Capital Flow Management when Capital Controls Leak

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Abstract

What are the implications of limited capital controls enforcement for the optimal design of capital flow management policies? We address this question in an environment where pecuniary externalities call for prudential capital controls, but financial regulators lack the ability to enforce them on the "shadow economy." While regulated agents reduce their risk-taking decisions in response to capital controls, *unregulated* agents respond by taking more risk, thereby undermining the effectiveness of the controls. We characterize the choice of a planner who sets capital controls optimally, taking into account the leakages arising from limited regulation enforcement. Our findings indicate that leakages do not necessarily make macroprudential policy on the regulated sphere less desirable, and that large stabilization gains remain despite leakages. Finally, there can be significant redistributive effects across the regulated and unregulated spheres.

Keywords: Macroprudential policy, financial crises, capital controls, limited regulation enforcement

1 Introduction

Central banks in emerging markets have responded to the recent surge in capital inflows by pursuing active capital flow management policies. The hope is that current efforts to curb capital inflows will reduce the vulnerability of the economy to sudden reversals in capital flows. While this macroprudential view of capital controls has gained considerable grounds in academic and policy circles, the debate about their effectiveness remains unsettled.[1] In fact, a growing empirical literature argues that there are important leakages in the implementation of capital controls, casting doubt on the effectiveness of such policies in fostering macroeconomic and financial stability.[2]

Against this backdrop, the literature has not addressed what are the precise consequences of imperfect regulation enforcement of capital controls: To what extent do leakages in regulation undermine the effectiveness of capital controls? Are capital controls desirable in the presence of imperfect regulation enforcement?

To tackle these questions, we use a dynamic model of endogenous sudden stops, that builds on Mendoza (2002) and Bianchi (2011), where households are subject to an occasionally binding credit constraint that links their credit market access to the value of their current income, composed of tradable goods and non-tradable goods. When the economy has accumulated a large stock of debt and an adverse shock hits, the economy falls into a vicious circle by which a contraction in capital flows and the real exchange rate mutually reinforce each other. Because households fail to internalize that higher borrowing leads to a higher exposure to these systemic episodes, this creates a pecuniary externality that can be corrected using appropriately designed capital controls (see e.g. Bianchi (2011) and Korinek (2011)). The existing literature, however, has restricted to the case where capital controls are perfectly enforceable.

In our model, the financial regulator can only enforce capital controls on a subset of the population. We show that unregulated agents respond to tighter regulation in the economy,

[1] For the views of the IMF see Ostry et al. (2010).

[2] See for example Klein (2012), Forbes, Fratzscher, and Straub (2013), Magud, Reinhart, and Rogoff (2011). There is also a related literature on leakages on macroprudential policy (see e.g. Aiyar, Calomiris, and Wieladek (2012), Jiménez, Ongena, Peydró, and Saurina (2012), Camors and Peydró (2013).

i.e. higher capital controls, by taking more risk, due to an implicit insurance provided by regulated agents. As the financial regulator tightens regulation on the "regulated sphere," this reduces overall risk-taking decisions, given borrowing decisions of unregulated agents. Unregulated agents, however, perceive now that crises are less likely and hence respond by taking more debt. That is, they reduce their precautionary savings as the likelihood of a severe contraction in their borrowing capacity falls with higher capital controls. As a result, these leakages in regulation undermine the effectiveness of capital controls.

In our normative analysis, we consider a financial regulator, subject to the same credit market frictions as the private economy, who chooses directly borrowing decisions of *regulated* agents. On the other hand, borrowing decisions remain a private choice for *unregulated* agents. A key aspect of the financial regulator's problem is that it internalizes the leakages from tighter regulation on the unregulated sphere.

We show that the planner's decision to impose capital controls on the regulated sphere when the economy is exposed to the risk of a future financial crisis results from the resolution of a tradeoff that involves a key feedback mechanism. Capital controls on the regulated sphere are socially desirable because they contribute to correct a pecuniary externality that causes an inefficiency due to a constraint linking credit limits to a market price. But capital controls on the regulated sphere encourage more borrowing by the unregulated sphere, and this borrowing pattern of the two spheres drives a wedge between the marginal utilities of the two sets of agents: relative to regulated agents, unregulated agents borrow "too much" ahead of potential future crises and therefore are overly exposed to the risk of crises. This excessive exposure of the unregulated sphere calls for tighter capital controls and therefore even less borrowing by the regulated sphere, which feeds back on more borrowing by the unregulated sphere, and so on. The opening of this wedge is costly to the planner from an allocative efficiency perspective. Therefore, in deciding about the optimal extent of capital controls, the planner trades off the benefits arising from the correction of an inefficiency due to a pecuniary externality with the costs arising from the creation of an otherwise non-existing allocative inefficiency.

Our results indicate that the spillover effects are such that the unregulated sphere responds to capital controls on the regulated sphere by borrowing unambiguously more than under laisser-faire. The results also indicate that the welfare gains from capital controls accrue disproportionately to unregulated agents. This is explained by the non-discriminatory character of the planner's financial crisis prevention policy, that operates through the stabilization of a market price. The planner's intervention on the borrowing of regulated agents entails costs and benefits. The costs arise from lower consumption by regulated agents (due to lower borrowing) when the economy is in a state where it is exposed to the risk of a future crisis. The benefits take the form of a lower probability of occurrence of such crises. Unlike regulated agents who pay a cost in exchange for enjoying the benefits, unregulated agents enjoy the financial stabilization benefits without incurring any cost. In other words, unregulated agents get access to a public good paid for by regulated agents.

This paper relates to the growing literature on capital controls and macroprudential policies. A first strand of this literature examines pecuniary externalities due to incomplete markets and prices that affect financial constraints.[3] In particular, we build on Bianchi (2011)'s normative analysis but consider the case where controls can be enforced only on a fraction of the population. We show that capital controls remain largely effective even in the presence of significant leakages and spillovers from regulated to unregulated agents. Spillovers from regulated to unregulated agents are also analyzed in Bengui (2013), who shows in a stylized two-country model of liquidity demand that an exogenous tightening of liquidity regulation at home discourages liquidity provision abroad.

A second strand of the literature examines prudential capital controls for macroeconomic stabilization in the presence of nominal rigidities. In Farhi and Werning (2012) and Schmitt-Grohé and Uribe (2013), there is a wedge between the private and social value of income due to Keynesian effects that arise when monetary policy is unable to achieve full economic stabilization. These papers also assumes that capital controls are perfectly enforceable.

The paper is organized as follows. Section 3 presents a three period model that shows analytical results for the main mechanisms in the paper. Sections 3 presents the infinite horizon model. Section 4 presents results from calibrated versions of the model. Section 5 concludes.

[3]Examples include Caballero and Krishnamurthy (2001), Lorenzoni (2008), Korinek (2011), Bianchi (2011), Bianchi and Mendoza (2013),Jeanne and Korinek (2011), Benigno, Chen, Otrok, Rebucci, and Young (2013), Bengui (2013).

2 A Three-Period Model

In this section, we present a three period model of sudden stops and optimal capital flow management in the presence of leakages.

2.1 Economic Environment

The economy is populated by a continuum of agents of one of two types $i = U, R$, present in respective proportions γ and $1 - \gamma$, who live for three dates: $t = 0, 1, 2$. Both types of agents have identical preferences and endowments. Preferences are given by:

$$U_i = c_{i0}^T + \mathbb{E}_0 \left[\beta \ln \left(c_{i1}(s) \right) + \beta^2 \ln \left(c_{i2}(s) \right) \right]. \tag{1}$$

with

$$c = \left(c^T \right)^\omega \left(c^N \right)^{1-\omega}.$$

$\mathbb{E}[\cdot]$ is the expectation operator and $\beta < 1$ is a discount factor. Date 0 utility linear in tradable consumption c^T, while date 1 and 2 utility is logarithmic in the consumption basket c, which is a Cobb-Douglas aggregator with unitary elasticity of substitution between tradable goods c^T and nontradable goods c^N. ω is the share of tradables in total consumption. Agents receive endowments of tradable goods and nontradable goods of $\left(y_t^T(s), \bar{y}^N \right)$ at date 1 and 2, but do not receive any endowment at date 0. The date 1 endowment of tradables $y_1^T(s)$ is a random variable depending on the event $s \in S$, which can be interpreted as the aggregate state of the economy. We define $\bar{y}^T \equiv \mathbb{E}_0 \left[y_1^T(s) \right]$ and for simplicity, we assume that $y_2^T(s)$ is a constant equal to $\bar{y}^T$, and that $\bar{y}^N = 1$.

Agents have access to a single one period, non-state contingent bond denominated in units of tradable goods that pays a fixed interest rate r, determined exogenously in the world market. Normalizing the price of tradables to 1 and denoting the price of nontradables by

p^N, the budget constraints are:

$$
\begin{aligned}
c_{i0}^T + b_{i1} &= 0 && (2)\\
c_{i1}^T(s) + p_1^N(s)c_{i1}^N(s) + b_{i2}(s) &= (1+r)b_{i1} + y_1^T(s) + p_1^N(s)\bar{y}^N && (3)\\
c_{i2}^T(s) + p_2^N(s)c_{i2}^N(s) &= (1+r)b_{i2}(s) + \bar{y}^T + p_2^N(s)\bar{y}^N && (4)
\end{aligned}
$$

where b_{t+1} denotes bond holdings an agent chooses at the beginning of period t.

At date 1, agents are subject to a credit constraint preventing them from borrowing more than a fraction κ of their current income:

$$
b_{i2}(s) \geq -\kappa\left(p_1^N(s)\bar{y}^N + y_1^T(s)\right). \tag{5}
$$

This form of credit constraint captures the empirical fact that income is critical to determine credit market access. Moreover, this has been used extensively in the literature on sudden stops following Mendoza (2002) to capture the contractionary effects from depreciations on balance sheets when debt is denominated in foreign currency.

We make the following assumptions on parameters.

Assumption 1. *The domestic agents' discount factor and the international interest rate satisfy* $\beta(1+r) = 1$.

This assumption, common in small open economy models, states that domestic agents are as patient as international investors. It implies that there is no intrinsic motivation for consumption tilting in the domestic economy.

Assumption 2. *The consumption shares and collateralizable fraction of income are such that* $\kappa < \frac{\omega}{1-\omega}$.

This assumption guarantees that a one dollar increase in aggregate consumption relaxes the borrowing constraint by less than a dollar, and therefore rules out multiple equilibria.[4]

Agents choose consumption and savings to maximize their utility (1) subject to budget constraints (2), (3), (4), and to their credit constraint (31), taking $p_1^N(s)$ and $p_2^N(s)$ as given.

[4]See Korinek and Mendoza (2013) for a general discussion of this assumption.

An agent's optimality conditions are given by

$$p_t^N(s) = \frac{1-\omega}{\omega} \frac{c_{it}^T(s)}{c_{it}^N(s)} \tag{6}$$

$$1 = \beta(1+r)\mathbb{E}_0 \left[\frac{\omega}{c_{i1}^T(s)} \right] \tag{7}$$

$$\frac{\omega}{c_{i1}^T(s)} = \beta(1+r)\frac{\omega}{c_{i2}^T(s)} + \mu_{i1}(s) \tag{8}$$

$$b_{i2}(s) + \kappa \left[p_1^N(s)\bar{y}^N + y_1^T(s) \right] \geq 0, \qquad \text{with equality if } \mu_{i1}(s) > 0. \tag{9}$$

where $\mu_{i1}(s)$ is the agent's non-negative multiplier associated with his date 1 credit constraint. Equation (6) is a static optimality condition equating the marginal rate of substitution between tradable and nontradable goods to their relative price. Equation (7) is the Euler equation for bonds at date 0, and equation (8) is the Euler equation for bonds at date 1. When the credit constraint is binding, there is a wedge between the current shadow value of wealth and the expected value of reallocating wealth to the next period, given by the shadow price of relaxing the credit constraint $\mu_{i1}(s)$. Equation (9) is the complementary slackness condition.

If an agent is unconstrained at date 1, he chooses a consumption plan given by

$$c_{i1}^T(s) = c_{i2}^T(s) = \frac{\omega}{1+\beta} we_{i1}(s), \quad c_{i1}^N(s) = \frac{1-\omega}{1+\beta}\frac{we_{i1}(s)}{p_1^N(s)}, \quad \text{and} \quad c_{i2}^N(s) = \frac{1-\omega}{1+\beta}\frac{we_{i1}(s)}{p_2^N(s)} \tag{10}$$

where $we_{i1}(s)$ is the agent's date 1 lifetime wealth

$$we_{i1}(s) \equiv (1+r)b_{i1} + y_1^T(s) + p_1^N(s)\bar{y}^N + \frac{\bar{y}^T + p_2^N(s)\bar{y}^N}{1+r}.$$

To finance this consumption plan, the agent borrows the shortfall between his expenditures $\frac{1}{1+\beta} we_{i1}(s)$ and cash on hand $(1+r)b_{i1} + y_1^T(s) + p_1^N(s)$ at date 1:

$$b_{i2}(s) = b_{i2}^{unc}(s) \equiv \frac{\beta}{1+\beta} \left[(1+r)b_{i1} + y_1^T(s) + p_1^N(s)\bar{y}^N - \bar{y}^T - p_2^N(s)\bar{y}^N \right]. \tag{11}$$

An agent is constrained at date 1 if the bond position in (11) violates the credit constraint

(31). In this case, he borrows the maximum amount:

$$b_{i2}(s) = b_{i2}^{con}(s) \equiv -\kappa \left[y_1^T(s) + p_1^N(s)\bar{y}^N \right] \tag{12}$$

and chooses a consumption plan given by

$$\begin{array}{cc} c_{i1}^T(s) = \omega \tilde{we}_{i1}(s) & c_{i2}^T(s) = \omega(1+r)\left[we_{i1}(s) - \tilde{we}_{i1}(s)\right] \\ c_{i1}^N(s) = (1-\omega)\frac{\tilde{we}_{i1}(s)}{p_1^N(s)} & c_{i2}^N(s) = (1-\omega)(1+r)\frac{we_{i1}(s)-\tilde{we}_{i1}(s)}{p_2^N(s)}, \end{array} \tag{13}$$

where $\tilde{we}_{i1}(s)$ is the agent's date 1 constrained wealth

$$\tilde{we}_{i1}(s) \equiv (1+r)b_{i1} + (1+\kappa)\left[y_1^T(s) + p_1^N(s)\bar{y}^N \right],$$

which corresponds to the sum of actual date 1 wealth and the maximum amount that can be borrowed.

A decentralized equilibrium of the model is a set of decisions $\{c_{i0}^T, b_{i1}\}_{i\in\{U,R\}}$, decision rules $\{c_{i1}^T(s), c_{i2}^T(s), c_{i1}^N(s), c_{i2}^N(s), b_{i2}(s)\}_{i\in\{U,R\}}$ and prices $p_1^N(s), p_2^N(s)$ such that (1) given prices, the agents' decisions are optimal, and (2) markets for the nontradable goods clear at all date. In what follows we proceed by backward induction. We first analyze the date 1 continuation equilibrium for given date 0 bond choices, and we then turn to the date 0 borrowing decisions.

2.2 Date 1 continuation equilibrium

The nontradable goods market clearing condition for $t = 1, 2$ is

$$\gamma c_{Ut}^N(s) + (1-\gamma)c_{Rt}^N(s) = \bar{y}^N = 1. \tag{14}$$

and the aggregation of the two sets of agents' intertemporal budget constraints yield economy's intertemporal resource constraint

$$C_1^T(s) + \frac{C_t^T(s)}{1+r} = (1+r)\left[\gamma B_{U1} + (1-\gamma)B_{R1}\right] + y_1^T(s) + \frac{\bar{y}^T}{1+r} \tag{15}$$

where $C_t^T(s) \equiv \gamma C_{Ut}^T(s) + (1-\gamma)C_{Rt}^T(s)$ is aggregate tradable consumption and upper case letters with U or R subscripts denote aggregates over an agent type.

Combining the nontradable market clearing condition (14) with the agents' static optimality condition (6) delivers a simple expression for the equilibrium price of nontradables:

$$p_t^N(s) = \frac{1-\omega}{\omega} C_t^T(s). \tag{16}$$

Hence, the equilibrium price of nontradables is proportional to the economy's absorption of tradables. Intuitively, when aggregate consumption of tradables is high, nontradables are relatively scarce and their relative price is high. All else equal, an increase in c_{R1}^T generates in equilibrium an increase in p_1^N, which by equation (31) increases the collateral value for all agents. Similarly, a reduction in c_{U1}^T reduces the collateral value for all agents. This mechanism will be a key source of interaction between the behavior of the regulated and unregulated spheres in the regulated equilibrium considered below.

At date 1, the economy's aggregate state variables are given by the tradable goods endowment $y_1^T(s)$ and by the respective aggregate bond positions of type U and type R agents, B_{U1} and B_{R1}. Depending on which set(s) of agents is (are) credit constrained, the economy can be in four regions at date 1: *cc* where both types of agents are constrained, *cu* where U agents are constrained and R agents are unconstrained, *uc* where U agents are unconstrained and R agents are constrained, and *uu* where both types of agents are unconstrained. Conveniently, in each of these cases the continuation equilibrium takes a particularly simple form, as stated in the following lemma.

