elementary example of stabilizing speculation whereby, as famously argued by Friedman (1953), arbitrageurs lean against mispricings until they eliminate them. If larger carry-trade inflows exert downward pressure on domestic asset returns, and thus reduce the expected return on the carry trade, then the equilibrium carry-trade activity is uniquely determined as that at which carry traders are indifferent between investing or not.

The picture is different in the presence of our two key ingredients. First, by ignoring that carry-trade inflows bid up asset prices and thus reduce the domestic real rate, the central bank acts as if it was inadvertently introducing positive policy shocks to the interest rule in response to these inflows. This implies that carry-trade inflows result in a realized inflation that is below target.

Second, given our assumption that the prices of nontradables do not adjust much, this deflationary impact of carry-trade inflows must operate through the prices of tradables. If, as documented by Burstein, Eichenbaum, and Rebelo (2005), the fraction of pure tradables in consumption services is small, then small deflationary shocks translate into large swings in tradables prices, and thus into a large appreciation of the nominal exchange rate.

Overall, the anticipation of future carry-trade activity raises the current expected return on the carry trade through an anticipated appreciation of the nominal exchange rate. If nontradables prices are sufficiently sticky and the official rate sufficiently stable, this positive effect of future inflows on the current return on the carry trade is larger than the negative effect of current inflows on domestic asset returns. In this case, there are multiple self-justified equilibria. There exists an equilibrium in which the anticipation of excessive future capital inflows fuels excessive current inflows. In this equilibrium, the domestic currency keeps appreciating, and the carry trade generates a positive excess return. There also exists a symmetric equilibrium associated with insufficient foreign lending, a negative return on the carry trade, and a depreciating currency.

In a version of the model with exogenous shocks on the interest-rate differential, the equilibrium is unique and the interest-rate differential acts as a coordination device among carry traders. Positive shocks on the interest-rate differential set off dynamics in which capital inflows increase, and the domestic currency keeps appreciating. This generates a prolonged series of

positive returns on the carry trade, that ends abruptly only after a sufficiently long series of negative shocks on the interest differential leads carry traders to coordinate on large and rapid capital outflows. These dynamics are reminiscent of the large and prolonged reactions of exchange rates to monetary shocks described in Eichenbaum and Evans (1995) in the context of the U.S. dollar.

The qualitative properties of these equilibrium paths relate to several well-documented empirical facts, such as the profitability of FX momentum and carry-trade strategies. Further, since equilibrium paths feature rare but dramatic fluctuations in carry-trade activity, finite sample paths generated by the model would likely generate a peso problem, and lead to an overestimation of the expected return on the carry trade. Our model also generates a new set of yet untested predictions on the relationship between the stance of monetary policy and the patterns of carry-trade returns.

Related Literature

Our theory of carry-trade returns as self-fulfilling genuine excess returns bears little relationship to the existing theories that seek to explain the return on carry trades as a compensation for (possibly mismeasured) risk. Farhi and Gabaix (2013) thoroughly survey this existing literature. We do not deny that a significant fraction of carry-trade returns may reflect risk premia. We abstract from risk considerations here for tractability only, and view our theory as a complement to such considerations rather than a competing alternative.

Our approach is more closely related to models of financial instability in which speculators earn rents if they successfully coordinate on a collective course of action that triggers a policy response that benefits them. In international economics, static models of self-fulfilling currency attacks pioneered by Obstfeld (1996) have this flavor. Farhi and Tirole (2012) or Schneider and Tornell (2004) offer models of "collective moral hazard" in which the government bails out speculators if their aggregate losses are sufficiently large, which creates a coordination motive among speculators. In this paper, we invoke related arguments in order to rationalize carry-trades returns. We contribute to this literature on coordination-driven financial instability in two ways.

First, our paper is the first, to our knowledge, in which speculators seek to game an interest-rate rule that is directly borrowed from New-Keynesian textbooks. We believe that this is a fruitful way to contribute to the important debate on the relationship between inflation targeting and financial

stability using a well-known framework.

Second, we formalize the dynamic coordination game among carry traders using the tools developed by Frankel and Pauzner (2000) and Burdzy, Frankel, and Pauzner (2001) in order to obtain a unique predictable outcome. We show that their setup can be adapted to the situation in which strategic complementarities among agents coexist with congestion effects. This is important because most financial models with strategic complementarities also feature congestion effects. We also show that the equilibrium paths resulting from this model square well, at least qualitatively, with many empirical patterns of carry-trade returns.

1 A perfect-foresight model of destabilizing carry trades

Time is discrete and is indexed by $t \in \mathbb{Z}$. There are two types of agents, international investors deemed "carry traders", and households populating a small open economy. There is a single tradable good that has a fixed unit price in the world currency. Carry traders consume only this tradable good.

Households

The households live in a small open economy. They use a domestic currency that trades at S_t units of the world currency per unit at date t . At each date, a unit mass of households are born. Households live for two dates, consume when young and old, and work when old. The cohort that is born at date t has quasi-linear preferences over bundles of consumption and labor (C_t, C_{t+1}, N_{t+1})

$$\ln C_t + \frac{C_{t+1} - N_{t+1}^{1+\eta}}{R}, \quad (1)$$

where $\eta, R > 0$. Domestic consumption services C_t are produced combining the tradable good C_t^T and two nontradable goods $C_t^{N_1}$ and $C_t^{N_2}$ according to the technology

$$C_t = \frac{(C_t^T)^\alpha (C_t^{N_1})^\beta (C_t^{N_2})^\gamma}{\alpha^\alpha \beta^\beta \gamma^\gamma}, \quad (2)$$

where $\alpha, \beta, \gamma \in (0, 1)$ and $\alpha + \beta + \gamma = 1$. Domestic firms set by old households use labor input to produce. The exact specification of the production processes are immaterial for our analysis. All that is needed is that both nontradable goods are produced in finite, non zero quantities at each date.

Households' endowment is their labor income, and the profits from their firms, that they both collect when old.

The first important ingredient of the model is that the prices of nontradable goods are less flexible than that of the tradable good. We introduce rigidity in the pricing of nontradable goods in the straightforward following way. The first nontradable good has a fully flexible price. A technology enables the transformation of each date- t unit of this first nontradable good into F units of the tradable good, where $F > 0$. The second nontradable good has a fully rigid price that we normalize to 1 without loss of generality. Thus, the parameter γ measures the degree of price rigidity in the domestic economy. The case $\gamma = 0$ corresponds to the fully flexible benchmark.

Carry traders

The unit mass of carry traders have access to investments denominated both in the world and in the domestic currency. The gross per period return on those denominated in the world currency is $Re^{-\delta}$.

The financing of households by carry traders

Households and carry traders trade risk-free one-period bonds in zero net supply that are denominated in the domestic currency. The nominal interest rate is set by the domestic central bank according to a rule described below.

Each carry trader can take any position in the bond market within $[P_t e^{\underline{l}}, P_t e^{\bar{l}}]$, where these limits are denominated in the domestic currency, P_t is the domestic consumption price level, and

$$\underline{l} < 0 < \bar{l}.$$

We denote L_t the aggregate real borrowing by young households at date t .

A natural interpretation of our model is that the carry traders are domestic banks that finance loans to the domestic economy by investing their own equity $P_t e^{\underline{l}}$, and if they wish to do so by borrowing in the world currency from the global banking sector. The domestic prudential regulation imposes a capital requirement $P_t e^{\underline{l} - \bar{l}}$ - a minimum fraction of their assets that banks must finance with their own capital. Thus banks can lend at most $P_t e^{\bar{l}}$. This interpretation of carry traders as local banks funded in foreign currencies by the global banking sector squares well with recent analyses of the role of international credit flows in destabilizing carry trades (Bruno and Shin, 2013; Rey, 2013).

Alternatively, one may also simply interpret $P_t e^l$ as an exogenous endowment received by households at birth, and $\left[0, P_t (e^{\bar{l}} - e^l)\right]$ as the trading limits of the carry traders.²

Perfect-foresight equilibria

We are interested in determining the excess return that carry traders earn from lending to the households rather than investing in world-currency denominated assets in the perfect foresight equilibria of this economy. We simply deem it the "return on carry trades". This return between two dates t and $t + 1$ is

$$\Theta_{t+1} = \frac{S_{t+1} I_{t+1}}{R e^{-\delta} S_t}, \quad (3)$$

where I_{t+1} is the domestic nominal interest rate. A perfect-foresight equilibrium must be such that carry traders exhaust their lending limit when the return on carry trades (3) is strictly larger than 1, lend the minimum amount if it is strictly smaller than 1, and are indifferent otherwise.³ The rest of the equilibrium is determined by four standard conditions. First, we impose that households optimally allocate their resources across dates, which yields a Fisher relation:

$$I_{t+1} = \frac{R P_{t+1}}{L_t P_t}, \quad (4)$$

The Euler equation (4) depends only on date- t real borrowing L_t because preferences are quasi-linear. Second, we also impose that households optimally spend across goods at each date, which implies:

$$P_t = (P_t^T)^\alpha (P_t^{N_1})^\beta (P_t^{N_2})^\gamma, \quad (5)$$

or

$$P_t = (P_t^T)^{1-\gamma} F^\beta, \quad (6)$$

where P_t^T is the price level of the tradable good. Relation (6) stems from our assumptions about the price levels of the nontradable goods:

²Setting lending limits in real terms simplifies the exposition but is not crucial. Nominal rigidities in trading limits would actually amplify our results.

³In this perfect-foresight model, returns on carry trades different from 1 are pure arbitrage opportunities. This is the reason the preferences of the carry traders are irrelevant, and also the reason we must impose exogenous limits on carry-trade sizes.

$$P_t^{N_2} = 1, \tag{7}$$

and

$$P_t^{N_1} = F P_t^T. \tag{8}$$

Third, we assume that purchasing power parity holds for the pure tradable good ("PPP at the dock"):

$$P_t^T S_t = 1. \tag{9}$$

Finally, we assume that the domestic monetary authority follows an interest-rate rule of the form:

$$I_{t+1} = R \left(\frac{P_t}{P_{t-1}} \right)^{1+\Phi} \tag{10}$$

where

$$\Phi > 0. \tag{11}$$

Rule (10) is a textbook interest-rate rule that follows the Taylor principle from (11).⁴ It responds to carry-trade inflows only insofar as these flows affect domestic inflation. This is the other important ingredient of the model together with rigidity in nontradables prices. The interpretation is that the central bank does not respond to the asset price fluctuations induced by flows of "hot money."

We introduce

$$\begin{aligned} r &= \ln R, \\ \theta_t &= \ln \Theta_t, \\ i_t &= \ln I_t, \\ s_t &= \ln S_t, \\ l_t &= \ln L_t, \\ \pi_{t+1} &= \ln \left(\frac{P_{t+1}}{P_t} \right). \end{aligned}$$

⁴Rule (10) sets a zero-inflation target for notational simplicity only, this plays no role in the analysis.

As is standard, the Fisher relation (4) and the interest-rate rule (10) define a linear-difference system for the path of inflation that has a unique non-exploding solution:

$$\pi_t = - \sum_{k \geq 0} \frac{l_{t+k}}{(1 + \Phi)^{k+1}}. \quad (12)$$

The current price level reflects all future expected "shocks" caused by carry trades on the real rate $(l_{t+k})_{k \geq 0}$. Using (6) and (9), one has

$$s_{t+1} - s_t = - \frac{1}{1 - \gamma} \pi_{t+1}. \quad (13)$$

Plugging (13) and (4) in (3) yields

$$\begin{aligned} \theta_{t+1} &= s_{t+1} - s_t + i_{t+1} - r + \delta, \\ &= - \frac{1}{1 - \gamma} \pi_{t+1} + \pi_{t+1} - l_t + \delta, \end{aligned} \quad (14)$$

$$= \frac{\gamma}{1 - \gamma} \sum_{k \geq 0} \frac{l_{t+k+1}}{(1 + \Phi)^{k+1}} - l_t + \delta. \quad (15)$$

Suppose for simplicity that $\delta = 0$. We now determine the steady-states in which the debt level l is constant over time. We have:

Lemma 1. *There exists a steady-state $l = 0$ in which the domestic real rate is R , the nominal exchange rate and the price level are constant, and the carry trade earns no excess return ($\theta = 0$).*

If $\Phi(1 - \gamma) > \gamma$, this is the only steady-state.

If $\Phi(1 - \gamma) < \gamma$, there also exists a steady-state with maximum lending ($l = \bar{l}$) in which the excess return on the carry trade is positive, and the nominal exchange rate constantly appreciates. There also exists a steady-state with minimum lending ($l = \underline{l}$), a negative excess return on the carry trade, and a constant depreciation of the exchange rate.

Proof. For $\delta = 0$ and a fixed l , expression (15) becomes

$$\theta = \frac{\gamma - \Phi(1 - \gamma)}{(1 - \gamma)\Phi} l. \quad (16)$$

Thus the steady-state $l = 0$ is unique if $\Phi(1 - \gamma) > \gamma$, it is not otherwise. ■

The situation $\Phi(1 - \gamma) < \gamma$ in which there are multiple steady-states may be interpreted as one that is prone to destabilizing speculation. In this situation, the stable steady-state in which carry traders earn the same return

on all investments, and in which domestic nominal variables are constant is only one possibility. Because current and future capital inflows reinforce each other, there is also the possibility that carry traders create and exploit a self-justified arbitrage opportunity. They may also enter into the self-defeating strategy of lending too little. Notice that the only steady-state is the one in which $l = 0$ regardless of the monetary rule when prices are fully flexible ($\gamma = 0$).

The intuition behind Lemma 1 is best seen from the return on carry trade as given in equation (14). This expression decomposes the impact of current lending l_t and that of future lending $(l_{t+k})_{k \geq 1}$ on the current excess return on carry trade θ_{t+1} . Current lending has a negative impact on the current return on carry trade simply because it lowers the real interest rate (term $-l_t$ in (14)). In contrast, the return on carry trade increases in future lending for the following reason. First, future anticipated carry trades are deflationary, very much like future positive policy shocks would be. This deflationary effect reduces the nominal domestic exchange rate and thus the profitability of the current carry trade (term π_{t+1} decreasing in future lending). This is more than offset, however, by the impact of future lending on nominal exchange rate appreciation (term $\frac{-\pi_{t+1}}{1-\gamma}$ increasing in future lending). Future lending leads to a current nominal exchange rate appreciation that is larger than the reduction in the CPI because of the assumption that nontradable prices are less flexible than that of the tradable good. Thus, the nominal exchange rate is more sensitive to shocks on the real rate than the CPI.

In sum, there are multiple steady-states when lending by other carry traders makes lending more appealing to each carry trader. This occurs when exchange rate appreciation due to anticipated future lending more than offsets the negative impact of current lending on the real rate. This is in turn the case when the nominal exchange rate is sensitive to future capital inflows because nontradables prices are rigid (γ large), and the official rate does not respond too aggressively to realized inflation (Φ small).

This multiplicity of steady-states sheds light on the circumstances under which carry trades reinforce each other and are destabilizing in a simple perfect-foresight environment. Yet this environment is admittedly problematic along several dimensions:

1. The multiplicity of steady-states leaves unclear how agents can coordinate on any equilibrium behavior at all.
2. If carry traders hold the same position forever, then domestic prices and the real-rate target of the central bank should eventually adjust.