Lemma 1. *For $x \in \{cc, uc, cu, uu\}$, aggregate date 1 consumption in region x is given by $C_1^T(s) = \alpha_y^x y_1^T(s) + \alpha_U^x B_{U1} + \alpha_R^x B_{R1} + \alpha_{\bar{y}}^x \bar{y}^T$.*

Lemma 1 says that, within each region, date 1 aggregate consumption is linear in each of the aggregate state variables $(y_1^T(s), B_{U1}, B_{R1})$. Date 2 aggregate consumption follows from the economy's intertemporal resource constraint (15) and is therefore also linear in $(y_1^T(s), B_{U1}, B_{R1})$. Finally, according to (16) the equilibrium prices $p_1^N(s)$ and $p_2^N(s)$ are linear in $C_1^T(s)$ and $C_2^T(s)$ (respectively) and therefore also linear in $(y_1^T(s), B_{U1}, B_{R1})$.

Lemma 2. *The coefficients of the decision rule for $C_1^T(s)$ are such that:*

1. $\alpha_y^x > 0$ *and* $\alpha_U^x, \alpha_R^x, \alpha_{\bar{y}}^x \geq 0$ *with* $\alpha_U^x = 0$ *(resp.* $\alpha_R^x = 0$*) if and only if* $\gamma = 0$ *(resp.* $\gamma = 1$*), and* $\alpha_{\bar{y}}^x = 0$ *if and only if* $x = cc$.

2. *if* $0 \leq \gamma \leq 0.5$ *(resp.* $0.5 \leq \gamma \leq 1$*), then* $\alpha_y^{uu} \leq \alpha_y^{cu} \leq \alpha_y^{uc} \leq \alpha_y^{cc}$, *(resp.* $\alpha_y^{uu} \leq \alpha_y^{uc} \leq \alpha_y^{cu} \leq \alpha_y^{cc}$*), with strict inequalities if* $0 < \gamma < 0.5$ *(resp.* $0.5 < \gamma < 1$*).*

3. $\alpha_U^{uu} \leq \alpha_U^{cu} \leq \alpha_U^{cc}$ *and* $\alpha_U^{uu} \leq \alpha_U^{uc} \leq \alpha_U^{cc}$, *with strict inequalities if and only if* $\gamma > 0$.

4. $\alpha_R^{uu} \leq \alpha_R^{cu} \leq \alpha_R^{cc}$ *and* $\alpha_R^{uu} \leq \alpha_R^{uc} \leq \alpha_R^{cc}$, *with strict inequalities if and only if* $\gamma < 1$.

Part 1. of Lemma 2 establishes that aggregate consumption is increasing in each of the three aggregate state variables $(y_1^T(s), B_{U1}, B_{R1})$, always strictly for $y_1^T(s)$, and strictly for B_{U1} unless $\gamma = 0$ and for B_{R1} unless $\gamma = 1$. Higher tradable income or higher wealth leads to higher aggregate tradable consumption. Part 2. of the lemma says that aggregate tradable consumption is more sensitive to tradable income in the regions where the credit constraints are binding. When no credit constraint binds, consumption is increasing in income due to a traditional permanent income effect. When credit constraints bind, the sensitivity of consumption to income is higher because every extra dollar of income is spent in current consumption, and because in addition there is a financial amplification effect working through the price of nontradables at work. The larger the mass of constrained agents, the stronger these binding constraint and financial amplification effects relative to the permanent income effect. Similarly, parts 3. and 4. establishes that aggregate tradable consumption is more sensitive to the two sets of agents' wealth positions in the regions where the credit constraints are binding.

An individual's credit constraint set is defined as the set of tradable endowment realisations such that her credit constraint is binding

$$\mathcal{Q}(b_{i1}; B_{U1}, B_{R1}, x) \equiv \left\{ y_1^T(s) \in \mathbb{R}^+ | b_{i2}^{unc}\left(b_{i1}; y_1^T(s), B_{U1}, B_{R1}, x\right) < b_{i2}^{con}\left(b_{i1}; y_1^T(s), B_{U1}, B_{R1}, x\right) \right\}.$$

where $x \in \{cc, uc, cu, uu\}$ denotes the region in which the economy is and determines the mapping between $(y_1^T(s), B_{U1}, B_{R1})$ and $(p_1^N(s), p_2^N(s))$ relevant to compute b_{i2}^{unc} and b_{i2}^{con}.

The four regions can hence be represented by the following sets:

$$\mathcal{X}^{cc}(B_{U1}, B_{R1}) \equiv \mathcal{Q}(B_{U1}; B_{U1}, B_{R1}, cc) \cap \mathcal{Q}(B_{R1}; B_{U1}, B_{R1}, cc), \tag{17}$$

$$\mathcal{X}^{uc}(B_{U1}, B_{R1}) \equiv \mathcal{Q}^{c}(B_{U1}; B_{U1}, B_{R1}, uc) \cap \mathcal{Q}(B_{R1}; B_{U1}, B_{R1}, uc), \tag{18}$$

$$\mathcal{X}^{cu}(B_{U1}, B_{R1}) \equiv \mathcal{Q}(B_{U1}; B_{U1}, B_{R1}, cu) \cap \mathcal{Q}^{c}(B_{R1}; B_{U1}, B_{R1}, cu), \tag{19}$$

$$\mathcal{X}^{uu}(B_{U1}, B_{R1}) \equiv \mathcal{Q}^{c}(B_{U1}; B_{U1}, B_{R1}, uu) \cap \mathcal{Q}^{c}(B_{R1}; B_{U1}, B_{R1}, uu). \tag{20}$$

Further, we define unions of some of these sets as $\mathcal{X}^{c\star} \equiv \mathcal{X}^{cc} \cup \mathcal{X}^{cu}$, $\mathcal{X}^{\star c} = \mathcal{X}^{cc} \cup \mathcal{X}^{uc}$, $\mathcal{X}^{u\star} = \mathcal{X}^{uu} \cup \mathcal{X}^{uc}$ and $\mathcal{X}^{\star u} = \mathcal{X}^{uu} \cup \mathcal{X}^{cu}$. These sets have some intuitive properties, summarized in the following lemmas.

Lemma 3. *There exists thresholds a_x and b_x satisfying $0 \leq a_x \leq b_x$ (with $a_x = b_x$ iff $B_{U1} = B_{R1}$) such that $y_1^T(s) \in \mathcal{X}^{cc}$ iff $y_1^T(s) < a_x$, $y_1^T(s) \in \mathcal{X}^{uu}$ iff $y_1^T(s) \geq b_y$, $y_1^T(s) \in \mathcal{X}^{cu}$ iff $a_x \leq y_1^T(s) < b_x$ and $B_{U1} < B_{R1}$; and $y_1^T(s) \in \mathcal{X}^{uc}$ iff $a_x \leq y_1^T(s) < b_x$ and $B_{U1} > B_{R1}$.*

Lemma 3 says that for a given pair (B_{U1}, B_{R1}), the regions are ordered along the real line, that the poorest type of agents is never unconstrained when the other type is constrained, and that when both types of agents have the same wealth only the symmetric regions cc and uu can arise. It notably implies that $\mathcal{X}^{cc}$, $\mathcal{X}^{uc}$, $\mathcal{X}^{cu}$ and $\mathcal{X}^{uu}$ are disjoint, and that their union is $\mathbb{R}^+$, meaning that for any triplet $(y_1^T(s), B_{U1}, B_{R1})$ the economy is always in one and only one region.

Lemma 4. *For a given B_{U1} (resp. B_{R1}) and any two $B_{R1}, \tilde{B}_{R1}$ (resp. $B_{U1}, \tilde{B}_{U1}$) such that $B_{R1} < \tilde{B}_R$ (resp. $B_{U1} < \tilde{B}_U$):*

1. *for $\mathcal{X} = \{\mathcal{X}^{cc}, \mathcal{X}^{c\star}, \mathcal{X}^{\star c}\}$, if $y_1^T(s) \in \mathcal{X}(B_{U1}, \tilde{B}_{R1})$ (resp. $y_1^T(s) \in \mathcal{X}(\tilde{B}_{U1}, B_{R1})$), then $y_1^T(s) \in \mathcal{X}(B_{U1}, B_{R1})$.*

2. *for $\mathcal{X} = \{\mathcal{X}^{uu}, \mathcal{X}^{u\star}, \mathcal{X}^{\star u}\}$, if $y_1^T(s) \in \mathcal{X}(B_{U1}, B_{R1})$, then $y_1^T(s) \in \mathcal{X}(B_{U1}, \tilde{B}_{R1})$ (resp. $y_1^T(s) \in \mathcal{X}(\tilde{B}_{U1}, B_{R1})$).*

Part 1. of Lemma 4 says that the region $\mathcal{X}^{cc}$ where both types of agents are credit constrained, and the regions $\mathcal{X}^{c\star}$ and $\mathcal{X}^{\star c}$ where at least one type of agents is constrained are all shrinking in B_{R1} and B_{U1}. Part 2. says that the region $\mathcal{X}^{uu}$ where both types of agents are unconstrained, and the regions $\mathcal{X}^{u\star}$ and $\mathcal{X}^{\star u}$ where at least one type of agents is unconstrained are all expanding in B_{R1} and B_{U1}.

2.3 Date 0 decentralized equilibrium

In an unregulated decentralized equilibrium, date 0 bond choices are symmetric[5]: $B_{U1} = B_{R1} \equiv b_1^{DE}$. The date 0 bond choice in the decentralized equilibrium is characterized by the private Euler equation:

$$1 = \mathbb{E}_0 \left[\frac{\omega}{C_1^T(y_1^T(s), b_1^{DE}, b_1^{DE})} \right], \tag{21}$$

where we have used the fact that given symmetric bond choices, agents' date 1 consumption of tradable goods coincides with aggregate tradable consumption.

Lemma 5. *The date 0 symmetric decentralized equilibrium exists and is unique.*

2.4 Regulated Equilibria

We now consider equilibria where R (regulated) agents face a tax on date 0 borrowing, but U (unregulated) agents don't, and interpret this tax as a capital control. We start by assuming exogenous taxes to streamline the private sector's response to the tax. We then solve for the optimal tax chosen by a constrained social planner.

2.4.1 Exogenous Capital Controls

U agents are not subject to the tax and their problem is the same as in the unregulated decentralized equilibrium of section 2.3. For R agents, however, the date 1 budget constraint (3) is replaced by

$$c_{R1}^T(s) + p_1^N(s)c_{R1}^N(s) + b_{R2}(s) = (1+r)(1+\tau)b_{R1} + y_1^T(s) + p_1^N(s)\bar{y}^N + T, \tag{22}$$

where τ is the tax rate on date 0 borrowing and T is a lump-sum transfer.[6] We assume that the planner rebates the tax proceeds to the agents who pay the tax, so that the government

[5] More precisely, given symmetric primitives, we choose to focus on symmetric equilibria.
[6] τ is a tax on capital inflows or a subsidy on capital outflows at date 0.

budget constraint is

$$T = -\tau b_{R1}. \tag{23}$$

In the presence of capital controls, the unregulated agent's date 0 Euler equation remains given by (7), but the regulated agent's date 0 Euler equation is replaced by

$$1 = \beta(1+r)(1+\tau)\mathbb{E}_0\left[\frac{\omega}{c_{R1}^T(s)}\right]. \tag{24}$$

A regulated equilibrium with an exogenous capital control τ is a set of decisions $\{c_{i0}^T, b_{i1}\}_{i\in\{U,R\}}$, decision rules $\{c_{i1}^T(s), c_{i2}^T(s), c_{i1}^N(s), c_{i2}^N(s), b_{i2}(s)\}_{i\in\{U,R\}}$ and prices $p_1^N(s), p_2^N(s)$ such that (1) given prices and given the tax, the agents' decisions are optimal, and (2) markets for the tradable and nontradable goods clear at all date. Such an equilibrium is characterized by the pair of Euler equations $h_U(b_{U1}^*, b_{R1}^*) = 0$ and $h_R(b_{U1}^*, b_{R1}^*; \tau) = 0$, and

$$h_U(b_{U1}, b_{R1}) \equiv 1 - \mathbb{E}_0\left[\frac{\omega}{c_{U1}^T(y_1^T(s), b_{U1}, b_{R1})}\right], \qquad h_R(b_{U1}, b_{R1}; \tau) \equiv \frac{1}{1+\tau} - \mathbb{E}_0\left[\frac{\omega}{c_{R1}^T(y_1^T(s), b_{U1}, b_{R1})}\right] \tag{25}$$

The functions $c_{U1}^T(y_1^T(s), B_{U1}, B_{R1})$ and $c_{R1}^T(y_1^T(s), B_{U1}, B_{R1})$ are obtained by combining the relevant expressions in (10) and (13) with the equilibrium price expressions obtained from (16) and the solution for aggregate tradable consumption from Lemma 1.

Each of the two Euler equations can be thought of as representing the best response of one type of agents to the other type's savings behavior.

Proposition 1. *For a given tax rate, the best response of type U (resp. R) agents to the borrowing choice of type R (resp. U) agents is unique and decreasing (strictly if and only if $\gamma < 1$, resp. $\gamma > 0$).*

Proposition 1 establishes that borrowing decisions by the two sets of agents are strategic substitutes. The less U (resp. R) agents borrow, the more R (resp. U) agents find it optimal to borrow. The mechanism generating this result does not hinge of the presence of credit constraints at date 1, but binding constraints act to strengthen it. The intuition is that less borrowing by R (resp. U) agents at date 0, by increasing the wealth of R (resp. U) agents in any state at date 1, pushes up the demand for both goods. This leads to an increase in the price of nontradables, which generates a positive wealth spillover on U (resp. R) agents at date 1. This wealth spillover induces U (resp. R) to borrow more at date 0. Note that the wealth spillover is stronger in states of the nature where credit constraints bind, because aggregate consumption (and therefore the price of nontradables) is more sensitive to the two sets of agents' wealth in these cases.

We write the best response functions as $b_{U1} = \phi_U(b_{R1})$ and $b_{R1} = \phi_R(b_{U1};\tau)$, and note that $\phi'_U(\cdot) \leq 0$ and $\phi'_R(\cdot;\tau) \leq 0$. The following proposition describes how borrowing by the two sets of agents responds to changes in the tax rate near the decentralized equilibrium.

Proposition 2. *For small taxes, b^*_{R1} is increasing in τ and b^*_{U1} is decreasing in τ (strictly if $\gamma < 1$).*

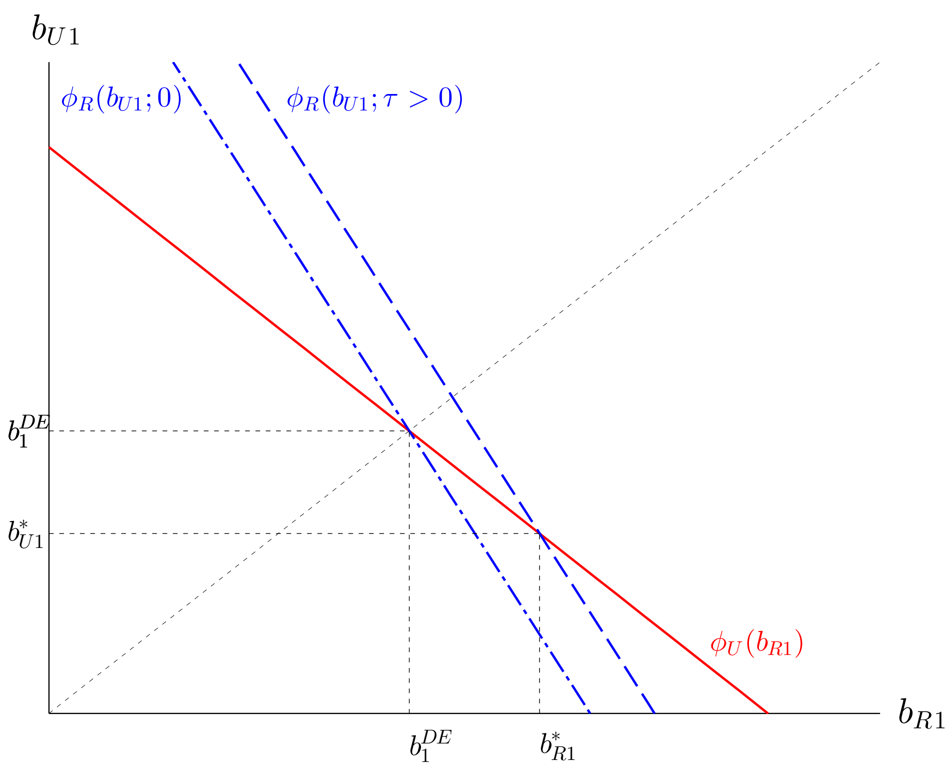

Figure 1: Best response functions of regulated and unregulated agents in equilibrium with exogenous tax ($0 < \gamma < 1$).