3. Perhaps more important, in this environment, the interest-rate differential plays no role in setting off steady-states with excessive or insufficient capital inflows. To see this, notice that the total return on the carry trade is:

$$\underbrace{-\frac{1}{1-\gamma}\pi_{t+1}}_{\text{exchange-rate appreciation}} + \underbrace{\pi_{t+1} - l_t + \delta}_{\text{interest-rate differential}} . \quad (17)$$

Thus, either steady-state can be sustained regardless of the value of the interest-rate differential provided γ is sufficiently close to 1, so that the sign of (17) is entirely driven by that of the exchange rate fluctuation regardless of the value of the interest-rate differential.

The next section develops a model that addresses these three issues. Loosely speaking for now, we will write down a version of the model in which carry traders switch from maximum to minimum lending at points that are uniquely determined by the paths of a stochastic interest rate differential.

2 Destabilizing carry trades

We now assume that time is continuous. The fixed integer dates of the previous section are replaced by the arrival times of a Poisson process with intensity 1. Namely, at each arrival time T_n , a new cohort of households are born, and die at the next arrival time T_{n+1} . They value consumption and labor only at these two dates, with preferences that are the same as that in the previous section:⁵

$$\ln C_{T_n} + \frac{1}{R} E_{T_n} \left[C_{T_{n+1}} - N_{T_{n+1}}^{1+\eta} \right].$$

At each arrival date T_n , the central bank sets a nominal rate $I_{T_{n+1}}$ between T_n and T_{n+1} according to the rule:

$$I_{T_{n+1}} = R \left(\frac{P_{T_n}}{P_{T_{n-1}}} \right)^{1+\Phi} . \quad (18)$$

Carry traders also value consumption at these arrival dates $(T_n)_{n \in \mathbb{Z}}$ only. They are risk-neutral, and have the same discount rate R as that of the

⁵Replacing integer dates with dates that arrive at a constant rate is only for tractability. It entails that the carry traders' problem studied below is time homogeneous. That agents do not care about the length of the time interval during two arrival dates slightly simplifies the exposition.

households between two arrival dates. They are penniless but have the ability to borrow in the world currency against bonds denominated in the domestic currency.

We depart from the perfect foresight model developed in the previous section in two important ways.

First, we assume that the interest rate at which carry traders borrow in the world currency between two arrival dates T_n and T_{n+1} is given by

$$R(1 - w_{T_n}), \quad (19)$$

where w_t is a Wiener process with no drift and volatility σ^2 . In other words, we introduce an exogenous stochastic component in the interest rate differential.⁶

Second, we assume that the capital supplied by carry traders is slow-moving in the following sense. Each carry trader can revise his lending policy only at switching dates that are generated by a Poisson process with intensity λ . These switching dates are independent across carry traders. In between two switching dates, each carry trader commits to lend a fixed real amount within $[e^l, e^{\bar{l}}]$ to each new cohort of households. We deem "active" a carry trader who committed to maximum lending $e^{\bar{l}}$ at his last switching date, and "inactive" one who committed to the minimum lending e^l .⁷

This model of slow-moving capital has two key properties that will yield equilibrium uniqueness. First, the aggregate supply of foreign capital obeys a continuous process. Second, every carry trader knows that some other carry traders will revise their lending strategy almost surely between his current switching date and the next one.

Suppose that a carry trader has a chance to revise his position at a date t such that

$$T_{n-1} < t < T_n. \quad (20)$$

Denoting T_λ his next switching date, his expected unit return from the carry trade—the expected value from committing to lend one additional real unit to each future cohort until T_λ —is

⁶Clearly, the world interest rate (19) is determined outside the model. We could alternatively introduce exogenous shocks to the domestic economy such as policy shocks at the cost of some analytical complexity.

⁷Risk-neutral carry traders choose corner strategies. As in the previous section, we could alternatively interpret e^l as the young households' endowment and suppose that carry traders cannot borrow in the domestic currency.

$$\Theta_t = E_t \left[\sum_{m \geq 0} \frac{1_{\{T_\lambda > T_{n+m}\}} P_{T_{n+m}}}{R^{m+1}} (S_{T_{n+m+1}} I_{T_{n+m+1}} - S_{T_{n+m}} R (1 - w_{T_{n+m}})) \right]. \quad (21)$$

Expression (21) states that the carry trader earns the carry-trade return associated with each cohort that borrows until he gets a chance to revise his position.

We let x_t denote the fraction of active carry traders at date t . Note that the paths of the process $(x_t)_{t \in \mathbb{R}}$ must be Lipschitz continuous, with a Lipschitz constant smaller than λ . The aggregate real lending L_{T_n} taking place at an arrival date T_n is then equal to

$$L_{T_n} = x_{T_n} e^{\bar{l}} + (1 - x_{T_n}) e^{\underline{l}}. \quad (22)$$

The evolution of the economy is fully described by two state variables, the exogenous state variable w_t that measures the interest rate differential, and the endogenous state variable x_t . The exogenous state variable affects only the expected return on carry trade Θ_t while the endogenous one affects both the carry trade return and the equilibrium variables $(L_{T_n}, I_{T_n}, P_{T_n}, S_{T_n})$ of the domestic economy. We are now equipped to define an equilibrium.

Definition. *An equilibrium is characterized by a process x_t that is adapted to the filtration of w_t and has Lipschitz-continuous paths such that:*

$$L_{T_n} = x_{T_n} e^{\bar{l}} + (1 - x_{T_n}) e^{\underline{l}}, \quad (23)$$

$$I_{T_{n+1}} = R \left(\frac{P_{T_n}}{P_{T_{n-1}}} \right)^{1+\Phi}, \quad (24)$$

$$P_{T_n} = (P_{T_n}^T)^{1-\gamma} F^\beta, \quad (25)$$

$$P_{T_n}^T S_{T_n} = 1, \quad (26)$$

$$E_{T_n} \left[\frac{I_{T_{n+1}} P_{T_n}}{P_{T_{n+1}}} \right] = \frac{R}{L_{T_n}}, \quad (27)$$

$$\frac{dx_t}{dt} = \begin{cases} -\lambda x_t & \text{if } \Theta_t < 0, \\ \lambda(1 - x_t) & \text{if } \Theta_t > 0. \end{cases} \quad (28)$$

Equations (24) to (27) state that the domestic economy is in equilibrium given the paths of x_t . Equation (28) states that carry traders make optimal

individual decisions. They become active at switching dates at which the expected return on the carry trade is positive (or remain active if this was their previous positions), and inactive if this is negative.

Notice that relations (24) to (26) are identical to their counterparts in the perfect foresight case except for the re-labelling of dates. They are in particular log-linear. Conversely, the Fisher relation (27) now features an expectation over the inverse of inflation given the stochastic environment. We will assume for the remainder of the paper that \underline{l} and \bar{l} are sufficiently close to 0 that we can log-linearize (27) around the steady-state

$$L = \frac{I}{R} = P = S = 1, \quad (29)$$

and write

$$\ln E_t \left[\frac{P_{T_n}}{P_{T_{n+1}}} \right] \simeq -E_t \left[\ln \frac{P_{T_{n+1}}}{P_{T_n}} \right]. \quad (30)$$

This implies of course that we restrict the analysis to the impact of relatively small capital inflows. Up to this log-linearization, we have

Proposition 2. *Suppose that*

$$\gamma > \Phi(1 - \gamma). \quad (31)$$

For λ sufficiently small, there exists a unique equilibrium defined by a decreasing Lipschitz function f such that

$$\frac{dx_t}{dt} = \begin{cases} -\lambda x_t & \text{if } w_t < f(x_t), \\ \lambda(1 - x_t) & \text{if } w_t > f(x_t). \end{cases} \quad (32)$$

Figure 1 illustrates the equilibrium dynamics described in Proposition 2.