Proposition 2 indicates that an increase in the tax on borrowing imposed on R agents generates a decrease in their date 0 borrowing, but an increase in the date 0 borrowing of U agents. This effect can be traced back to the shift in the the regulated agents' best response function caused by an increase in the tax rate. Figure 1 represents the best response functions of the two sets of agents in the (b_{R1}, b_{U1}) pace. The solid (red) line is the best response of U agents, and the dash-dotted and dashed (blue) lines are the best responses of R agents associated with a zero tax (dash-dotted) and to a positive tax (dashed). The intersection between the U agents' best response and the R agents' best response associated with a zero tax coincides by definition with the unregulated symmetric competitive equilibrium. A positive tax causes a shift of the R agents' best response to the right: for a given b_{U1} choice, R agents respond to the tax by borrowing less (since the tax makes borrowing more costly). But U agents respond to this lower borrowing by R agents by borrowing more themselves. This extra borrowing by U agents in turn induces R agents to borrow less. This process continues until equilibrium is reached at point (b^*_{R1}, b^*_{U1}).

2.4.2 Welfare Effects of Capital Controls

So far we have considered regulated equilibria and characterized the private sector's response to an exogenous tax, but we have not provided welfare theoretic foundations for this tax. We now establish that there is scope for Pareto improving capital controls under the condition that credit constraints might bind at date 1 in the decentralized equilibrium.

Proposition 3. *A small positive tax on regulated agents' borrowing leads to Pareto improvements if and only if credit constraints bind with non-zero probability in the unregulated decentralized equilibrium.*

Locally, a tax on capital inflows imposed on regulated agents decreases regulated agents' borrowing and increases unregulated agents' borrowing. However, the net effect on the price of nontradable goods is unambiguously positive in any state of the world at date 1, as the direct effect of a higher b_{R1} always outweighs the leakage effect of a lower b_{U1}. Supporting a higher price of nontradable generally has two effects: to relax binding credit constraints, and to redistribute wealth from net buyers to net sellers of nontradables. This second effect is of second order, so for a small tax the constraint relaxation effect prevails. This explains why both sets of agents benefit from a small positive tax on regulated agents' borrowing.

Figure 2 illustrates the welfare implications of alternative bond choices by plotting the two sets of agents' iso-utility curves in the (b_{R1}, b_{U1}) space in a case where credit constraints

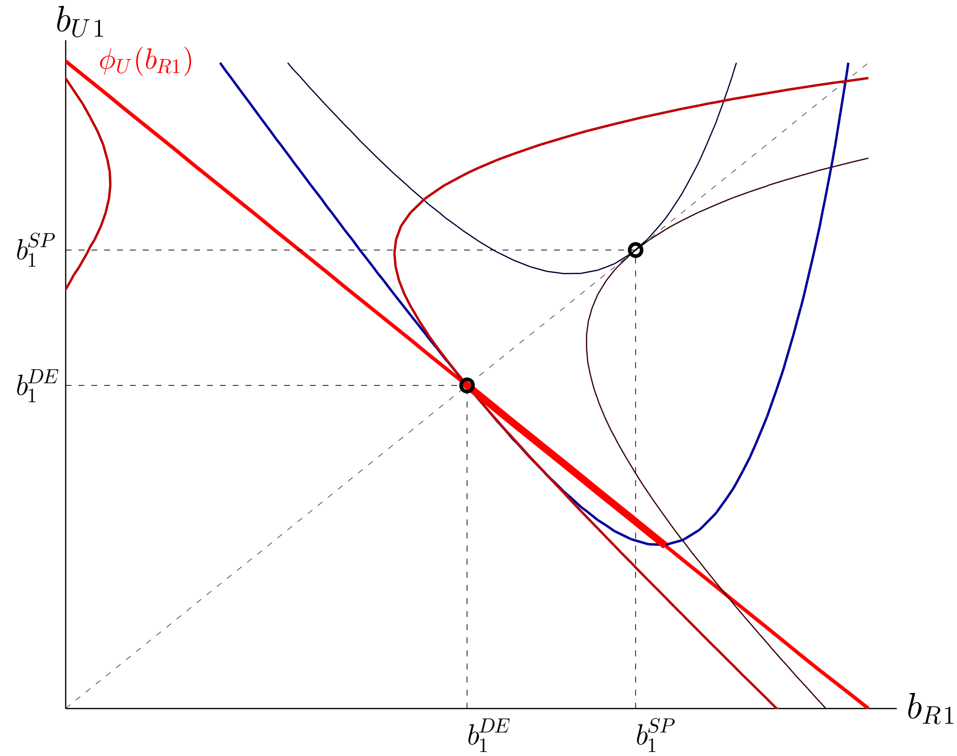

Figure 2: Implementable bond choice pairs leading to Pareto improvements.

bind with non-zero probability in the unregulated decentralized equilibrium. The constrained inefficiency of the decentralized equilibrium implies that the iso-utility curves of the two sets of agents are not tangent at the point (b_1^{DE}, b_1^{DE}). The set of points inside the lense formed by the two iso-utility curves going through (b_1^{DE}, b_1^{DE}) represents bond choice pairs that Pareto dominate the decentralized equilibrium. The pair (b_1^{SP}, b_1^{SP}) corresponds to the constrained efficient allocation that could be reached by a planner who has the ability to control both sets of agents' bond choices. But with imperfectly enforced capital controls, this point is not implementable. Implementable bond choice pairs are located along the downward sloping unregulated agents' best response map $\phi_U(b_{R1})$. The set of implementable bond choice pairs that Pareto dominate the decentralized equilibrium therefore belong to the part of the $\phi_U(b_{R1})$ line that falls inside the lense (thick line in the Figure).

2.4.3 Optimal Capital Controls

Motivated by the fact that in some circumstances, i.e. when credit constraints bind in the unregulated decentralized equilibrium, capital controls can lead to Pareto improvement, we now endogenize the level of the tax by considering the optimal choice of a planner who has the ability to tax the date 0 borrowing choice of R (regulated) agents but not of U (unregulated) agents. To abstract from pure wealth transfer considerations, we continue to assume that the planner rebates the proceeds of the tax to the agents that pay it, i.e. the tax is purely distortionary.

As we show in Appendix B, the optimal tax problem is equivalent to a problem where the planner chooses directly allocations and prices subject to implementability constraints, so we focus on that latter problem in what follows.

Problem 2.1 (Planner's problem)**.**

$$\max_{\{c_{i0}^T, b_{i1}, c_{i1}^T(s), c_{i2}^T(s), c_{i1}^N(s), c_{i2}^N(s), b_{i2}(s)\}_{i\in\{U,R\}}, p_1^N, p_2^N} \gamma U_U + (1-\gamma) U_R \tag{26}$$

subject to (1), (2), (3), (4), (6), (8), (9) for $i = U, R$, (7) for $i = U$ and (14).

The planner, assumed to behave like a utilitarian planner, maximizes a weighted sum of the agents' utility, subject to a number of constraints. (1) is the definition of the utility function. (2), (3) and (4) are the agents' date 0, date 1 and date 2 respective budget constraints. (6) is the agents' static optimality condition between tradable and nontradable goods (which must hold both at date 1 and 2). (8) is the agents' date 1 Euler equation for bonds. (9) is the agents' credit constraint and associated complementary slackness condition. (7) for $i = U$ is the U agents' date 0 Euler equation for bonds and (14) is the nontradable goods market clearing condition (which must hold both at date 1 and 2).

The planner's optimal choice of b_{R1} can be shown to be characterized by the following

Generalized Euler equation (GEE)

$$
\begin{aligned}
1 &= \beta(1+r)\,\mathbb{E}_0\left[\frac{\omega}{c_{R1}^T}\right] \\
&+\beta\mathbb{E}_0\left[\left(\mu_{R1}+\frac{\gamma}{1-\gamma}\mu_{U1}\right)\kappa\left(\frac{\partial p_1^N}{\partial b_{R1}}+\frac{\partial p_1^N}{\partial b_{U1}}\frac{\partial b_{U1}}{\partial b_{R1}}\right)\right] \\
&+\sum_{t=1}^{2}\beta^t\mathbb{E}_0\left[\left(\frac{\omega}{c_{Rt}^T}-\frac{\omega}{c_{Ut}^T}\right)\left(\bar{y}^N-c_{Rt}^N\right)\left(\frac{\partial p_t^N}{\partial b_{R1}}+\frac{\partial p_t^N}{\partial b_{U1}}\frac{\partial b_{U1}}{\partial b_{R1}}\right)\right]
\end{aligned} \tag{27}
$$

where $\mu_{i1}=\frac{\omega}{c_{i1}^T}-\frac{\omega}{c_{i2}^T}\geq 0$ are the shadow costs associated with the credit constraints at date 1. This GEE resembles the private Euler equation (7), but it contains additional terms reflecting the planner's internalization of pecuniary externalities. The first term on the right-hand side corresponds to the private valuation of wealth, also present in (7). The term on the second line reflects the benefits the planner derives from a relaxation of the agents' credit constraints at date 1 through supporting higher prices of nontradables by saving more at date 0. This term is common in the normative analysis of models with credit constraints linked to market prices. However, in contrast to models with perfect financial regulation enforcement, it now embeds the unregulated agents' response to the borrowing choice of the planner for the regulated agents, through the negative derivative $\frac{\partial b_{U1}}{\partial b_{R1}}$ (see Proposition 1). This pure leakage effect lowers the marginal value of saving on the behalf of regulated agents for the planner and therefore pushes in the direction of a weaker intervention. In addition, the last terms on the third line of the GEE reflect the benefits the planner derives from wealth transfers caused by the pecuniary externality on the size of a wedge between the marginal rates of substitution of regulated and unregulated agents. This wedge does not exist when date 0 borrowing levels are the same for both sets of agents, so it's opening is a direct consequence of regulation under imperfect enforcement. Supporting higher future prices of nontradables via higher date 0 savings causes a wealth transfer from the (rich) net buyers to the (poor) net sellers of nontradables. The former have a lower marginal utility of tradable consumption than the latter, so such a transfer is valued positively by the planner.[7] In other words, the leakage effects of regulation result in the emergence of a new distortion,

[7] The static optimality condition (6) together with market clearing for nontradable goods (14) implies that $c_{Rt}^T > c_{Ut}^T$ iff $c_{Rt}^N > \bar{y}^N$, so the product $\left(\frac{\omega}{c_{Rt}^T}-\frac{\omega}{c_{Ut}^T}\right)\left(\bar{y}^N-c_{Rt}^N\right)$ is always positive.

which contrary to the pure leakage effect discussed above, calls for a stronger intervention.

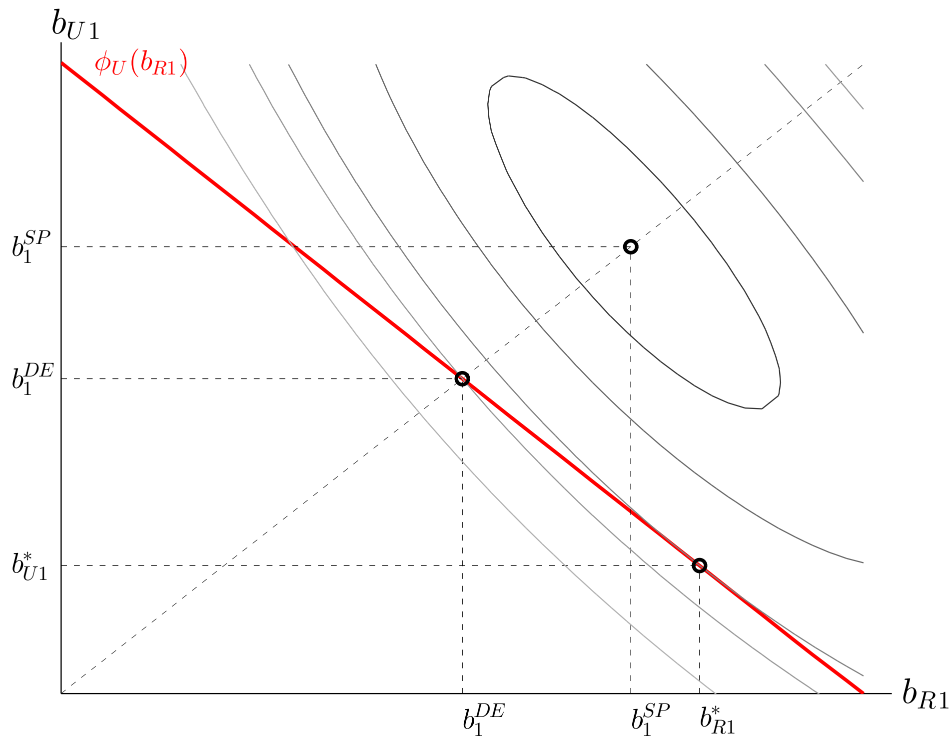

Figure 3: Borrowing choices in equilibrium with optimal capital controls, with planner's iso-utility curves.

The optimal borrowing choices commanded by the planner are displayed in Figure 3, together with the planner's iso-utility curves. The point (b_1^{DE}, b_1^{DE}) corresponds to the unregulated decentralized equilibrium, the point (b_1^{SP}, b_1^{SP}) is the constrained efficient choice which the planner would achieve if he could regulate everyone, and the point (b_{R1}^*, b_{U1}^*) is the borrowing pair chosen by planner in the presence of leakages. Acting as a Stackelberg leader in the latter case, the planner picks the point tangent to his so-utility curves on the unregulated agents' best response map. In the case drawn, it happens that $b_{R1}^* > b_1^{SP}$, but the relationship between b_{R1}^* and b_1^{SP} generally depends on parameters. Figure 4 represents the same borrowing choices, but this time together with the regulated (left) and unregulated (right) agents' so-utility curves.

Combining the GEE (27) with the regulated agents' Euler equation (24) yields an ex-

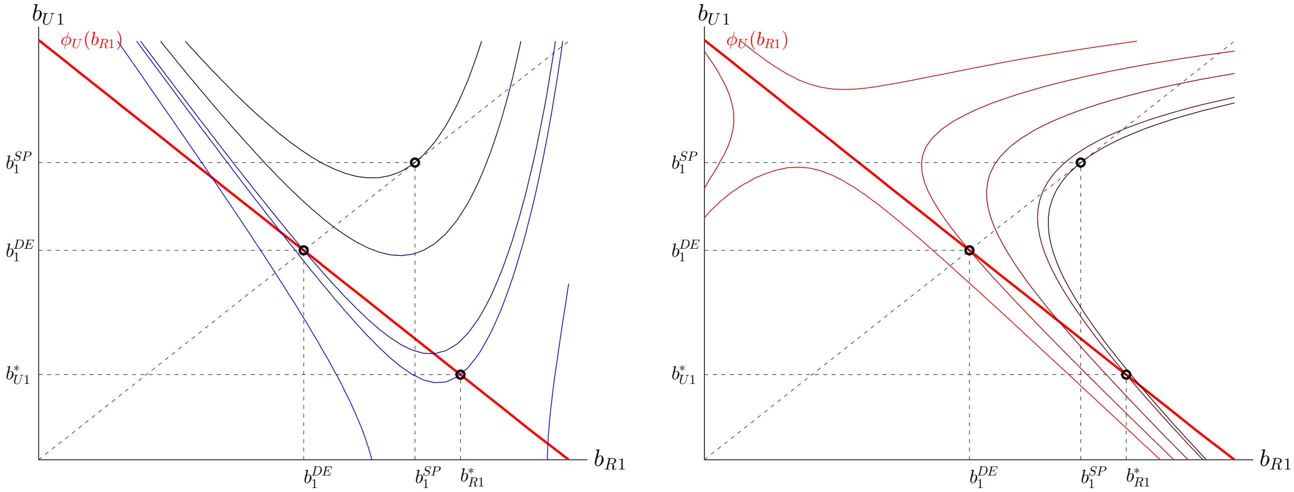

Figure 4: Borrowing choices in equilibrium with optimal capital controls, regulated (left) and unregulated (right) agents' so-utility curves.

pression for the optimal tax:

$$\tau = \frac{\beta \mathbb{E}_0 \left[\left(\mu_{R1} + \frac{\gamma}{1-\gamma} \mu_{U1} \right) \kappa \bar{y}^N \frac{dp_1^N}{db_{R1}} \right] + \sum_{t=1}^{2} \beta^t \mathbb{E}_0 \left[\left(\frac{\omega}{c_{Rt}^T} - \frac{\omega}{c_{Ut}^T} \right) \left(\bar{y}^N - c_{Rt}^N \right) \frac{dp_t^N}{db_{R1}} \right]}{\mathbb{E}_0 \left[\frac{\omega}{c_{R1}^T} \right]} \tag{28}$$

This expression can be thought of as defining the planner's desired capital control τ associated with the private sector's bond choices (b_{U1}, b_{R1}), while the regulated equilibrium with an exogenous capital control defined the private sector's desired bond choices as a function of the capital control. The optimal capital control is the fixed point of this mapping.

As is well known from the literature, if the planner could tax all agents in the economy, he would do so if and only if credit constraints bind in some states of the world at date 1. As stated in the following proposition, we find that this result continues to hold when the regulation leaks as long as the planner can tax a subset of agents, irrespective of how large (or small) this subset is.

Proposition 4. *The optimal tax on regulated agents' borrowing is zero if and only if credit constraints never bind in the unregulated decentralized equilibrium.*

The intuition for this result is straightforward from the expression for the optimal tax in (28). At a zero tax, the distortion represented by the second term of the numerator in (28) is nonexistent, and the only motive for taxing capital inflows is to address the inefficiency caused by the pecuniary externality that arises due to credit constraints being linked to a market price. Hence, for a zero tax to be optimal, it must be that credit constraints never bind under laissez-faire. Conversely, if credit constraints bind under laissez-faire, it is always optimal to impose some tax on regulated agents because for small taxes leakage considerations are of second order.

2.5 Insights from Three Period Model

This section developed a heavily stylized model of imperfectly enforced capital flow management policies, where the inherent motivation for capital controls derived from a pecuniary externality caused by financial constraints linked to a market price. The key prediction of the model is that in response to capital controls on the regulated sphere, capital inflows to the unregulated sphere increase. Our main normative insight is that this leakage phenomenon exerts two counteracting forces on the magnitude of optimal capital controls. On the one hand, a pure leakage effect makes capital controls on the regulated sphere less desirable because the reduction in the regulated sphere's indebtedness is partially offset by an increase in borrowing by the unregulated sphere. On the other hand, the leakages make capital controls introduce a new distortion that takes the form of an excessive relative indebtedness of the unregulated sphere. Correcting this distortion requires reducing the economy's indebtedness further and therefore calls, paradoxically, for even tighter controls on the regulated sphere.

In the next section, we embed the leakage phenomenon into a quantitative model of emerging market crises with the aim of assessing the quantitative relevance of the above mentioned mechanisms.

3 Infinite horizon model

3.1 Economic Environment

The economy is populated by a continuum of infinitely lived agents of one of two types $i \in \{U, R\}$, present in (constant) respective proportions γ and $1 - \gamma$. Both types of agents have identical preferences and endowments. Preferences are given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{it}). \tag{29}$$

In this expression, $\mathbb{E}(\cdot)$ is the expectation operator, and β is the discount factor. The period utility function $u(\cdot)$ is a standard concave, twice-continuously differentiable function that satisfies the Inada condition. The consumption basket c is an Armington-type CES aggregator with elasticity of substitution $1/(\eta+1)$ between tradable goods c^T and nontradable goods c^N, given by:

$$c = \left[\omega \left(c^T\right)^{-\eta} + \left(c^N\right)^{-\eta}\right]^{-\frac{1}{\eta}}, \eta > 1, \omega \in (0,1).$$

In each period t, agents receive an endowment of tradable goods y_t^T and an endowment of nontradable goods y_t^N. The vector of endowments $\mathbf{y} \equiv \left(y^T, y^N\right) \in Y \subset R_{++}^2$ follows a first-order Markov process.

Agents have access to a single one period, non-state contingent bond denominated in units of tradable goods that pays a fixed interest rate r, determined exogenously in the world market. Normalizing the price of tradable goods to 1 and denoting the price of nontradable goods by p^N, the budget constraint is:

$$b_{it+1} + c_{it}^T + p_t^N c_{it}^N = b_{it}(1+r) + y_t^T + p_t^N y_t^N, \tag{30}$$

where b_{it+1} denotes bond holdings that a type i agent chooses at the beginning of period t.

We assume that agents are subject to a credit constraint preventing them from borrowing more than a fraction κ of their current income:

$$b_{it+1} \geq -\kappa_t \left(p_t^N y_t^N + y_t^T\right). \tag{31}$$

This constraint is the same as in the three period model, but κ_t is financial shock that hits exogenously the borrowing capacity of agents. This shock is introduced to capture disturbances in financial markets that are exogenous to domestic fundamentals, i.e., variations of domestic income. This shock follows a two-state Markov process with values given by κ^H and κ^L.

Agents choose stochastic processes $\{c_{it}^T, c_{it}^N, b_{it+1}\}_{t\geq 0}$ to maximize the expected present discounted value of utility (29) subject to sequences of budget constraints (30) and credit constraints (31), taking b_{i0} and $\left\{p_t^N\right\}_{t\geq 0}$ as given. This maximization problem yields the following first-order conditions:

$$\lambda_{it} = u_T^i(t) \tag{32}$$

$$p_t^N = \left(\frac{1-\omega}{\omega}\right)\left(\frac{c_{it}^T}{c_{it}^N}\right)^{\eta+1} \tag{33}$$

$$\lambda_{it} = \beta(1+r)\mathbb{E}_t\lambda_{it+1} + \mu_{it} \tag{34}$$

$$b_{it+1} + \kappa_t\left(p_t^N y_t^N + y_t^T\right) \geq 0, \qquad \text{with equality if } \mu_{it} > 0, \tag{35}$$

where λ is the non-negative multiplier associated with the budget constraint and μ is the non-negative multiplier associated with the credit constraint. Condition (32) equates the marginal utility of tradable consumption to the shadow value of current wealth. Condition (33) equates the marginal rate of susbtitution between tradable and nontradable goods to their relative price. Equation (34) is the Euler equation for bonds. When the credit constraint is binding, there is a wedge between the current shadow value of wealth and the expected value of reallocating wealth to the next period, given by the shadow price of relaxing the credit constraint μ_{it}. Equation (35) is the complementary slackness condition.

Market clearing conditions are given by:

$$\gamma c_{Ut}^N + (1-\gamma)c_{Rt}^N = y_t^N \tag{36}$$

$$\gamma c_{Ut}^T + (1-\gamma)c_{Rt}^T = y_t^T + [\gamma b_{Ut} + (1-\gamma)b_{Rt}](1+r) - [\gamma b_{Ut+1} + (1-\gamma)b_{Rt+1}]. \tag{37}$$

Combining equation (33) for $i \in \{U, R\}$ with the market clearing condition (36) yields

$$p_t^N = \frac{1-\omega}{\omega}\left(\frac{\gamma c_{Ut}^T + (1-\gamma)c_{Rt}^T}{y_t^N}\right)^{\eta+1},$$

from which it is apparent that the equilibrium price of nontradable goods p^N depends positively on the aggregate consumption of tradable goods $C^T \equiv \gamma c_U^T + (1-\gamma)c_R^T$.

3.2 Recursive Competitive Equilibrium

We now consider the optimization problem of a representative agent in recursive form. The aggregate state vector of the economy is $X = \{B_U, B_R, y^T, y^N, \kappa\}$. The state variables for a type i agent's problem is the individual state b_i and the aggregate states X. Agents need to forecast the future price of nontradables. To this end, they need to forecast future aggregate bond holdings. We denote by $\Gamma_i(\cdot)$ the forecast of aggregate bond holdings for the set of type i agents for every current aggregate state X, i.e., $B_i' = \Gamma_i(X)$. Combining equilibrium conditions (33), (36) and (37), the forecast price function for nontradable can be expressed as

$$p^N(X) = \frac{1-\omega}{\omega}\left(\frac{y_t^N + [\gamma B_U + (1-\gamma)B_R](1+r) - [\gamma\Gamma_U(X) + (1-\gamma)\Gamma_R(X)]}{y^N}\right)^{\eta+1}. \quad (38)$$

The problem of a type i agent can then be written as:

$$V(b_i, X) = \max_{b_i', c_i^T, c_i^N} u\left(c\left(c_i^T, c_i^N\right)\right) + \beta\mathbb{E}V(b_i', X') \quad (39)$$

subject to

$$b_i' + p^N(X)c_i^N + c_i^T = b_i(1+r) + p^N(X)y^N + y^T$$

$$b_i' \geq -\kappa\left(p^N(X)y^N + y^T\right)$$

$$B_j' = \Gamma_j(X) \quad \text{for} \quad j = \{U, R\}$$

The solution to this problem yields decision rules for individual bond holdings $\hat{b}(b_i, X)$, tradable goods consumption $\hat{c}^T(b_i, X)$ and nontradable goods consumption $\hat{c}^N(b_i, X)$. The decision rule for bond holdings induces actual laws of motion for aggregate bonds, given by $\hat{b}(B_i, X)$. In a recursive rational expectations equilibrium, as defined below, these two laws of motion must coincide.

Definition 1 (Recursive Competitive Equilibrium)**.** *A recursive competitive equilibrium is defined by a pricing function $p^N(X)$, perceived laws of motions $\Gamma_i(X)$ for $i \in \{U, R\}$, and* *decision rules* $\hat{b}(b_i, X)$*,* $\hat{c}^T(b_i, X)$*,* $\hat{c}^N(b_i, X)$ *with associated value function* $V(b_i, X)$ *such that:*

1. *Agents' optimization:* $\left\{\hat{b}(b_i, X), \hat{c}^T(b_i, X), \hat{c}^N(b_i, X)\right\}$ *and* $V(b_i, X)$ *solve the agent's* i *recursive optimization problem for* $i \in \{U, R\}$*, taking as given* $p^N(X)$ *and* $\Gamma_i(X)$ *for* $i = \{U, R\}$*.*

2. *Consistency: the perceived laws of motion for aggregate bonds are consistent with the actual laws of motion:* $\Gamma_i(X) = \hat{b}(B_i, X)$ *for* $i = \{U, R\}$*.*

3. *Market clearing:*

$$\gamma \hat{c}^N(B_U, X) + (1-\gamma)\hat{c}^N(B_R, X) = y^N$$

and

$$\gamma\left[\Gamma_U(X) + \hat{c}^T(B_U, X) - B_U(1+r)\right] + (1-\gamma)\left[\Gamma_R(X) + \hat{c}^T(B_R, X) - B_R(1+r)\right] = y^T.$$

3.3 Regulated Equilibrium

We now consider a constrained social planner who makes debt choices for a subset of agents. We subject the planner to the same collateral constraint as private agents, deprive him of the ability to commit to future policies, and let him control credit operations of R (regulated) agents and rebate the proceeds of the transactions in a lump-sum fashion to these agents.

As opposed to atomistic agents, the planner internalizes the effect of borrowing decisions on the price of nontradables. He also internalizes the effect of its borrowing decision for R agents on U agents' borrowing choices.

3.3.1 Regulated Agents Optimization Problem

Since the planner chooses R agents' bond holdings, the optimization problem faced by private agents reduces to choosing tradable and nontradable consumption, taking as given a government transfer T_{Rt}, which corresponds to the resources added or subtracted by the planner's debt choices:

Problem 3.1 (Regulated agent's problem in regulated equilibrium)**.**

$$\max_{\{c_{Rt}^T, c_{Rt}^N\}_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c\left(c_{Rt}^T, c_{Rt}^N\right)\right)$$

$$s.t. \qquad c_{Rt}^T + p_t^N c_{Rt}^N = y_t^T + p_t^N y_t^N + T_{Rt}. \tag{40}$$

This problem is static and its first-order condition is the intra-temporal optimality condition (33) for $i = R$. This condition, together with the budget constraint (40), enters as an implementability condition in the planner's problem.

3.3.2 Unregulated Agents Optimization Problem

When the planner makes decision about R agents choices, the laws of motion used by U agents to forecast aggregate bond positions do not coincide with the laws of motion used in (and induced by) the recursive competitive equilibrium. We denote these laws of motion by $\mathcal{B}_i(\cdot)$. The price forecast function associated with these laws of motion is accordingly obtained by replacing $\Gamma_i(\cdot)$ by $\mathcal{B}_i(\cdot)$ in (38), and is denoted by $\mathcal{P}^N(\cdot)$.

Problem 3.2 (Unregulated agent's problem in regulated equilibrium)**.**

$$V(b_U, X) = \max_{b_U', c_U^T, c_U^T} u\left(c\left(c_U^T, c_U^N\right)\right) + \beta \mathbb{E} V(b_U', X')$$

subject to

$$b_U' + \mathcal{P}^N(X) c_U^N + c_U^T = b_U(1+r) + \mathcal{P}^N(X) y^N + y^T \tag{41}$$

$$b_U' \geq -\kappa\left(\mathcal{P}^N(X) y^N + y^T\right)$$

$$B_i' = \mathcal{B}_i(X) \quad \textit{for} \quad i = \{U, R\}$$

Using sequential notation, the first-order conditions of this problem are given by the conditions (32)-(35) for $i = U$. Conditions (32), (34) and (35) can be alternatively expressed as two inequalities

$$u_T\left(c_{Ut}^T, c_{Ut}^N\right) \geq \beta(1+r) \mathbb{E}_t u_T\left(c_{Ut+1}^T, c_{Ut+1}^N\right) \tag{42}$$

$$b_{Ut+1} \geq -\kappa_t\left(p_t^N y_t^N + y_t^T\right) \tag{43}$$

and an equality

$$\left[b_{Ut+1} + \kappa_t \left(p_t^N y_t^N + y_t^T\right)\right] \left[u_T\left(c_{Ut}^T, c_{Ut}^N\right) - \beta(1+r)\mathbb{E}_t u_T\left(c_{Ut+1}^T, c_{Ut+1}^N\right)\right] = 0. \tag{44}$$

The intra-temporal optimality condition (33) and the inter-temporal conditions (42)-(44), together with the sequential version of the budget constraint (41), will enter as implementability constraints in the planner's problem.

The constraints (42)-(44) are crucial constraints, because they embed the spillover effects from the planner's debt choices for R agents to the U agents' choices. In particular, they imply that the planner's current borrowing choice for R agents influences U agents' current borrowing by affecting their future marginal utility. For instance, a more cautious borrowing choice by the planner today softens the U agents' credit constraint tomorrow, and, if the latter binds, thereby encourages more borrowing by these agents today.

3.3.3 Social Planner's Optimization Problem

Let $\mathcal{B}_R(X)$ be the policy rule for bond holdings of future planners that the current planner takes as given, and let $\mathcal{B}_U(X)$, $\mathcal{C}_R^T(X)$, $\mathcal{C}_U^T(X)$, $\mathcal{C}_R^N(X)$, $\mathcal{C}_U^N(X)$ and $\mathcal{P}^N(X)$ be the associated recursive functions that return U agents' bond holdings, consumption allocations and the price of nontradables under this policy rule.

Problem 3.3 (Recursive representation of the planner's problem in regulated equilibrium)**.** *Given the policy rule of future planners $\mathcal{B}_R(X)$ and the associated bond decision $\mathcal{B}_U(X)$, consumption allocations $\mathcal{C}_R^T(X)$, $\mathcal{C}_U^T(X)$, $\mathcal{C}_R^N(X)$, $\mathcal{C}_U^N(X)$ and nontradable price $\mathcal{P}^N(X)$, the planner's problem is characterized by the following Bellman equation:*

$$\mathcal{V}(X) = \max_{\{c_i^T, c_i^N, b_i'\}_{i\in\{U,R\}}, p^N} \delta\gamma u\left(c\left(c_U^T, c_U^N\right)\right) + (1-\gamma)u\left(c\left(c_R^T, c_R^N\right)\right) + \beta\mathbb{E}\mathcal{V}(X') \tag{45}$$

subject to

$$c_i^T + p^N c_i^N + b_i' = b_i(1+r) + y^T + p^N y^N \qquad \textit{for} \quad i \in \{U, R\} \tag{46}$$

$$b_i' \geq -\kappa\left(p^N y^N + y^T\right) \textit{for} \quad i \in \{U, R\} \tag{47}$$

$$c_i^N = \frac{c_i^T}{\gamma c_U^T + (1-\gamma) c_R^T} y^N \qquad \textit{for} \quad i \in \{U, R\} \tag{48}$$

$$p^N = \frac{1-\omega}{\omega}\left(\frac{\gamma c_U^T + (1-\gamma) c_R^T}{y^N}\right)^{\eta+1} \tag{49}$$

$$u_T\left(c_U^T, c_U^N\right) \geq \beta(1+r)\mathbb{E}_{\mathbf{y}'|\mathbf{y}} u_T\left(\mathcal{C}_U^T(X'), \mathcal{C}_U^N(X')\right) \tag{50}$$

$$\left[b_U' + \kappa\left(p^N y^N + y^T\right)\right] \times \left[u_T\left(c_U^T, c_U^N\right) - \beta(1+r)\mathbb{E} u_T\left(\mathcal{C}_U^T(X'), \mathcal{C}_U^N(X')\right)\right] = 0 \tag{51}$$

In the above problem, the planner chooses $b_R'(X)$ optimally to maximize a welfare criterion subject to nine constraints. We allow the planner to assign different weights to the utility of the two sets of agents. (46) represents the agents' budget constraint, with respective multipliers $\gamma\lambda_U$ and $(1-\gamma)\lambda_R$, which states that the consumption plan must be consistent w ith what agents choose optimally given their budget constraint and the planner's transfer. (47) represents the agents' collateral constraints, with respective multipliers $\gamma\mu_U$ and $(1-\gamma)\mu_R$, faced by the planner for the agents' borrowing. (48) represents static implementability constraints, with respective multipliers $\gamma\xi_U$ and $(1-\gamma)\chi_R$, stating that agents' consumption bundles must be consistent with their optimal intra-temporal choice between c^T and c^N, and equilibrium on the nontradable goods market. (49) is a static implementability constraint, with multiplier χ, stating that the nontradable price must be consistent with the optimal intra-temporal choice of both types of agents and equilibrium on the nontradable goods market. (50) and (51) are a set of dynamic implementability constraints, with respective multiplier $\gamma\nu$ and $\gamma\psi$, stating that U agents' bond choice must satisfy their inter-temporal Euler equation. (51) indicates that among the collateral constraint (47) for U agents and the Euler equation (50), at most one can hold with strict inequality.