[Figure 1 here]

The frontier f divides the (w, x) -space into two regions. Proposition 2 states that in the unique equilibrium, any trader decides to be active when the system is to the right of the frontier f at his switching date, and inactive when it is on the left of the frontier. Thus, lending positions (and therefore the exchange rate) will tend to rise in the right-hand region, and tend to fall in the left-hand region, as indicated by the arrows in Figure 1. The expected return on the carry trade at date t is zero if and only if $w_t = f(x_t)$.

The dynamics of x_t implied by the unique equilibrium are given by:

$$dx_t = \lambda (1_{\{w_t > f(x_t)\}} - x_t) dt, \quad (33)$$

where $1_{\{\cdot\}}$ denotes the indicator function that takes the value 1 when the condition inside the curly brackets is satisfied. These processes are known as *stochastic bifurcations*, and are studied in Bass and Burdzy (1999) and Burdzy et al. (1998). These mathematics papers establish in particular that for almost every sample path of w_t , there exists a unique Lipschitz solution x_t to the differential equation (33) defining the price dynamics for f Lipschitz decreasing.

The main features of these dynamics can be seen from Figure 1. Starting on the frontier, a positive shock on w will pull the system on the right of it. Unless the path of w_t is such that a larger negative shock brings it back on the frontier immediately, a more likely scenario is that lending grows for a while so that x_t becomes close to 1, in which case $\frac{dx_t}{dt}$ becomes close to 0. If cumulative negative shocks on w eventually lead the system back to the left of the frontier, then there are large outflows

$$\frac{dx_t}{dt} \simeq -\lambda.$$

Condition (31) is the same as the one that generates multiple steady-states in the perfect-foresight case. It is worthwhile commenting on the additional condition that capital move sufficiently slowly (λ sufficiently small). This condition guarantees that the frontier f is decreasing, and thus that carry trades are destabilizing. To better grasp its role, notice that if a carry trader expects other carry traders to become active in the future, then he expects the exchange rate to appreciate. This implies that on one hand, the currency will be expensive when he will purchase it to lend. On the other hand, it will keep appreciating over the duration of the loan, thereby generating a positive return. The former effect is akin to a congestion effect. Other traders make the trade more expensive and thus less desirable. Conversely, the latter effect is destabilizing as future carry trades make becoming active more appealing. That λ be sufficiently small ensures that this latter effect offsets the former congestion effect because aggregate lending does not converge too quickly to its maximum value. Thus the carry trader with a current switching date is more likely to have a chance to lend before the currency becomes too expensive, and its upside potential too small. This congestion effect is the salient difference between our setup and that studied by Burdzy, Frankel, and Pauzner.

Proof of Proposition 2

The proof of Proposition 2 essentially extends to this stochastic environment the logic leading to the perfect-foresight results in Lemma 1. In a first step, we solve for the nominal exchange rate and interest rate as a function of future capital inflows. This will yield an expression of the expected return on carry trades (21) as a function of these inflows and of the interest-rate differential w_t that is the stochastic counterpart of equation (15). Second, we use this expression to solve for a Lipschitz process that satisfies (28). This latter step is the equivalent of the one that consisted in solving for feasible steady-states given the expected return for carry traders (15) under perfect foresight.

More precisely, the first step consists in using relations (24) to (27) to express the nominal exchange rate and interest rate as functions of the expected future paths of capital inflows L_t . This yields in turn a relatively simple expression for the expected return on the carry trade Θ_t as a function of these expected capital inflows:

Lemma 3. *At first-order, the expected return on the carry trade is*

$$\Theta(w_t, x_t) = \int_0^{+\infty} \left(\left(\frac{\chi\omega}{\omega - \rho - \lambda} - 1 \right) e^{-(\lambda+\rho)v} - \frac{\chi\omega}{\omega - \rho - \lambda} e^{-\omega v} \right) E_t[l_{t+v}] dv + \frac{w_t}{\lambda + \rho}, \quad (34)$$

where

$$l_t = \ln L_t \simeq x_t \bar{l} + (1 - x_t) \underline{l}, \quad (35)$$

$$\rho = 1 - \frac{1}{R}, \quad (36)$$

$$\omega = \frac{\Phi}{1 + \Phi}, \quad (37)$$

$$\chi = \frac{\gamma}{(1 - \gamma)\Phi}. \quad (38)$$

Proof. See the Appendix. ■

The factor that discounts future capital inflows in (34):

$$\left(\frac{\chi\omega}{\omega - \rho - \lambda} - 1 \right) e^{-(\lambda+\rho)v} - \frac{\chi\omega}{\omega - \rho - \lambda} e^{-\omega v}, \quad (39)$$

is first negative, then positive as v spans $[0, +\infty]$.⁸ This formalizes the above comment that future active traders create congestion effect for the current

⁸Notice that this is so regardless of the sign of $\omega - \lambda - \rho$.

trader. The earliest inflows have a negative impact on Θ because they make the domestic currency expensive. The more remote inflows are desirable as the current trader is more likely to have lent before they push up the exchange rate. The following lemma establishes conditions under which the congestion effect is not too important.

Lemma 4. *Suppose that $\chi > 1$. There exists $\bar{\lambda}$ such that for all $\lambda \leq \bar{\lambda}$, the following is true. Suppose that two processes x_t^1 and x_t^2 satisfy*

$$0 < x_0^1 \leq x_0^2 < 1,$$

$$\text{For } i = 1, 2, dx_t^i = \lambda \left(1_{\{w_t > f^i(x_t^i)\}} - x_t^i \right) dt,$$

where f^i is decreasing Lipschitz and $f^2 \leq f^1$. Then

$$\Theta(w_t, x_0^2) \geq \Theta(w_t, x_0^1). \quad (40)$$

The inequality is strict if $f^1 \neq f^2$ and/or $x_0^1 \neq x_0^2$.

Proof. See the Appendix. ■

Lemma 4 states that if (31) holds and λ is sufficiently small, then future carry trades make current carry trades more attractive because the reinforcing effect overcomes the congestion effect. In the balance of the paper, we suppose that the conditions in Lemma 4 are satisfied. We now show that there is in this case a unique Lipschitz process x_t that satisfies the equilibrium conditions.

First, the proof of Lemma 4 also shows that the case in which x_t obeys $\frac{dx_t}{dt} = -\lambda x_t$ for all $u \geq 0$ corresponds to a lower bound on the expected carry-trade return. When x_t obeys such dynamics, there exists a frontier f_0 such that

$$w_t = f_0(x_t) \implies \Theta(x_t, w_t) = 0 \quad (41)$$

The frontier f_0 is decreasing from Lemma 4 (with $f^1 = f^2 = +\infty$) and is clearly affine and thus Lipschitz.⁹ Thus an admissible equilibrium process must be such that traders who have a chance to switch when the system is on the right of f_0 become active.

Define now f_1 such that

$$w_t = f_1(x_t) \implies \Theta(x_t, w_t) = 0 \quad (42)$$

⁹The frontier simply obtains from writing $E_t[l_{t+v}] = \underline{l} + (\bar{l} - \underline{l}) x_t e^{-\lambda v}$ in (34).

if for all $u \geq 0$,

$$\frac{dx_{t+u}}{du} = \begin{cases} -\lambda x_{t+u} & \text{if } w_{t+u} < f_0(x_{t+u}), \\ \lambda(1 - x_{t+u}) & \text{if } w_{t+u} > f_0(x_{t+u}). \end{cases} \quad (43)$$

That is, f_1 is such that a carry trader is indifferent between being active or inactive when the system is on f_1 at his switching date if he believes that other traders become active if and only if they are on the right of f_0 . This function f_1 must be decreasing. Suppose otherwise that two points (w, x) and (w', x') on f_1 satisfy

$$\begin{aligned} x' &> x, \\ w' &\geq w. \end{aligned}$$

Then applying Lemma 4 with $f^2 = f_0, f^1 = f_0 + w' - w$ contradicts that both points generate the same expected carry trade return. We also show in the appendix that f_1 is Lipschitz, with a Lipschitz constant smaller than that of f_0 .