We now turn to a formal definition of a regulated equilibrium.

Definition 2 (Recursive Regulated Equilibrium)**.** *The recursive regulated equilibrium is defined by the policy rule $b_R'(X)$ with associated bond decision $b_U'(X)$, consumption allocations $c_R^T(X)$, $c_U^T(X)$, $c_R^N(X)$, $c_U^N(X)$, nontradable price $p^N(X)$ and value function $\mathcal{V}(X)$, and* *the conjectured functions characterizing the policy rule of future planners $\mathcal{B}_R(X)$ and its associated bond decision $\mathcal{B}_U(X)$, consumption allocations $\mathcal{C}_R^T(X)$, $\mathcal{C}_U^T(X)$, $\mathcal{C}_R^N(X)$, $\mathcal{C}_U^N(X)$, nontradable price $\mathcal{P}^N(X)$ such that the following conditions hold:*

1. *Planner's optimization: $\mathcal{V}(X)$, $b_i'(X)$, $c_i^T(X)$, $c_i^N(X)$ for $i \in \{U, R\}$ and $p^N(X)$ solve the Bellman equation defined in Problem 3.3 given $\mathcal{B}_i(X)$, $\mathcal{C}_i^T(X)$, $\mathcal{C}_i^N(X)$ for $i \in \{U, R\}$ and $\mathcal{P}^N(X)$.*

2. *Time consistency: The conjectured policy rule and associated bond decision, consumption allocations and pricing function that represent choices of future planners coincide with the corresponding recursive functions that represent optimal plans of the current regulator: $\mathcal{B}_i(X) = b_i'(X)$, $\mathcal{C}_i^T(X) = c_i^T(X)$, $\mathcal{C}_i^N(X) = c_i^N(X)$ for $i \in \{U, R\}$ and $\mathcal{P}^N(X) = p^N(X)$.*

Note that the requirements that 1. the consumption allocations are optimal for regulated agents, and 2. the bond choice and consumption allocations are optimal for unregulated agents, are redundant because they are embedded into the planner's various implementability constraints.

4 Quantitative Analysis

This section contains a preliminary quantitative analysis of the model. The model is solved numerically using global non-linear methods.

4.1 Calibration

The calibration largely follows the baseline calibration of Bianchi (2011). The main features of the calibration are the following. The model is calibrated to annual data from Argentina. The coefficient of relative risk aversion is set to 2 and the international interest rate is set to 4 percent, both standard values in DSGE models. The discount factor is set to $\beta = 0.91$, and the share of tradable goods in consumption is set to $\omega = 0.32$. The non-tradable endowment is (temporarily) taken to be constant. The tradable endowment shock is taken to be a first-order univariate autoregressive process. This process is estimated with the

HP-filtered component of tradable GDP from the World Development Indicators for the 1965-2007 period. The vector of shocks is discretized into a first-order Markov process, with three points, using the quadrature-based procedure of Tauchen and Hussey (1991). The intratemporal elasticity of substitution between tradable and nontradable $1/(\eta+1)$, is set to a value of 1.[8] The process for κ is assumed to be independent from the process for y_t^T. κ_t follows a regime-switching Markov process with regime values given by $\{\kappa^L, \kappa^H\}$ and transition matrix

$$P = \begin{bmatrix} P_{LL} & 1-P_{LL} \\ 1-P_{HH} & P_{HH} \end{bmatrix}.$$

The value of κ^L is set to 0.25, and the value of κ^H is set to 0.5. The continuation probability P_{HH} is set to $P_{HH} = 0.9$ so as to produce a mean duration of the κ^H regime of 10 years. The value of P_{LL} is to $P_{LL} = 0.1$ so as generate, given P_{HH}, a long-run probability of the κ^L regime of 10%. This parametrization of the κ_t process yields Sudden Stops events with a frequency of around 5%. Sudden Stops are defined as events where the credit constraint binds and where this leads to an increase in net capital outflows that exceeds one standard deviation.

The relative weight of U agents in the planner's welfare criterion is tentatively set to $\delta = 1$, corresponding to a utilitarian social planner. We solve the regulated equilibrium for different values of γ between 0 and 1 in order to investigate the role of the size of the shadow economy for the effectiveness, desirability and welfare implications of financial regulation.

4.2 Borrowing Decisions

4.2.1 Zero measure of unregulated agents

Figure 5 plots the unregulated agents' bond decision rule for a mean realizaton of the tradable endowment shock and a low realization of the financial shock in the unregulated competitive equilibrium (CE), in the constrained efficient allocation (SP) and in the regulated equilibrium when unregulated agents exist with zero measure (i.e. when $\gamma = 0$). This is a useful case

[8] Standard values are around 0.8, but for these values we have experienced difficulties in having our Markov Perfect Equilibrium converge.

to consider, because it allows us to concentrate on the spillover effects from the planner's regulation of regulated agents to the borrowing decisions of unregulated, while abstracting from the effect of the presence of unregulated agents on the planner's behavior[9]. In order to display decision rules in two dimensions, we focus on the points in the state space where $b_R = b_U$.

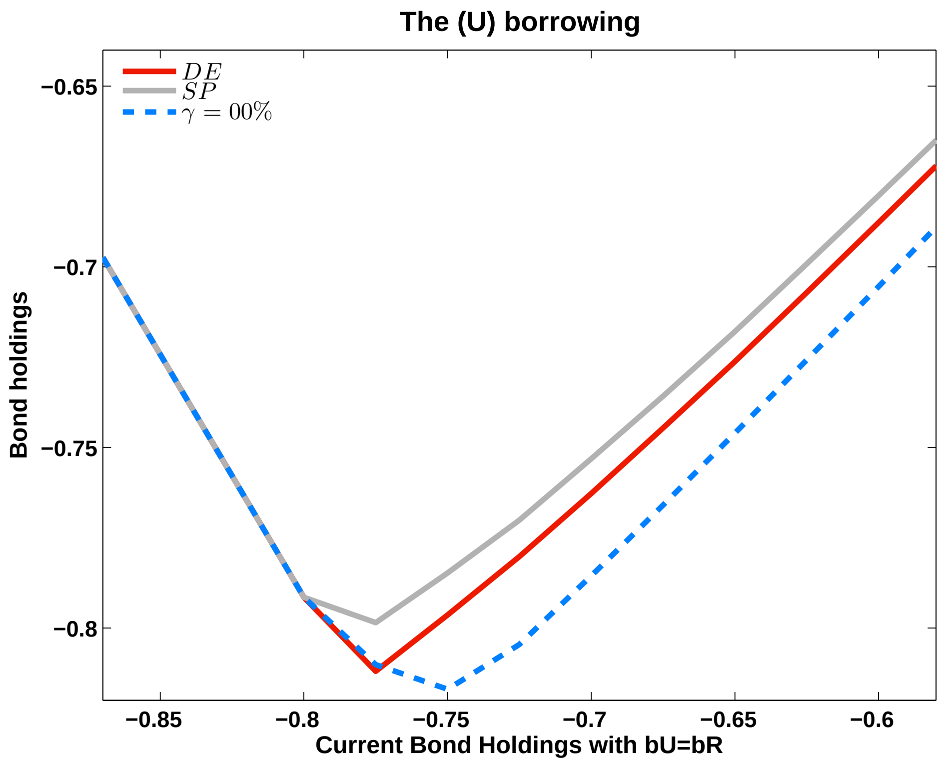

Figure 5: Bond decision rules of unregulated agents for mean y^T realization and κ^L, in states where $b_U = b_R$, for $\gamma = 0$.

The dark (red) solid line corresponds to the bond decision rule in the decentralized equilibrium. In the absence of the collateral constraint, the bond decision rule would be monotonically increasing in current bond holdings. The collateral constraint generates a kink in the policy function, to the right of which the agent is unconstrained and to the left of which the agent is constrained. In the unconstrained region, the bond policy function is increasing in current bond holdings. In the constrained region it is decreasing in current

[9]When unregulated agents exist with zero measure, neither do they enter into the planner's welfare criterion and nor do they affect aggregates. Therefore, they do not influence the planner's behavior. This means that the bond policy for regulated agents in a partially regulated equilibrium coincides with its counterpart in the fully regulated equilibrium.

bond holdings. This pattern is due to the endogeneity of the borrowing limit. When the constraint is binding, a lower level of current bond holdings induces a lower consumption level, which itself induces a lower price of nontradable goods. This lower price of nontradable goods tightens the constraint, and therefore requires a smaller amount of borrowing (or a larger bond position).

The light (grey) solid line corresponds to the bond decision rule in the constrained efficient allocation (i.e. hypothetical situation where planner controls borrowing choices of all agents). This decision rule approximately coincides with the decision rule of the decentralized equilibrium except in the region of the state space where the agent's collateral constraint is currently non-binding, but might bind with positive probability next period. In this region, the social planner chooses to set aside an extra amount of precautionary savings above and beyond the amount set aside by private agents, because the planner internalizes the financial amplification mechanism that takes place when the constraint is binding.

The dashed (blue) line corresponds to the bond decision rule of unregulated agents in the regulated equilibrium. The plot shows that unregulated agents respond to the regulation by increasing their borrowing level relative to the decentralized equilibrium in the precise region of the state space where the planner chooses to reduce regulated agents' borrowing relative to the decentralized equilibrium. In addition, the magnitude of this increase in borrowing is relatively important, as indicated by the larger distance between the dashed and dark solid lines than between the light solid and dark solid lines.

4.2.2 Positive measure of unregulated agents

When there is a non-zero measure of U agents, a two way interaction takes place between the regulated and unregulated spheres in the regulated equilibrium. U agents respond to the safer environment by borrowing more, and the planner responds to this extra borrowing by U agents by borrowing even less on the behalf of R agents. This is illustrated in Figures 6 and 7.

Figure 6 represents the unregulated agents' bond decision rule for a mean realizaton of the tradable endowment shock and a low realization of the financial shock in the unregulated competitive equilibrium (CE), in the constrained efficient allocation (SP) and in the regulated equilibrium for various measures of unregulated agents. As in Figure 5, the CE and SP lines represent the bond policy rules in the unregulated competitive equilibrium and constrained efficient allocation, respectively. The other lines are the bond policy rules in the regulated equilibrium. It is apparent that the spillover effects operate in the same direction for values of γ ranging from 0 to 0.4, and that size of the spillover effects, as measured by the distance between the decision rule in the competitive equilibrium and the regulated equilibrium, decreases with size of the shadow economy γ.

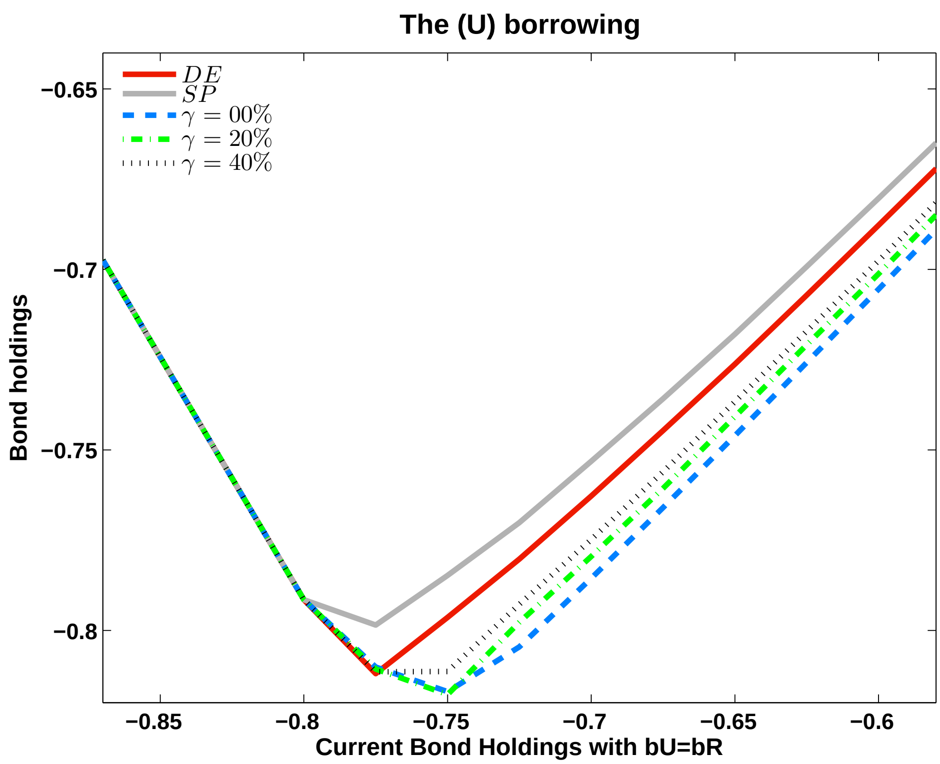

Figure 6: Bond decision rules of unregulated agents for mean y^T realization and κ^L, in states where $b_U = b_R$, for $\gamma \geq 0$.

Figure 7 represents the regulated agents' bond decision rule for a mean realizaton of the tradable endowment shock and a low realization of the financial shock in the unregulated competitive equilibrium (CE), in the constrained efficient allocation (SP) and in the regulated equilibrium for various measures of unregulated agents. The CE and SP lines represent the bond policy rules in the unregulated competitive equilibrium and constrained efficient allocation, respectively. The other lines are the bond policy rules in the regulated equilibrium. Here, the figure indicates that a larger size of the shadow economy (a larger γ) generates an incentive for the planner to deviate more from the decentralized equilibrium bond decision rules in the region of the state space where unregulated agents borrow more. In other words, the larger the shadow economy, the more the planner decreases regulated agents' borrowing.

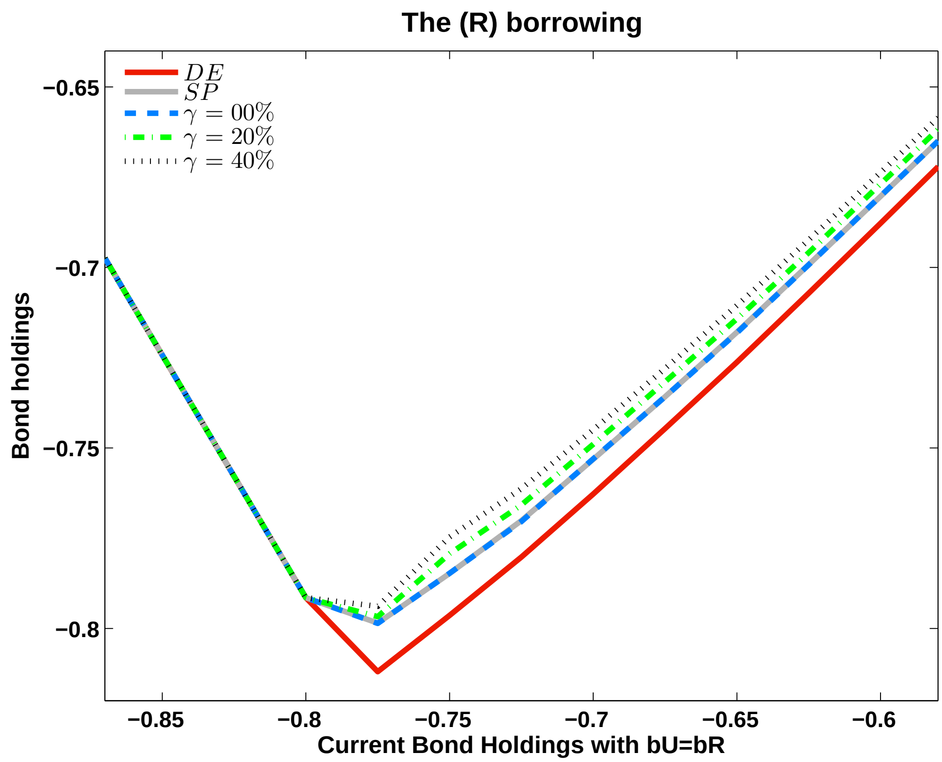

Figure 7: Bond decision rules of regulated agents for mean y^T realization and κ^L, in states where $b_U = b_R$, for $\gamma \geq 0$.

4.3 Magnitude of Capital Controls

Figure 8 shows that the average tax on borrowing decreases markedly with γ for $0 \leq \gamma \leq 0.6$.