By iterating this process, we obtain a limit f_∞ of the sequence of frontiers $(f_n)_{n \geq 0}$ that is decreasing Lipschitz as a limit of decreasing Lipschitz functions with decreasing Lipschitz constants. The process

$$\frac{dx_t}{dt} = \begin{cases} -\lambda x_t & \text{if } w_t < f_\infty(x_t), \\ \lambda(1 - x_t) & \text{if } w_t > f_\infty(x_t). \end{cases} \quad (44)$$

is an admissible equilibrium since by construction, if all traders switch to inactivity to the left of f_∞ and to activity to the right, the indifference point for a trader also lies on f_∞ . We now show that this is the only equilibrium process.

Consider a translation to the left of the graph of f_∞ in (w, x) so that the whole of the curve lies in a region where w_t is sufficiently small that inactivity is dominant regardless of the dynamics of x_t . Call this translation f'_0 . To the left of f'_0 , inactivity is dominant. Then construct f'_1 as the rightmost translation of f'_0 such that a trader must choose inactivity to the left of f'_1 if he believes that other traders will play according to f'_0 . By iterating this process, we obtain a sequence of translations to the right of f'_0 . Denote by f'_∞ the limit of the sequence. Refer to Figure 2.

[Figure 2 here]

The boundary f'_∞ does not necessarily define an equilibrium strategy, since it was merely constructed as a translation of f'_0 . However, we know

that if all others were to play according to the boundary f'_∞ , then there is at least one point A on f'_∞ where the trader is indifferent. If there were no such point as A , this would imply that f'_∞ is not the rightmost translation, as required in the definition.

We claim that f'_∞ and f_∞ coincide exactly. The argument is by contradiction. Suppose that we have a gap between f'_∞ and f_∞ . Then, choose point B on f_∞ such that A and B have the same height - i.e. correspond to the same x . But then, since the shape of the boundaries of f'_∞ and f_∞ and the values of x are identical, the paths starting from A must have the same distribution as the paths starting from B up to the constant difference in the initial values of w . This contradicts the hypothesis that a trader is indifferent between the two actions both at A and at B . If he were indifferent at A , he would strictly prefer maximum lending at B , and if he is indifferent at B , he would strictly prefer minimum lending when in A . But we constructed A and B so that traders are indifferent in both A and B . Thus, there is only one way to make everything consistent, namely to conclude that $A = B$. Thus, there is no “gap”, and we must have $f'_\infty = f_\infty$. ■

Proposition 2 shows that adding exogenous shocks w_t to the carry return eliminates the indeterminacy of the perfect-foresight case. More precisely, equilibrium uniqueness stems from the interplay of these shocks with the fact that each carry trader, when he receives a switching opportunity, needs to form beliefs about the decisions of the carry traders that will have an opportunity to switch between now and his next switching date. Suppose that (w_t, x_t) is close to a dominance region in which carry traders would prefer a course of action for sure, but just outside it. If w_t was fixed, it may be possible to construct an equilibrium for both actions, but when w_t moves around stochastically, it will wander into the dominance region between now and the next opportunity that the trader gets to switch with some probability. This gives the trader some reason to hedge his bets and take one course of action for sure. But then, this shifts out the dominance region, and a new round of reasoning takes place given the new boundary, and so on.

Remark 1. We model the interest-rate differential as a Brownian motion for expositional simplicity. It is easy to see that we could write it as $d(w_t)$, where w_t is a standard Brownian motion, and d a Lipschitz increasing function, possibly bounded as long as there are still dominant actions for w_t sufficiently large or small.

Remark 2. While a strong persistence in target rates is undoubtedly realistic, extensions of this framework can also accommodate for various forms of mean-reversion (Burdzy, Frankel, and Pauzner, 2001, or Frankel

and Burdzy, 2005).

Remark 3. The condition that λ be sufficiently small seems particularly relevant for the carry trades that involved many retail investors, such as those targeting New Zealand dollar or Icelandic krona. The glacier bonds denominated in Icelandic krona or the uridashi bonds used by Japanese investors to invest in New Zealand had a typical maturity of 1 to 5 years, and were principally purchased by retail investors. More generally, Bacchetta and van Wincoop (2009) claim an average two-year rebalancing frequency to be plausible in FX markets in general, and assume it in order to quantitatively explain the forward discount bias. Also, well-documented price pressure and illiquidity in currency markets, especially for small currencies, may force FX speculators to build-up or unwind large positions more gradually than they would like to.¹⁰

The case of small shocks

The limiting case in which the volatility σ of the interest-rate differential tends to zero yields useful insights. It is possible to characterize the shape of the frontier f in this case.

In this section we denote the frontier f_σ to emphasize its dependence on σ . Suppose the economy is in the state $(f_\sigma(x_t), x_t)$ at date t . That is, it is on the equilibrium frontier. For some arbitrarily small $\varepsilon > 0$, introduce the stopping times

$$\begin{aligned} T_1 &= \inf_{u \geq 0} \{x_{t+u} \notin (\varepsilon, 1 - \varepsilon)\}, \\ T_0 &= \sup_{0 \leq u < T_1} \{w_{t+u} \neq f_\sigma(x_{t+u})\}. \end{aligned}$$

In words, T_1 is the first date at which x_t gets close to 0 or 1, and T_0 is the last date at which x_t crosses the frontier before T_1 . If T_0 is small in distribution, it means that the economy is prone to bifurcations. That is, it never stays around the frontier for long. Upon hitting it, it quickly heads towards extreme values of x . The next proposition shows that this is actually the most likely scenario when σ is small. This, in turn, yields a simple explicit determination of the frontier.

Proposition 5.

¹⁰In fact, our model is identical to one in which a single large carry trader can move his capital only at the rate λ .

1. As $\sigma \rightarrow 0$, T_0 converges to 0 in distribution, and the probability that $\frac{dx_t}{dt} > 0$ (respectively $\frac{dx_t}{dt} < 0$) over $[T_0, T_1]$ converges to $1 - x_t$ (x_t respectively).
2. As $\sigma \rightarrow 0$, the frontier f_σ tends to an affine function. For λ sufficiently small, the slope of this function is increasing in Φ and decreasing in γ .

Proof. See the Appendix. ■

First, Proposition 5 clears the concern that in equilibrium, x would only exhibit small fluctuations around a fixed value because Brownian paths cross the frontier too often. As σ becomes smaller, the system exhibits more frequent bifurcations towards extremal values of x . When the system reaches the frontier, it is all the more likely to bifurcate towards capital outflows when cumulative inflows have been large (x large). Thus the model does generate "destabilizing carry trades," whereby carry traders generate durable self-justified excess returns on the carry trade followed by large reversals.

The second point in Proposition 5 relates the slope of the frontier f_σ to the monetary parameters of the model Φ and γ in this case of small shocks. The slope of the frontier affects the dynamics of capital inflows and in turn the exchange-rate dynamics. If the graph of the frontier is closer to being horizontal in the (w, x) plane, then the system should cross the frontier less often, and thus do so only for more extreme values of x . Carry-trade returns should in this case exhibit more serial correlation and fatter tails. Point 2 states that, at least for λ sufficiently small, the frontier is flatter when Φ is smaller, and γ larger. In other words, if the central bank fails to respond to inflows by sufficiently reducing its official rate, then carry trade returns should exhibit more skewness.

3 Empirical content

The model generates a rich set of qualitative empirical predictions. This suggests that a coordination motive among carry traders may be a common driving force behind several well-documented and seemingly unrelated empirical findings on carry-trade returns.

Profitability of FX momentum strategies

Proposition 5 shows that as $\sigma \rightarrow 0$, the system often bifurcates in one direction. This implies that, at least at a sufficiently short horizon, returns

are positively autocorrelated, so that momentum strategies in FX markets should generate a positive excess return.

It is important to stress that the profitability of momentum strategies is not a mechanical consequence of the assumption of slowly moving capital (λ sufficiently small). Returns on the carry trade are still positively autocorrelated if the system bifurcates quickly towards extreme values of activity x . The key economic force behind this profitability of momentum strategies is that once carry traders coordinate on a course of action, they stick to it until a sufficiently large reversal of the interest-rate differential leads them to switch to a different strategy. Such a rationalization of momentum returns with coordination motives is novel to our knowledge.

Profitability of FX carry trades

Lemma 4 implies that the equilibrium expected return on the carry trade $\Theta(w, x)$ increases with respect to x . It also implies that $\Theta(w, x)$ increases with respect to w , because an increase in w is equivalent to a leftward translation of the frontier f in the graph (w, x) . On the other hand, the interest-rate differential increases in w and decreases in x . We have indeed:

Lemma 6. *At first-order, the interest-rate differential at a given arrival date T_n is given by*

$$R \left(w_{T_n} - l_{T_n} - \frac{1}{1 + \Phi} \int_0^{+\infty} e^{-\omega s} E_{T_n} [l_{T_n+s}] ds \right). \quad (45)$$

Proof. See the Appendix. ■

The interest-rate differential increases w.r.t. w but decreases w.r.t. l (and thus x) because the current domestic real rate is lower and future deflation more likely when l is large. Thus the expected return on the carry trade is not unambiguously increasing in the interest-rate differential. For l, \bar{l} sufficiently small, however, most of the interest-rate differential is due to the exogenous component w rather than to the endogenous actions of the carry traders l . In this case, when the interest-rate differential is sufficiently large in absolute terms, it must be that the system is on the right (left) of the frontier when the differential is positive (negative). In other words, we have the following interesting prediction:

A positive (negative) interest-rate differential predicts a positive (negative) return on the carry-trade only for sufficiently large absolute differentials. The exchange rate must be more volatile when the interest-rate differential is small.

When w is small, so is the interest-rate differential, and the differential may correspond to values of (w, x) that are either on the left or on the right of the frontier. The expected return on the carry trade is thus unclear. Since the system is closer to the frontier in this case, future crossings of the frontier are more likely and thus the exchange rate should be more volatile. This is because close to the frontier, for a finite σ , it takes more time to carry traders to coordinate on a given course of action and bifurcate in one direction. This nonlinear impact of the interest-rate differential on the carry-trade return has not been tested to our knowledge.

The profitability of FX momentum and carry-trade strategies that we predict has been established in a large empirical literature, reviewed for example in Burnside, Eichenbaum, and Rebelo (2011).

Peso problem

A large literature argues that the return on the carry trade partly reflects a risk premium for rare and extreme events that may not show in finite samples (see, e.g., Burnside, Eichenbaum, Kleshchelski, and Rebelo, 2011, Farhi and Gabaix, 2013, Jurek, 2014, or Lewis, 2007, and the references herein.) We closely connect to this literature as follows. Fix $\epsilon > 0$ small. The expected return on the carry trade is 0 starting both from $(f(\epsilon), \epsilon)$ and $(f(1-\epsilon), 1-\epsilon)$ in the (w, x) plane. Yet from Proposition 5, as σ becomes small, most paths starting from $(f(\epsilon), \epsilon)$ will exhibit long periods of appreciation of the domestic currency ended with rare (and large) depreciations, while paths starting from $(f(1-\epsilon), 1-\epsilon)$ will feature a symmetric prolonged depreciation. The interest-rate differential is positive in the former case and negative in the latter. Thus, finite samples should yield that a positive interest-rate differential predicts a positive excess return on the carry trade even when the true return is zero.

Leverage and currency appreciation predict financial crises

Gourinchas and Obstfeld (2011) find that credit expansion and appreciation of the domestic currency predict financial crises. The build up of leverage and currency appreciation correspond to paths in which x increases for a long time in our model. Such paths are the ones in which sharp deleveraging and important capital outflows are most likely to occur soon other things being equal.

Monetary policy and carry-trade returns

In addition to relating to the above existing empirical findings, the model also generates a new range of predictions on the relationship between the stance of monetary policy and the distribution of the returns on momentum and carry trade strategies. Proposition 5 suggests that the frontier is flatter when Φ is smaller and γ larger. In words, the frontier is flatter when the central bank is more reluctant to respond to a surge in carry-trade activity with a large reduction in the official rate. This is in turn more likely to be the case when the prices of nontradables are very sticky. Otherwise stated, if an economy is such that the CPI is not too sensitive to the exchange rate, and/or the central bank not too aggressive, then this economy should be more prone to large fluctuations in carry-trade activity because it will experience more prolonged bifurcations. Thus the returns on carry-trade and momentum strategies should have fatter tails. These predictions are novel, to our knowledge.

Concluding remarks

As a conclusion, we briefly discuss two interesting avenues for future research.

- *More general preferences.* Assuming that households are risk-neutral over late consumption dramatically simplifies the analysis, because it implies that the impact of capital inflows on the real rate is straightforward. With strictly concave preferences, the current real rate would depend on consumption growth, so that we could no longer abstract from the impact of foreign lending on quantities and thus production in the domestic economy as we are able to do here. We find it useful to derive our novel mechanism for self-fulfilling profitable carry trades in a highly tractable framework that delivers clear intuitions. An interesting avenue for future research is the study of the impact of such carry trades on quantities under more standard preferences. For such a study, one should also introduce a more standard modelling of price adjustment.
- *Repelling carry traders.* We assume here that the domestic central bank does not use an appropriate rule. An interesting avenue for future research consists in explicitly modelling the commitment issues or welfare costs that prevent the monetary authority from using a larger Φ . This would pave the way to a normative analysis. Notice that the

central bank can repel carry traders in this framework in three other ways: using a measure of inflation that is tilted towards tradables, adding a term that is sufficiently decreasing in the exchange rate appreciation to the interest-rate rule, or simply targeting the realized real rate $r - l_t$. It is easy to see from the perfect-foresight model that these three measures are strictly equivalent in this simple environment, because they all amount to sufficiently reducing the official rate in response to carry-trade activity, thereby discouraging it. These different policies would probably each come with distinctive costs in a more general environment. In any case, a clear implication from this framework is that a decrease in the official rate is the appropriate response when foreign speculative inflows bid up domestic asset prices.