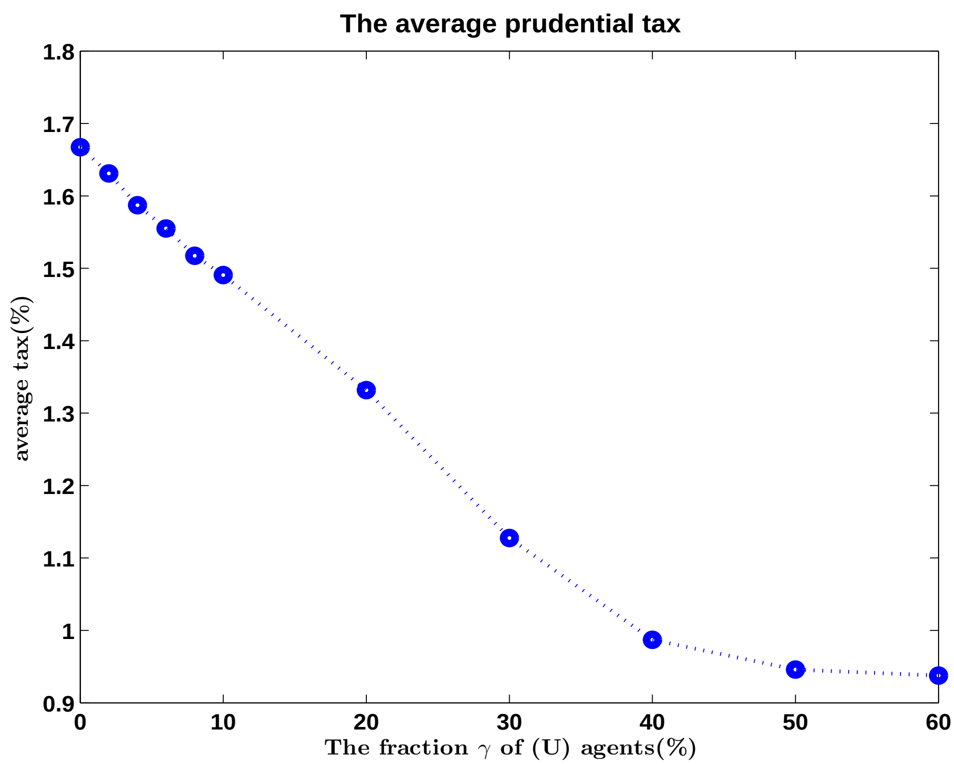

Figure 8: Average tax on borrowing for regulated agents.

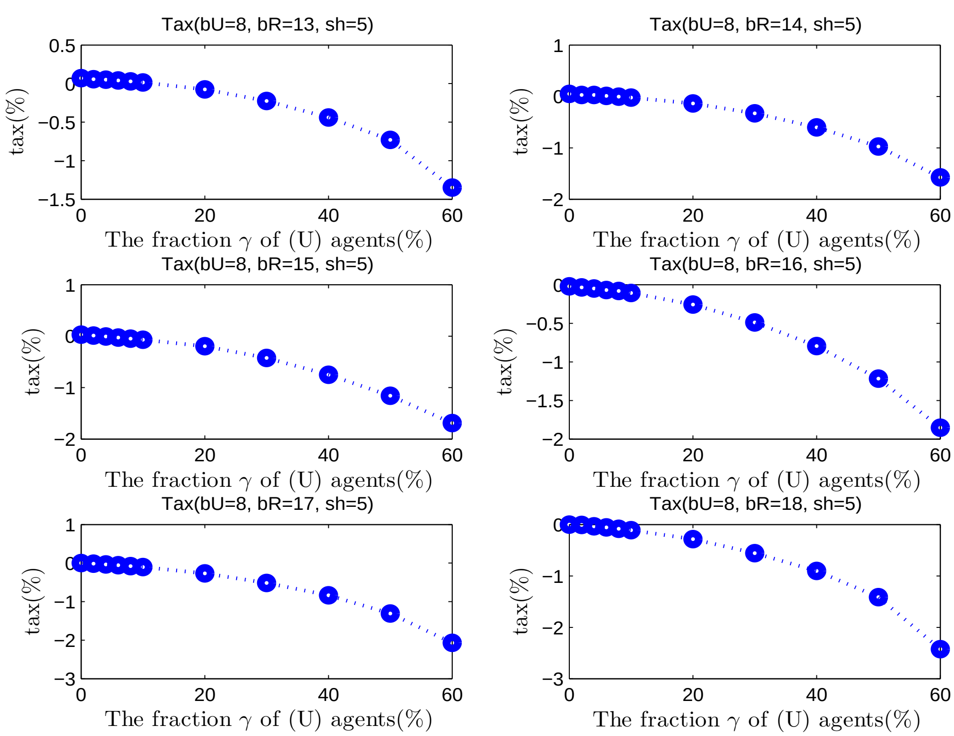

Figure 9: Examples of states where capital controls are decreasing in γ.

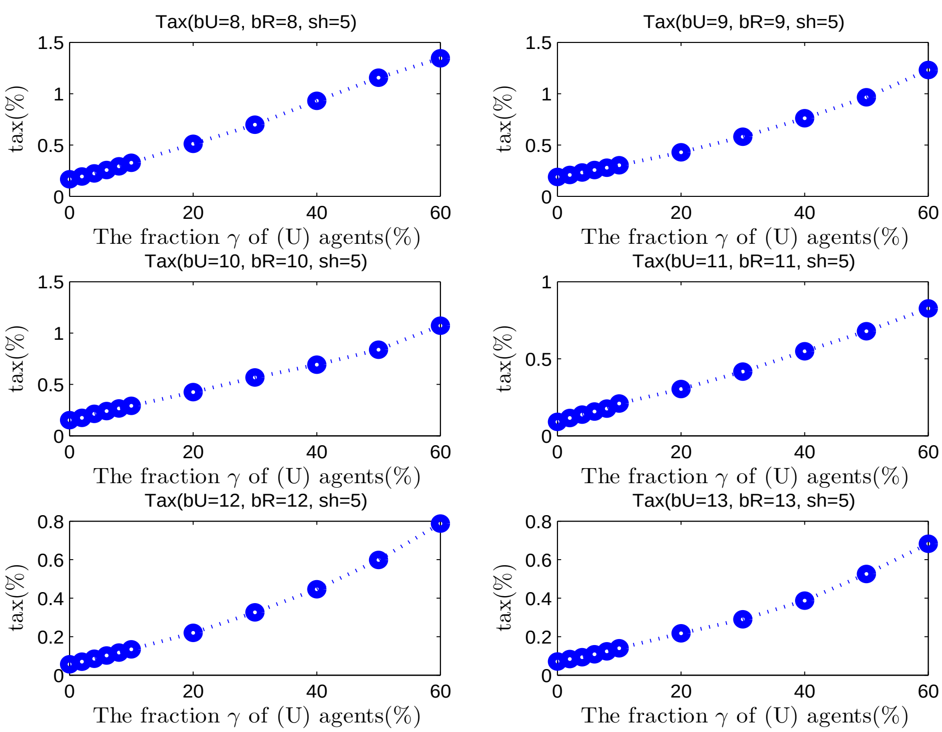

Figure 10: Examples of states where capital controls are increasing in γ.

4.4 Frequency and Severity of Crises

In this section, we show the extent to which leakages undermine the effectiveness of capital controls. In particular, we study how the probability and severity of sudden stops vary with the ability to enforce capital controls, as indexed by γ.

Figure 11 shows how the severity of financial crises change with the size of the leakages. When about half the population can avoid the controls, the probability of a sudden stop becomes very close to 5 percent, which is the probability of a sudden stop in the decentralized equilibrium.

We construct a comparable event analysis to show how the severity of sudden stops depend on γ in the following way. First, we simulate the decentralized equilibrium for a large number of periods, identify all the sudden stop episodes and construct nine-year event window events centered in the sudden stop. Second, we average the key variables across the window period for the decentralized equilibrium. Third, we feed the sequence of shocks and initial states —that characterize each sudden stop in the decentralized equilibrium— to the planner's problem for various values of γ. Finally, we average all the key variables across the window period. This experiment allows us to do a counterfactual analysis that highlights how differences in γ leads to different dynamics of sudden stops, despite having all the economies the same sequence of shocks and the same initial states.

As figures 12 and 13 show, higher leakages make the economy prone to more severe sudden stops, although the economy remains relatively more protected, even for values of γ close to one-half.

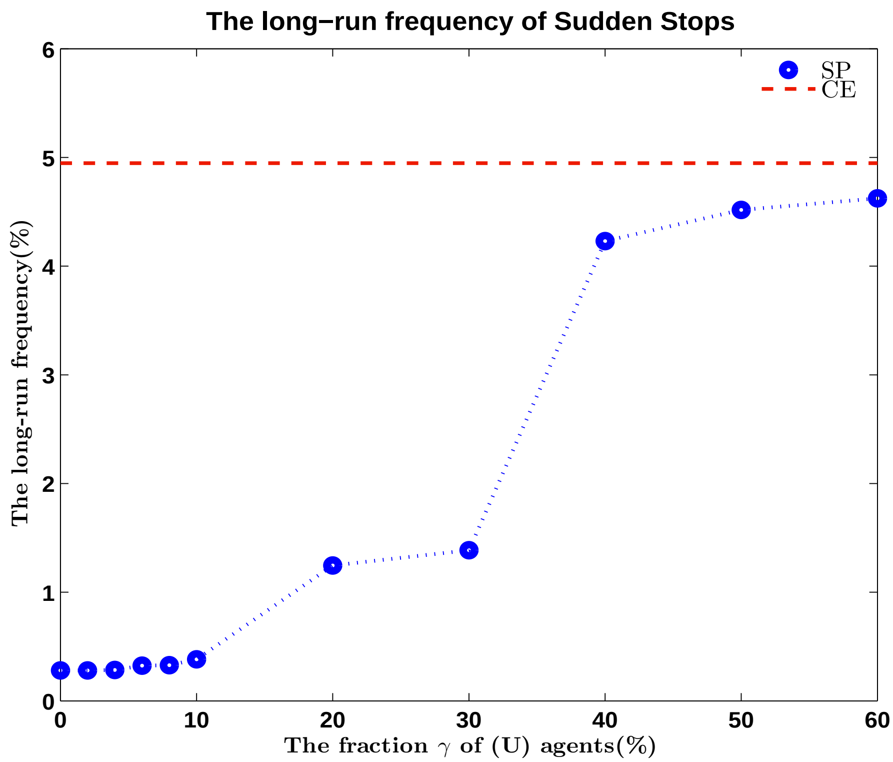

Figure 11: Long-run frequency of Sudden Stops.

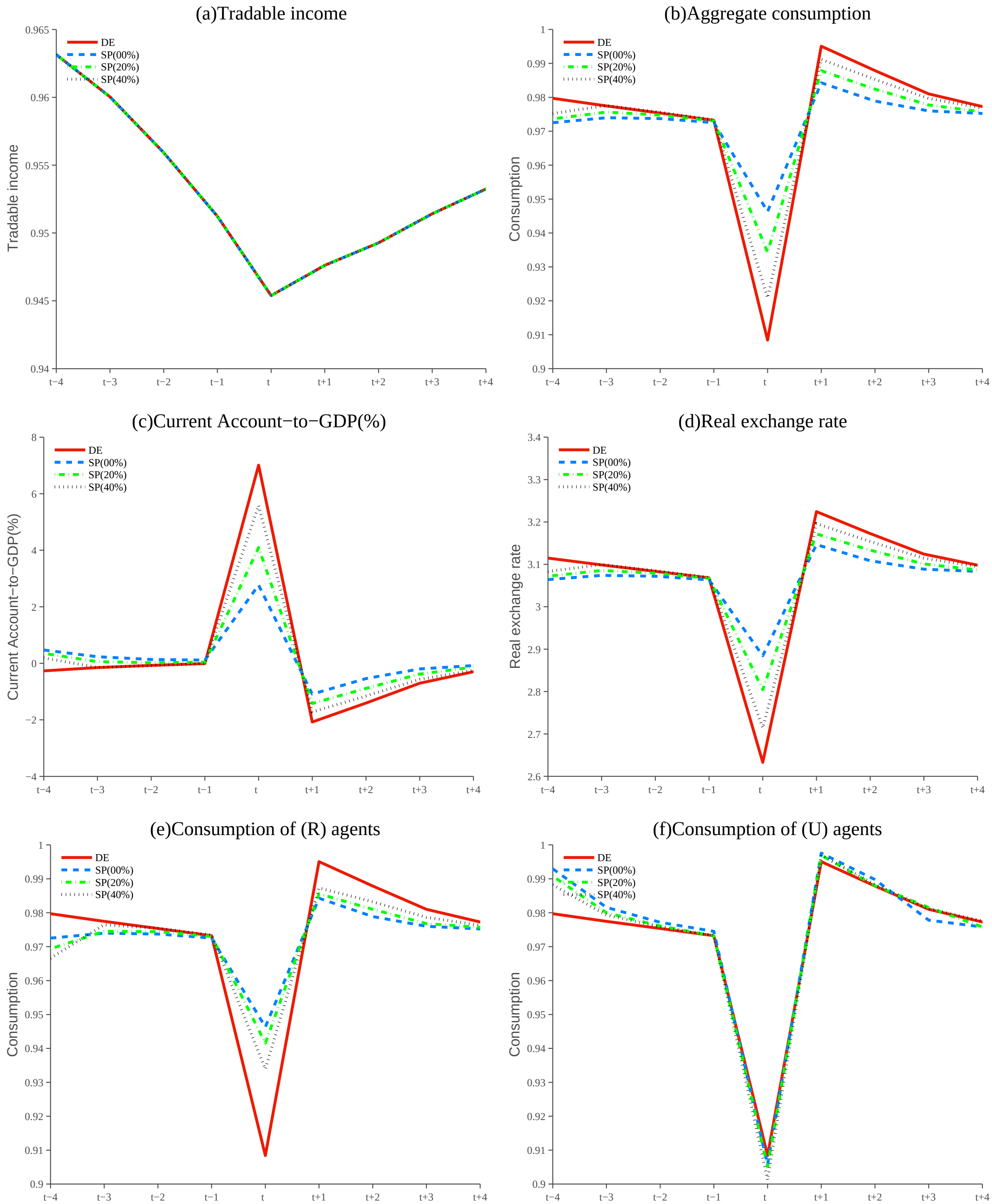

Figure 12: Event analysis.

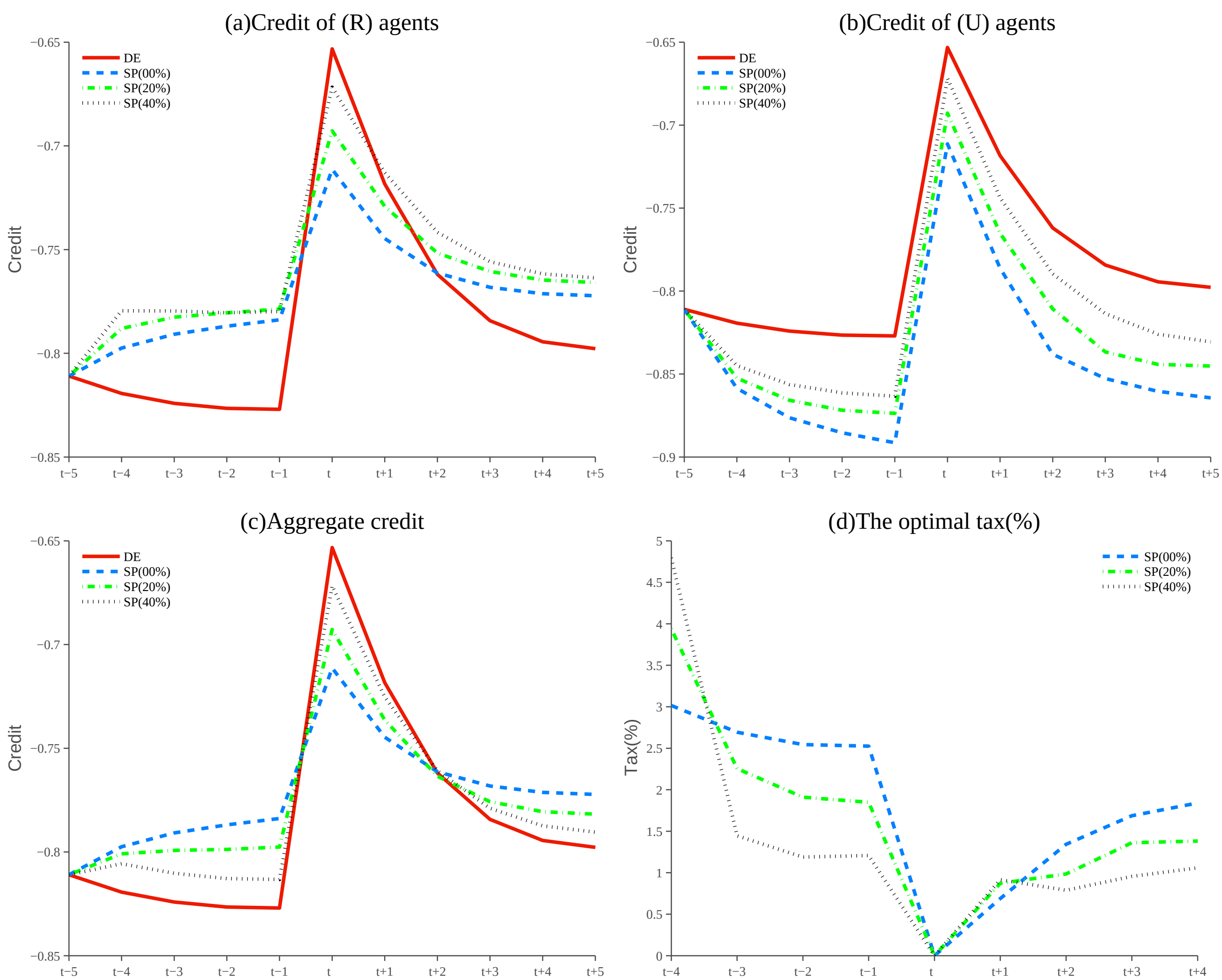

Figure 13: Event analysis (cont.).