References

Agrippino, Silvia Miranda and Helene Rey (2014) "World Asset Markets and the Global Liquidity," working paper

Bacchetta, Philippe and Eric van Wincoop (2010) "Infrequent Portfolio Decisions: A Solution to the Forward Discount Puzzle," *American Economic Review*, 100(3), 870-904

Bass, Richard and Krzysztof Burdzy (1999) "Stochastic bifurcation models," *Annals of Probability*, 27, 50-108

Bruno, Valentina and Hyun Song Shin (2013), "Capital Flows and the Risk-Taking Channel of Monetary Policy," NBER Working Paper No. 18942

Burdzy, Krzysztof, David Frankel and Ady Pauzner (2001) "Fast Equilibrium Selection by Rational Players Living in a Changing World.," *Econometrica*, 68, 163-190

Burdzy, Krzysztof, David Frankel and Ady Pauzner (1998) "On the Time and Direction of Stochastic Bifurcation." In B. Szyszkowicz, ed. *Asymptotic Methods in Probability and Statistics: A Volume in Honour of Miklos Csorgo*. Holland: Elsevier

Burnside, Craig, Martin Eichenbaum, Isaac Kleshchelski and Sergio Rebelo (2011) "Do Peso Problems Explain the Returns to the Carry Trade?," *Review of Financial Studies*, 24 (3), 853-891

Burnside, Craig, Martin Eichenbaum and Sergio Rebelo (2011) "Carry Trade and Momentum in Currency Markets," *Annual Review of Financial Economics, Annual Reviews*, vol. 3(1), 511-535

Burstein, Ariel, Martin Eichenbaum and Sergio Rebelo (2005) "Large Devaluations and the Real Exchange Rate," *Journal of Political Economy*, Vol. 113, No. 4, 742-784

Eichenbaum, Martin and Charles Evans (1995) "Some Empirical Evidence on the Effects of Shocks to Monetary Policy on Exchange Rates," *The Quarterly Journal of Economics*, Vol. 110, No. 4, 975-1009

Farhi, Emmanuel and Xavier Gabaix (2013) "Rare Disasters and Exchange Rates", working paper

Farhi, Emmanuel and Jean Tirole (2012) "Collective Moral Hazard, Maturity Mismatch and Systemic Bailouts," *American Economic Review*, vol. 102, 60-93

Frankel, David and Krzysztof Burdzy, (2005) "Shocks and Business Cycles," *Advances in Theoretical Economics*, v. 5 iss. 1 paper no. 2

Frankel, David and Ady Pauzner (2000) "Resolving Indeterminacy in Dynamic Settings: The Role of Shocks," *Quarterly Journal of Economics*, Vol. 115, 283-304

Friedman, Milton (1953) *Essays in Positive Economics*, Chicago University Press

Gourinchas, Pierre-Olivier and Maurice Obstfeld (2012) "Stories of the Twentieth Century for the Twenty-First," *American Economic Journal: Macroeconomics*, 4, 226-65

Jurek, Jakub (2014) "Crash-Neutral Currency Carry Trades," *Journal of Financial Economics*, forthcoming

Lewis Karen (2007) "Peso Problem," *The New Palgrave Dictionary of Money and Finance*

Obstfeld, Maurice (1996) "Models of currency crises with self-fulfilling features," *European Economic Review*, vol. 40(3-5), 1037-1047

Rey, Helene (2013) "Dilemma not Trilemma: The Global Financial Cycle and Monetary Policy Independence," paper presented at the Jackson Hole Symposium

Schneider, Martin and Aaron Tornell (2004), "Balance Sheet Effects, Bailout Guarantees and Financial Crises," *The Review of Economic Studies*, 71, 883-913

Appendix

Proof of Lemma 3

Using the first-order approximation (30) in (27), relations (24) and (27) yield domestic inflation as a function of future expected inflows as in the perfect-foresight case:

$$\ln \frac{P_{T_n}}{P_{T_{n-1}}} = - \sum_{k \geq 0} \frac{E_{T_n} [l_{T_{n+k}}]}{(1 + \Phi)^{k+1}}, \quad (46)$$

where $l_t = \ln L_t$. As in the perfect-foresight case, (25) and (26) yield in turn:

$$E_{T_n} \left[\ln \frac{S_{T_{n+1}} I_{T_{n+1}}}{R S_{T_n}} \right] = \frac{\gamma}{1 - \gamma} \sum_{k \geq 0} \frac{E_{T_n} [l_{T_{n+k+1}}]}{(1 + \Phi)^{k+1}} - l_{T_n} \quad (47)$$

One can write (21) as

$$\Theta_t = E_t \left[\sum_{m \geq 0} \frac{1_{\{T_\lambda > T_{n+m}\}} S_{T_{n+m}} P_{T_{n+m}}}{R^m} \left(\frac{S_{T_{n+m+1}} I_{T_{n+m+1}}}{R S_{T_{n+m}}} - 1 + w_{T_{n+m}} \right) \right]. \quad (48)$$

At first-order w.r.t. l_t ,

$$P_{T_{n+m}} S_{T_{n+m}} E_{T_{n+m}} \left[\frac{S_{T_{n+m+1}} I_{T_{n+m+1}}}{R S_{T_{n+m}}} - 1 \right] = E_{T_{n+m}} \left[\ln \frac{S_{T_{n+m+1}} I_{T_{n+m+1}}}{R S_{T_{n+m}}} \right] \quad (49)$$

$$= \frac{\gamma}{1 - \gamma} \sum_{k \geq 0} \frac{E_{T_{n+m}} [l_{T_{n+m+k+1}}]}{(1 + \Phi)^{k+1}} - l_{T_{n+m}}. \quad (50)$$

Thus,

$$\Theta_t = E_t \left[\int_0^{+\infty} \sum_{m \geq 1} \left(\frac{s}{R} \right)^{m-1} \frac{e^{-(\lambda+1)s}}{(m-1)!} \left[\int_0^{+\infty} \frac{\gamma}{1-\gamma} \sum_{k \geq 1} \frac{u^{k-1}}{(k-1)!} \frac{e^{-u}}{(1+\Phi)^k} l_{t+s+u} du \right] ds \right], \quad (51)$$

$$= E_t \left[\int_0^{+\infty} e^{-(\lambda+\rho)s} \left(\int_0^{+\infty} \chi \omega e^{-\omega u} l_{t+s+u} du - l_{t+s} + w_t \right) ds \right], \quad (52)$$

$$= \int_0^{+\infty} e^{-(\lambda+\rho)v} \left(\chi \omega \int_0^v e^{-(\omega-\lambda-\rho)u} du - 1 \right) E_t [l_{t+v}] dv + \frac{w_t}{\lambda + \rho}, \quad (53)$$

and integrating yields the result.

Proof of Lemma 4

Suppose $\chi > 1$. Consider two processes x_t^1 and x_t^2 that satisfy the conditions stated in Lemma 4 with $x_0^1 < x_0^2$. Lemma 2 in Burdzy, Frankel and Pautner (1998) states that almost surely,

$$x_t^2 \geq x_t^1 \text{ for all } t \geq 0. \quad (54)$$

This implies in particular that whenever traders switch to being active along a sample path of (w_t, x_t^1) , so do they along the sample path of (w_t, x_t^2) that corresponds to the same sample path of w_t . This is because it must be that (w_t, x_t^2) is on the right of the frontier f^2 whenever (w_t, x_t^1) is on the right of the frontier f^1 . Thus, the process

$$y_t = x_t^2 - x_t^1 \quad (55)$$

satisfies

$$0 < y_0 < 1, \quad (56)$$

$$\frac{dy_t}{dt} = \lambda(\epsilon_t - y_t), \quad (57)$$

where $\epsilon_t \in \{0; 1\}$.

In order to prove the Lemma, we only need to find $\bar{\lambda}$ such that for all $\lambda \leq \bar{\lambda}$,

$$\Delta = \int_0^{+\infty} \left(\left(\frac{\chi\omega}{\omega - \rho - \lambda} - 1 \right) e^{-(\lambda+\rho)v} - \frac{\chi\omega}{\omega - \rho - \lambda} e^{-\omega v} \right) y_v dv \geq 0. \quad (58)$$

for all deterministic process y_t that obeys (56) and (57). The result then obtains from taking expectations over all paths of w_t .

To prove (58), we introduce the function ζ that satisfies

$$\begin{cases} \frac{d\zeta(v)}{dv} = - \left(\left(\frac{\chi\omega}{\omega - \rho - \lambda} - 1 \right) e^{-(\lambda+\rho)v} - \frac{\chi\omega}{\omega - \rho - \lambda} e^{-\omega v} \right), \\ \lim_{+\infty} \zeta = 0. \end{cases}$$

Integrating by parts, we have

$$\Delta = \zeta(0)y_0 + \int_0^{+\infty} \zeta(v) \frac{dy_v}{dv} dv, \quad (59)$$

$$= \zeta(0)y_0 + \lambda \int_0^{+\infty} \zeta(v)(\epsilon_v - y_v) dv. \quad (60)$$

Further,

$$y_v = y_0 e^{-\lambda v} + \lambda \int_0^v e^{-\lambda(v-u)} \epsilon_u du, \quad (61)$$

and thus,

$$\Delta = y_0 \left(\zeta(0) - \lambda \int_0^{+\infty} \zeta(v) e^{-\lambda v} dv \right) \quad (62)$$

$$+ \lambda \left[\int_0^{+\infty} \epsilon_v \left(\zeta(v) - \lambda \int_v^{+\infty} \zeta(u) e^{-\lambda u} du \right) \right]. \quad (63)$$

We have

$$\lim_{\lambda \rightarrow 0} \zeta(0) = \frac{\chi - 1}{\rho} > 0, \quad (64)$$

ζ is increasing then decreasing beyond a value that stays bounded as λ tends to zero, and $\int_0^{+\infty} \zeta$ converges. Thus for λ sufficiently small,

$$\zeta(v) - \lambda \int_v^{+\infty} \zeta(u) e^{-\lambda u} du \quad (65)$$

is positive for all $v \geq 0$, which yields that Δ is positive, and concludes the proof.

Complement to the proof of Proposition 2

We prove here that f_1 is Lipschitz with a constant that is smaller than that of f_0 , that we denote K_0 . Suppose by contradiction that two points (w_t, x_t) and (w'_t, x'_t) on f_1 satisfy

$$x' > x, \quad (66)$$

$$\frac{x'_t - x_t}{w_t - w'_t} < \frac{1}{K_0}. \quad (67)$$

We compare the paths x'_{t+u} and x_{t+u} corresponding to pairs of paths of w'_{t+u} and w_{t+u} that satisfy for all $u \geq 0$

$$w_{t+u} - w'_{t+u} = w_t - w'_t. \quad (68)$$

It must be that for such pairs of paths:

$$x'_{t+u} - x_{t+u} \leq (x'_t - x_t) e^{-\lambda u}. \quad (69)$$

Otherwise it would have to be the case that (w', x') can be on the right of f_0 when (w, x) is not. Let T denote the first time at which this occurs. It must be that

$$K_0 e^{-\lambda T} (x'_t - x_t) \geq w_{t+T} - w'_{t+T} = w_t - w'_t, \quad (70)$$

a contradiction with (67).

Thus along such paths of $w'_{t+u} - w_{t+u}$, $x'_{t+u} - x_{t+u}$ shrinks at least as fast as when traders switch to inactivity all the time. Together with (67), this implies that the expected return on the carry trade cannot be the same in (w_t, x_t) and (w'_t, x'_t) , a contradiction.

Proof of Proposition 5

The first point is a particular case of Theorem 2 in Burdzy, Frankel, and Pautner (1998). To prove the second point, notice that as $\sigma \rightarrow 0$, starting from a point on the frontier,

$$E_t [x_{t+v}] \simeq (1 - x_t) \left(1 - (1 - x_t) e^{-\lambda v} \right) + x_t^2 e^{-\lambda v} \quad (71)$$

because the system bifurcates upwards with probability $1 - x_t$ and downwards with probability x_t in the limit. Plugging this in (34) and writing that the expected return is zero yields a slope of the frontier equal to

$$- (\bar{l} - \underline{l}) (\lambda + \rho) \left[\frac{\left(\frac{\chi \omega}{\omega - \lambda - \rho} - 1 \right) \left(\frac{2}{2\lambda + \rho} - \frac{1}{\lambda + \rho} \right)}{-\frac{\chi \omega}{\omega - \lambda - \rho} \left(\frac{2}{\omega + \lambda} - \frac{1}{\omega} \right)} \right], \quad (72)$$

which tends to

$$- (\bar{l} - \underline{l}) (\chi - 1) \quad (73)$$

as $\lambda \rightarrow 0$. This means that the absolute value of the slope of the frontier varies as χ w.r.t. γ, Φ for σ, λ sufficiently small.

Proof of Lemma 6

We have

$$I_{T_n+1} - R(1 - w_{T_n}) = R \left(\left(\frac{P_{T_n}}{P_{T_n-1}} \right)^{1+\Phi} - 1 + w_{T_n} \right), \quad (74)$$

$$\simeq R \left(w_{T_n} - E_{T_n} \left[\sum_{k \geq 0} \frac{l_{T_n+k}}{(1+\Phi)^k} \right] \right), \quad (75)$$

$$= R \left(w_{T_n} - l_{T_n} - \int_0^{+\infty} \sum_{k \geq 1} \frac{s^{k-1} e^{-s}}{(k-1)!(1+\Phi)^k} E_{T_n} [l_{T_n+s}] ds \right), \quad (76)$$

$$= R \left(w_{T_n} - l_{T_n} - \frac{1}{1+\Phi} \int_0^{+\infty} e^{-\omega s} E_{T_n} [l_{T_n+s}] ds \right). \quad (77)$$

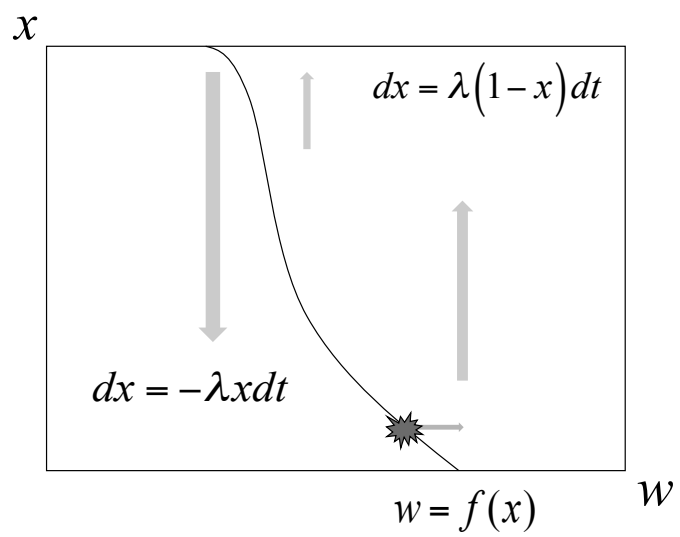


Figure 1

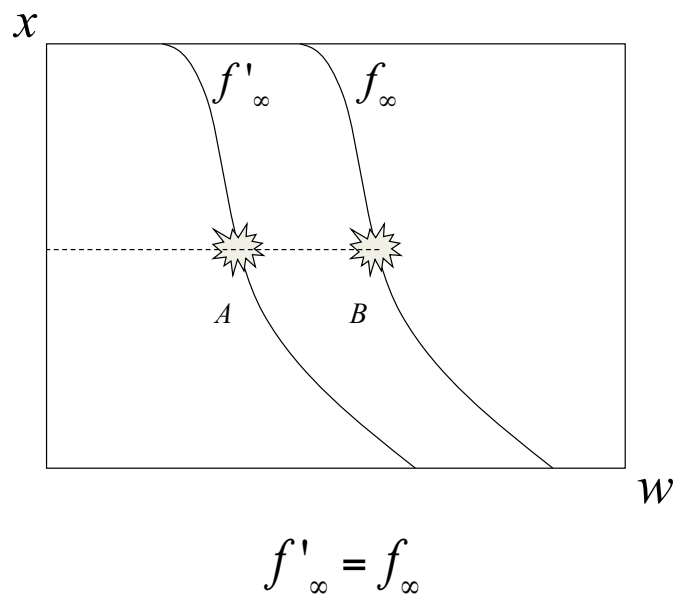


Figure 2