4.5 Welfare Effects

We compute the welfare gains of financial regulation with a shadow economy as the proportional increase in consumption for all possible future histories in the decentralized equilibrium that would make households indifferent between remaining in the decentralized equilibrium and being in the regulated equilibrium.

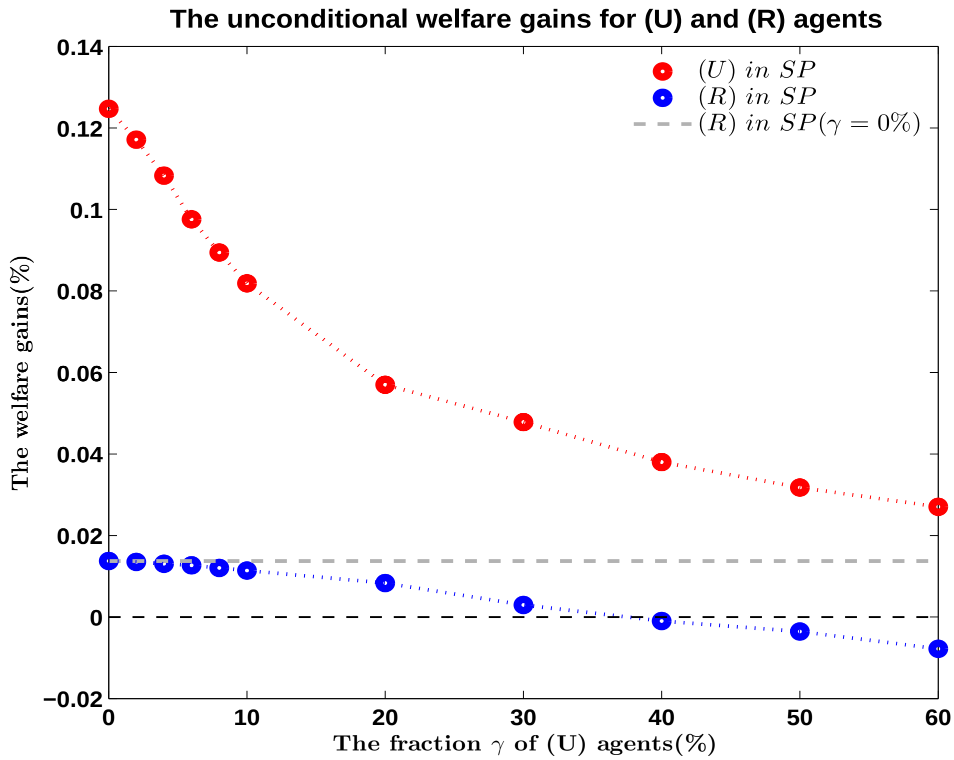

Figure 14: Welfare gains from capital controls in the presence of leakages for regulated (blue) and unregulated (red).

The welfare gains for the two types of agents are shown in Figure 14 for various sizes of the shadow economy (i.e. values of γ). The dotted line represents the welfare gain for any of the two types of agents in the constrained efficient allocation. The blue and red circles represent the respective welfare gains of being in a regulated equilibrium for R agents and U agents. Two main insights can be gained from this figure. First, and unsurprisingly, the welfare gains of being in the regulated equilibrium for both types of agents decrease with the size of the shadow economy for $\gamma < 1$. As the size of the shadow economy increases, the planner controls a smaller share of the economy and becomes less effective at correcting the inefficiency caused by pecuniary externalities and the endogenous collateral constraint. Further, as the size of the shadow economy increases, the planner's intervention generates increasing distortions, as the extra risk-taking behavior of the unregulated sector requires an increasingly precautionary behavior of the regulated sector. Second, unregulated agents benefit much more than regulated agents from the macroprudential intervention, and benefit the most when the size of the shadow economy is small. The intuition for this result is straightforward. The planner's intervention on the borrowing of regulated agents entails costs and benefits. The costs come in the form of lower consumption by R agents (due to lower borrowing) when the economy is in a state where it is exposed to the risk of a future crisis. The benefits comes in the form of a lower probability of occurrence of such crises. Unlike regulated agents who pay the cost in exchange for enjoying the benefits, unregulated agents enjoy the same benefits but without incurring any cost. The non-discriminatory character of the planner's financial crisis prevention policy makes it vulnerable to free-riding by unregulated agents.

5 Conclusion

We conducted an analysis of optimal capital flow management when capital controls leak. We characterize the optimal policy for different degrees of enforcement and show the extent to which leakages undermine the effectiveness of capital flow management. Our analysis indicates that while leakages create distortions that make capital controls undesirable, the planner may find optimal to tighten regulation on the regulated sphere to achieve higher stabilization effects. Overall, our findings indicate that there are important gains from capital controls despite the presence of leakages.

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A Proofs

Lemma 1

We consider each region in turn.

cc In this case equilibrium is given by the system (12), (13), (14), (15) and (16). This system is block recursive in a linear system in $C_1^T(s)$ and $p_1^N(s)$. Solving this linear system yields the following coefficients for $C_1^T(s)$: $\alpha_y^{cc} = (1+\kappa)/\left(1-\kappa\frac{1-\omega}{\omega}\right)$, $\alpha_U^{cc} = \gamma(1+r)/\left(1-\kappa\frac{1-\omega}{\omega}\right)$, $\alpha_R^{cc} = (1-\gamma)(1+r)/\left(1-\kappa\frac{1-\omega}{\omega}\right)$ and $\alpha_y^{cc} = 0$.

cu In this case equilibrium is given by the system (12) and (13) for $i = U$, (10) and (11) for $i = R$, (14), (15) and (16). This system is block recursive in a linear system in $C_1^T(s)$, $C_2^T(s)$, $p_1^N(s)$ and $p_1^N(s)$. Solving this linear system yields the following coefficients for $C_1^T(s)$: $\alpha_y^{cu} = \frac{\gamma(1+\kappa)+\frac{1-\gamma}{\omega}\frac{1}{1+\beta}}{\frac{1-\gamma}{\omega}+\gamma\left(1-\kappa\frac{1-\omega}{\omega}\right)}$, $\alpha_U^{cu} = \frac{\gamma(1+r)\frac{1}{1+\beta}\left(\beta+\frac{1-\gamma}{\omega}+\gamma\right)}{\frac{1-\gamma}{\omega}+\gamma\left(1-\kappa\frac{1-\omega}{\omega}\right)}$, $\alpha_R^{cu} = \frac{(1-\gamma)(1+r)\frac{1}{1+\beta}\left(\frac{1-\gamma}{\omega}+\gamma\right)}{\frac{1-\gamma}{\omega}+\gamma\left(1-\kappa\frac{1-\omega}{\omega}\right)}$, $\alpha_{\bar{y}}^{cu} = \frac{\frac{1-\gamma}{\omega}\frac{\beta}{1+\beta}}{\frac{1-\gamma}{\omega}+\gamma\left(1-\kappa\frac{1-\omega}{\omega}\right)}$.

uc In this case R agents are constrained, and equilibrium is given by the system (12) and (13) for $i = R$, (11) and (10) for $i = U$, (14), (15) and (16). This system is block recursive in a linear system in $C_1^T(s)$, $C_2^T(s)$, $p_1^N(s)$ and $p_1^N(s)$. Solving this linear system yields the following coefficients for $C_1^T(s)$: $\alpha_y^{uc} = \frac{(1-\gamma)(1+\kappa)+\gamma\frac{1}{\omega}\frac{1}{1+\beta}}{\frac{\gamma}{\omega}+(1-\gamma)\left(1-\kappa\frac{1-\omega}{\omega}\right)}$, $\alpha_U^{uc} = \frac{\gamma(1+r)\frac{1}{1+\beta}\left(\frac{\gamma}{\omega}+1-\gamma\right)}{\frac{\gamma}{\omega}+(1-\gamma)\left(1-\kappa\frac{1-\omega}{\omega}\right)}$, $\alpha_R^{uc} = \frac{(1-\gamma)(1+r)\frac{1}{1+\beta}\left(\beta+\frac{\gamma}{\omega}+1-\gamma\right)}{\frac{\gamma}{\omega}+(1-\gamma)\left(1-\kappa\frac{1-\omega}{\omega}\right)}$ and $\alpha_{\bar{y}}^{uc} = \frac{\gamma\frac{1}{\omega}\frac{\beta}{1+\beta}}{\frac{\gamma}{\omega}+(1-\gamma)\left(1-\kappa\frac{1-\omega}{\omega}\right)}$.

uu In this case equilibrium is given by the system (10), (11), (14), (15) and (16). This system is block recursive in a linear system in $C_1^T(s)$ and $C_2^T(s)$. Solving this linear system yields the following coefficients for $C_1^T(s)$: $\alpha_y^{uu} = 1/(1+\beta)$, $\alpha_U^{uu} = \gamma(1+r)/(1+\beta)$, $\alpha_R^{uu} = (1-\gamma)(1+r)/(1+\beta)$ and $\alpha_{y}^{uu} = \beta/(1+\beta)$.

Lemma 2

The proof of part 1. simply follows from an inspection of the expressions for the coefficients (see proof of Lemma 1 above), noting that Assumption 2 implies $0 < 1 - \kappa\frac{1-\omega}{\omega} < 1$.

The proof of part 2. follows directly from the observations that (1) $\alpha_y^{uu} < 1 < \alpha_y^{cc}$; (2) for $\gamma = 0$, $\alpha_y^{cc} = \alpha_y^{uc}$ and $\alpha_y^{cu} = \alpha_y^{uu}$; (3) for $\gamma = 1$, $\alpha_y^{cc} = \alpha_y^{cu}$ and $\alpha_y^{uc} = \alpha_y^{uu}$; (4) $\partial\alpha_y^{cu}/\partial\gamma > 0$ and $\partial\alpha_y^{uc}/\partial\gamma < 0$; and (5) for $\gamma = 0.5$, $\alpha_y^{cu} = \alpha_y^{uc}$.

For part 3. we observe that if $\gamma = 0$, then $\alpha_U^{uu} = \alpha_U^{cu} = \alpha_U^{uc} = \alpha_U^{cc} = 0$, and that if $\gamma > 0$ assuming that $\alpha_U^{uu} \geq \alpha_U^{cu}$, $\alpha_U^{uu} \geq \alpha_U^{uc}$, $\alpha_U^{cu} \geq \alpha_U^{cc}$ and $\alpha_U^{uc} \geq \alpha_U^{cc}$ individually lead to contradictions.

Similarly, for part 4. we observe that if $\gamma = 1$, then $\alpha_R^{uu} = \alpha_R^{cu} = \alpha_R^{uc} = \alpha_R^{cc} = 0$, and that if $\gamma < 1$ assuming that $\alpha_R^{uu} \geq \alpha_R^{cu}$, $\alpha_R^{uu} \geq \alpha_R^{uc}$, $\alpha_R^{cu} \geq \alpha_R^{cc}$ and $\alpha_R^{uc} \geq \alpha_R^{cc}$ individually leads to contradictions.

Lemma 3

Let us define the thresholds $a_x \equiv \min(0, \tilde{a}_x)$,

$$\tilde{a}_x \equiv -\omega \frac{1-\kappa \frac{1-\omega}{\omega}}{\theta} \max\left(B_{R1}, B_{U1}\right) - \left(1-\omega\right)\left(1+r\right)\left[\gamma B_{U1} + \left(1-\gamma\right) B_{R1}\right] + \beta \frac{1-\kappa \frac{1-\omega}{\omega}}{\theta} \bar{y}^T,$$

and

$$b_x \equiv -\frac{1}{\theta} \min\left(B_{R1}, B_{U1}\right) - \frac{\frac{1-\omega}{\omega}\kappa\left(1+r\right)}{\theta}\left[\gamma B_{U1} + \left(1-\gamma\right) B_{R1}\right] + \frac{\beta\left(1-\frac{1-\omega}{\omega}\kappa\right)}{\theta} \bar{y}^T,$$

where

$$\theta \equiv \left(1+\kappa\right)\beta + \kappa \frac{1}{\omega}.$$

It can be easily verified that

1. $b_{i2}^{unc}(B_{i1}; y_1^T(s), B_{U1}, B_{R1}, cc) < b_{i2}^{con}(B_{i1}; y_1^T(s), B_{U1}, B_{R1}, cc)$ for $i = U, R$ is equivalent $y_1^T(s) < a_x$,

2. $b_{i2}^{unc}(B_{i1}; y_1^T(s), B_{U1}, B_{R1}, uu) \geq b_{i2}^{con}(B_{i1}; y_1^T(s), B_{U1}, B_{R1}, uu)$ for $i = U, R$ is equivalent to $y_1^T(s) \geq b_x$,

3. $b_{U2}^{unc}(B_{U1}; y_1^T(s), B_{U1}, B_{R1}, cu) < b_{U2}^{con}(B_{U1}; y_1^T(s), B_{U1}, B_{R1}, cu)$ and $b_{R2}^{unc}(B_{R1}; y_1^T(s), B_{U1}, B_{R1}, cu) \geq b_{R2}^{con}(B_{R1}; y_1^T(s), B_{U1}, B_{R1}, cu)$ is equivalent to $a_x \leq y_1^T(s) < b_x$ iif $B_{U1} < B_{R1}$, and

4. $b_{U2}^{unc}(B_{U1}; y_1^T(s), B_{U1}, B_{R1}, uc) \geq b_{U2}^{con}(B_{U1}; y_1^T(s), B_{U1}, B_{R1}, uc)$ and $b_{R2}^{unc}(B_{R1}; y_1^T(s), B_{U1}, B_{R1}, uc) < b_{R2}^{con}(B_{R1}; y_1^T(s), B_{U1}, B_{R1}, uc)$ is equivalent to $a_x \leq y_1^T(s) < b_x$ iif $B_{U1} > B_{R1}$.

Lemma 4

The proof simply follows from the fact that a_x and b_x are non-increasing in B_{U1} and B_{R1} (see expressions in proof of Lemma 3).

Lemma 5

Existence follows from standard arguments. For uniqueness, note that the private Euler equation (21) is given by $g(b_1^{DE}) = 0$ where

$$\begin{aligned} g(b) &\equiv 1 - \int_{\underline{y}}^{a_x} \frac{\omega}{\alpha_y^{cc} y_1^T(s) + (\alpha_U^{cc} + \alpha_U^{cc}) b} dF(y_1^T(s)) - \int_{a_x}^{\infty} \frac{\omega}{\alpha_y^{uu} y_1^T(s) + (\alpha_U^{uu} + \alpha_U^{uu}) b + \alpha_{\bar{y}}^{uu} \bar{y}^T} dF(y_1^T(s)) \\ &= 1 - \int_{\underline{y}}^{a_x} \frac{\omega \left(1 - \kappa \frac{1-\omega}{\omega}\right)}{(1+\kappa) y_1^T(s) + (1+r) b} dF(y_1^T(s)) - \int_{a_x}^{\infty} \frac{\omega(1+\beta)}{y_1^T(s) + (1+r) b + \beta \bar{y}^T} dF(y_1^T(s)) \end{aligned}$$

$g(b)$ is continuous and satisfies $g'(b) > 0$ for the b range over which consumption is positive (where the existing equilibrium lies). It follows that there exists a single b_1^{DE} for which $g(b_1^{DE}) = 0$.

Proposition 1

Let us consider first the best response of U agents. Existence again follows from standard arguments. For uniqueness and decreasingness, we note that we can write h_U as

$$h_U(b_{U1}, b_{R1}) = 1 - \int_{\underline{y}}^{a_x} \frac{\omega}{c_{U1}^T} dF(y_1^T(s)) - \int_{a_x}^{b_x} \frac{\omega}{c_{U1}^T} dF(y_1^T(s)) - \int_{b_x}^{\infty} \frac{\omega}{c_{U1}^T} dF(y_1^T(s))$$

where the arguments of c_{U1}^T, a_x and b_x are omitted for space reasons. According to the implicit function theorem we have $\frac{db_{U1}}{db_{R1}} = -\frac{\partial h_U / \partial b_{R1}}{\partial h_U / \partial b_{U1}}$, with

$$\begin{aligned} \frac{\partial h_U}{\partial b_{R1}} &= \int_{\underline{y}}^{a_x} \frac{\omega}{(c_{U1}^T)^2} \frac{\partial c_{U1}^T}{\partial b_{R1}} dF(y_1^T(s)) + \int_{a_x}^{b_x} \frac{\omega}{(c_{U1}^T)^2} \frac{\partial c_{U1}^T}{\partial b_{R1}} dF(y_1^T(s)) + \int_{b_x}^{\infty} \frac{\omega}{(c_{U1}^T)^2} \frac{\partial c_{U1}^T}{\partial b_{R1}} dF(y_1^T(s)) \\ \frac{\partial h_U}{\partial b_{U1}} &= \int_{\underline{y}}^{a_x} \frac{\omega}{(c_{U1}^T)^2} \frac{\partial c_{U1}^T}{\partial b_{U1}} dF(y_1^T(s)) + \int_{a_x}^{b_x} \frac{\omega}{(c_{U1}^T)^2} \frac{\partial c_{U1}^T}{\partial b_{U1}} dF(y_1^T(s)) + \int_{b_x}^{\infty} \frac{\omega}{(c_{U1}^T)^2} \frac{\partial c_{U1}^T}{\partial b_{U1}} dF(y_1^T(s)) \end{aligned}$$

where we used the fact that terms containing derivatives of a_x and b_x drop out due to the continuity of c^T_{U1} across regions.[10] The derivatives in the various regions are given by

$$
\begin{aligned}
cc: \quad & \frac{\partial c^T_{U1}}{\partial b_{R1}} = \frac{\omega(1+r)(1+\kappa)\frac{1-\omega}{\omega}(1-\gamma)}{1-\kappa\frac{1-\omega}{\omega}}; \qquad \frac{\partial c^T_{U1}}{\partial b_{R1}} = \omega(1+r)\left[1+\frac{(1+\kappa)\frac{1-\omega}{\omega}\gamma}{1-\kappa\frac{1-\omega}{\omega}}\right]. \\
cu: \quad & \frac{\partial c^T_{U1}}{\partial b_{R1}} = \frac{\omega(1+r)(1+\kappa)\frac{1-\omega}{\omega}(1-\gamma)\left(\frac{1-\gamma}{\omega}+\gamma\right)}{(1+\beta)\left[\frac{1-\gamma}{\omega}+\gamma\left(1-\kappa\frac{1-\omega}{\omega}\right)\right]}; \\
& \frac{\partial c^T_{U1}}{\partial b_{U1}} = \omega(1+r)\left[1+\frac{(1+\kappa)\frac{1-\omega}{\omega}\gamma\left(\beta+\frac{1-\gamma}{\omega}+\gamma\right)}{(1+\beta)\left[\frac{1-\gamma}{\omega}+\gamma\left(1-\kappa\frac{1-\omega}{\omega}\right)\right]}\right] \\
uc: \quad & \frac{\partial c^T_{U1}}{\partial b_{R1}} = \frac{\omega(1+r)\frac{1-\omega}{\omega}(1-\gamma)}{1+\beta}; \qquad \frac{\partial c^T_{U1}}{\partial b_{U1}} = \frac{\omega(1+r)\left[1+\frac{1-\omega}{\omega}\gamma\right]}{1+\beta} \\
uu: \quad & \frac{\partial c^T_{U1}}{\partial b_{R1}} = \frac{\omega(1+r)\frac{1-\omega}{\omega}(1-\gamma)}{(1+\beta)}; \qquad \frac{\partial c^T_{U1}}{\partial b_{U1}} = \frac{\omega(1+r)\left[1+\frac{1-\omega}{\omega}\gamma\right]}{(1+\beta)}
\end{aligned}
$$

Therefore, in every region the term $\frac{\partial c^T_{U1}}{\partial b_{R1}}$ is non-negative (strictly positive if and only if $\gamma < 1$) and the term $\frac{\partial c^T_{U1}}{\partial b_{U1}}$ is strictly positive. It follows that for a given b_R, $\partial h_U/\partial b_{U1} > 0$ in the range of b_U for which c^T_{U1} is always positive. The best response of U agents to b_R is therefore unique, and can be written as $b_U = \phi_U(b_R)$. Further, in the range of b_U and b_R for which c^T_{U1} is always positive, we have $\partial h_U/\partial b_{R1} \geq 0$, with $>$ if and only if $\gamma < 1$. It follows that $\phi'_U(b_R) \leq 0$, with $<$ if and only if $\gamma < 1$.

For the best response of R agents, the proof is analogous and involves the derivatives of c^T_{R1} in the four regions. The best response of R agents to b_U is unique and decreasing, strictly if and only if $\gamma > 0$.

Proposition 2

The proof relies on the relationship between the slopes $\phi'_U(\cdot)$ and $1/\phi'_R(\cdot;\tau)$ at $\tau = 0$ and on the sign of the partial derivative $\partial\phi'_R(b_U;\cdot)/\partial\tau$.

The slopes are given by $\phi'_U(\cdot) = -\frac{\partial h_U/\partial b_{R1}}{\partial h_U/\partial b_{U1}}$ and $\phi'_R(\cdot;\tau) = -\frac{\partial h_R/\partial b_{U1}}{\partial h_R/\partial b_{R1}}$. At $\tau = 0$, $b^*_{U1} = b^*_{R1} = b^{DE}_1$, and therefore $a_x = b_x$ and $c^T_{U1} = c^T_{R1} = C^T_1$ in any date 1 state. Defining

[10] Note that if $b_{U1} < b_{R1}$ the relevant intermediate region between a_x and b_x is $x = cu$, while if $b_{U1} > b_{R1}$ the relevant region is $x = uc$. If $b_{U1} = b_{R1}$ then $a_x = b_x$ so this intermediate region drops out.

$\eta_{cc} = \int_{\underline{y}}^{a_x} \frac{1}{\left(C_1^T\right)^2} dF\left(y_1^T(s)\right)$ and $\eta_{uu} = \int_{a_x}^{\infty} \frac{1}{\left(C_1^T\right)^2} dF\left(y_1^T(s)\right)$, the slopes are given by

$$\phi_U'(b_1^{DE}) = -\frac{\eta_{cc}(1-\gamma)\frac{(1+\kappa)\frac{1-\omega}{\omega}}{1-\kappa\frac{1-\omega}{\omega}} + \eta_{uu}(1-\gamma)\frac{1}{1+\beta}\frac{1-\omega}{\omega}}{\eta_{cc}\left[1+\gamma\frac{(1+\kappa)\frac{1-\omega}{\omega}}{1-\kappa\frac{1-\omega}{\omega}}\right] + \eta_{uu}\frac{1}{1+\beta}\left[1+\gamma\frac{1-\omega}{\omega}\right]} \tag{52}$$

and

$$1/\phi_U'(b_1^{DE};0) = -\frac{\eta_{cc}\left[1+(1-\gamma)\frac{(1+\kappa)\frac{1-\omega}{\omega}}{1-\kappa\frac{1-\omega}{\omega}}\right] + \eta_{uu}\frac{1}{1+\beta}\left[1+(1-\gamma)\frac{1-\omega}{\omega}\right]}{\eta_{cc}\gamma\frac{(1+\kappa)\frac{1-\omega}{\omega}}{1-\kappa\frac{1-\omega}{\omega}} + \eta_{uu}\gamma\frac{1}{1+\beta}\frac{1-\omega}{\omega}} \tag{53}$$

For any value of γ, the numerator in (52) is smaller than the one in (53), and the denominator in (52) is larger than the one in (53). It follows that $|\phi_U'(b_1^{DE})| < |1/\phi_U'(b_1^{DE};0)|$ and therefore $\phi_U'(b_1^{DE}) > 1/\phi_U'(b_1^{DE};0)$.

The partial derivative $\partial\phi_R'(b_U;)/\partial\tau$ is given by

$$\frac{\partial\phi_R'(b_U;\cdot)}{\partial\tau} = -\frac{\partial h_R/\partial\tau}{\partial h_R/\partial b_R} = -\frac{-1/(1+\tau)^2}{\partial h_R/\partial b_R} > 0$$

since $\partial h_R/\partial b_R > 0$.

In the (b_{U1}, b_{R1}) space, the curve $\phi_R(b_U;\tau)$ crosses the curve $\phi_U(b_R)$ from above at $(b_{U1}, b_{R1}) = (b_1^{DE}, b_1^{DE})$, and it shifts to the right when τ rises. Hence, near (b_1^{DE}, b_1^{DE}), a rise in τ results in a downward movement of (b_{U1}^*, b_{R1}^*) along the downward sloping $\phi_U(b_R)$ curve. It follows that when τ is small, b_{U1}^* is decreasing and b_{R1}^* is increasing in τ.

Proposition 3

The proof is simply based on the derivatives of the two sets of agents' utility function evaluated at the unregulated decentralized equilibrium. The derivatives are given by

$$\begin{aligned}
\frac{dU_U}{db_{R1}} &= \frac{\partial U_U}{\partial b_{R1}} + \frac{\partial U_U}{\partial b_{U1}}\frac{\partial b_{U1}}{\partial b_{R1}} \\
&= \mathbb{E}_0\left[\sum_{t=1}^{2}\beta^t\frac{\omega}{c_{Ut}^T}\left(\bar{y}^N - c_{Ut}^N\right)\frac{\partial p_t^N}{\partial b_{R1}} + \beta\mu_{U1}\kappa\bar{y}^N\frac{\partial p_1^N}{b_{R1}}\right] \\
&\quad + \left\{-1 + \mathbb{E}_0\left[\frac{\omega}{c_{U1}^T} + \sum_{t=1}^{2}\beta^t\frac{\omega}{c_{Ut}^T}\left(\bar{y}^N - c_{Ut}^N\right)\frac{\partial p_t^N}{\partial b_{U1}} + \beta\mu_{U1}\kappa\bar{y}^N\frac{\partial p_1^N}{\partial b_{U1}}\right]\right\}\frac{\partial b_{U1}}{\partial b_{R1}}
\end{aligned}$$

and

$$
\begin{aligned}
\frac{dU_R}{db_{R1}} &= \frac{\partial U_R}{\partial b_{R1}} + \frac{\partial U_R}{\partial b_{U1}} \frac{\partial b_{U1}}{\partial b_{R1}} \\
&= -1 + \mathbb{E}_0 \left[\frac{\omega}{c^T_{R1}} + \sum_{t=1}^{2} \beta^t \frac{\omega}{c^T_{Rt}} \left(\bar{y}^N - c^N_{Rt} \right) \frac{\partial p^N_t}{\partial b_{R1}} + \beta \mu_{R1} \kappa \bar{y}^N \frac{\partial p^N_1}{\partial b_{R1}} \right] \\
&\quad + \mathbb{E}_0 \left[\sum_{t=1}^{2} \beta^t \frac{\omega}{c^T_{Rt}} \left(\bar{y}^N - c^N_{Rt} \right) \frac{\partial p^N_t}{\partial b_{U1}} + \beta \mu_{R1} \kappa \bar{y}^N \frac{\partial p^N_1}{b_{U1}} \right] \frac{\partial b_{U1}}{\partial b_{R1}}
\end{aligned}
$$

At the unregulated decentralized equilibrium, we have $c^N_{Ut} = c^N_{Rt} = \bar{y}^N$ for $t = 1, 2$ and $1 = \mathbb{E}_0 \left[\frac{\omega}{c_{i1}} \right]$ for $i \in \{U, R\}$, so that the derivatives are given by

$$
\frac{dU_i}{db_{R1}} = \beta \mathbb{E}_0 \left[\mu_{i1} \kappa \bar{y}^N \left(\frac{\partial p^N_1}{\partial b_{R1}} + \frac{\partial p^N_1}{\partial b_{U1}} \frac{\partial b_{U1}}{\partial b_{R1}} \right) \right]
$$

for $i \in \{U, R\}$. Evaluated at $(b_{U1}, b_{R1}) = (b^{DE}_1, b^{DE}_1)$, the derivative $\frac{dp^N_1}{db_{R1}} \equiv \frac{\partial p^N_1}{\partial b_{R1}} + \frac{\partial p^N_1}{\partial b_{U1}} \frac{\partial b_{U1}}{\partial b_{R1}}$ is given by

$$
\begin{aligned}
\frac{dp^N_1}{db_{R1}} &= \frac{1-\omega}{\omega} \left[\frac{(1-\gamma)(1+r)}{1-\kappa\frac{1-\omega}{\omega}} + \frac{\gamma(1+r)}{1-\kappa\frac{1-\omega}{\omega}} \phi'_U(b^{DE}_1) \right] \\
&= \frac{1-\omega}{\omega} \frac{(1-\gamma)(1+r)}{1-\kappa\frac{1-\omega}{\omega}} \left[\frac{\eta_{cc} + \eta_{uu}\frac{1}{1+\beta}}{\eta_{cc}\left[1+\gamma\frac{(1+\kappa)\frac{1-\omega}{\omega}}{1-\kappa\frac{1-\omega}{\omega}}\right] + \eta_{uu}\frac{1}{1+\beta}\left[1+\gamma\frac{1-\omega}{\omega}\right]} \right] > 0
\end{aligned}
$$

in region cc and by

$$
\begin{aligned}
\frac{dp^N_1}{db_{R1}} &= \frac{1-\omega}{\omega} \left[\frac{(1-\gamma)(1+r)}{1+\beta} + \frac{\gamma(1+r)}{1+\beta} \phi'_U(b^{DE}_1) \right] \\
&= \frac{1-\omega}{\omega} \frac{(1-\gamma)(1+r)}{1+\beta} \left[\frac{\eta_{cc} + \eta_{uu}\frac{1}{1+\beta}}{\eta_{cc}\left[1+\gamma\frac{(1+\kappa)\frac{1-\omega}{\omega}}{1-\kappa\frac{1-\omega}{\omega}}\right] + \eta_{uu}\frac{1}{1+\beta}\left[1+\gamma\frac{1-\omega}{\omega}\right]} \right] > 0
\end{aligned}
$$

in region uu.[11] Since for $(b_{U1}, b_{R1}) = (b^{DE}_1, b^{DE}_1)$, the only two relevant regions are cc and uu (because $a_x = b_x$), it must be that $\frac{dU_i}{db_{R1}} > 0$ if and only if $\mu_{U1} = \mu_{R1} > 0$ with non-zero probability at date 1.

[11] η_{cc} and η_{uu} were defined in the proof of Proposition 2.

Proposition 4

The proof of the "if" part is by construction. Assume that the tax is zero. $\tau = 0$ implies that $(b_{U1}, b_{R1}) = (b_1^{DE}, b_1^{DE})$, which implies symmetric allocations in all states of the world at date 1 and 2: $c_{Ut}^T = c_{Rt}^T$ and $c_{Ut}^N = c_{Rt}^N$ for $t = 1, 2$. The optimal tax expression (28) then implies

$$\tau = \frac{\beta E_0 \left[\left(\mu_{R1} + \frac{\gamma}{1-\gamma} \mu_{U1} \right) \kappa \bar{y}^N \frac{dp_1^N}{db_{R1}} \right]}{E_0 \left[\frac{\omega}{c_{R1}^T} \right]} = 0$$

since $\mu_{R1} = \mu_{U1} = 0$ in all states of the world. $\tau = 0$ is therefore indeed optimal.

The proof of the "only if" part is by contradiction. Assume that the tax is zero and that credit constraint binds, i.e. $\mu_{R1} > 0$ and/or $\mu_{U1} > 0$, in some states of the world in the decentralized equilibrium. $\tau = 0$ implies that $(b_{U1}, b_{R1}) = (b_1^{DE}, b_1^{DE})$, which induces symmetric allocations in all states of the world at date 1 and 2: $c_{Ut}^T = c_{Rt}^T$ and $c_{Ut}^N = c_{Rt}^N$ for $t = 1, 2$. The optimal tax expression (28) then implies

$$\tau = \frac{\beta \mathbb{E}_0 \left[\left(\mu_{R1} + \frac{\gamma}{1-\gamma} \mu_{U1} \right) \kappa \bar{y}^N \frac{dp_1^N}{db_{R1}} \right]}{\mathbb{E}_0 \left[\frac{\omega}{c_{R1}^T} \right]} \tag{54}$$

The proof of Proposition 3 established that when evaluated at $(b_{U1}, b_{R1}) = (b_1^{DE}, b_1^{DE})$, the derivative $\frac{dp_1^N}{db_{R1}} > 0$ in any state of the world. This, together with (54) and the assumption that $\mu_{R1} > 0$ and/or $\mu_{U1} > 0$ in some states, implies $\tau > 0$, a contradiction.

B Equivalence between planning problems

Consider the following problem:

Problem B.1 (Planner's problem with tax instrument)**.**

$$\max_{\{c_{i0}^T, b_{i1}, c_{i1}^T(s), c_{i2}^T(s), c_{i1}^N(s), c_{i2}^N(s), b_{i2}(s)\}_{i \in \{U,R\}}, \tau, T, p_1^N, p_2^N} \gamma U_U + (1-\gamma) U_R \tag{55}$$

subject to

$$1 = \beta(1+r)(1+\tau)E_0\left[\frac{\omega}{c^T_{R1}(s)}\right] \tag{56}$$

$$c^T_{R1}(s) + p^N_1(s)c^N_{R1}(s) + b_{R2}(s) = (1+r)(1+\tau)b_{R1} + y^T_1(s) + p^N_1(s)\bar{y}^N + T \tag{57}$$

$$T = -\tau b_{R1} \tag{58}$$

and (1), (2), (4), (6), (8) and (9) for $i \in \{U, R\}$, (7) for $i = U$, and (14).

We observe that after combining (58) with (57), the tax only appears in the private Euler equation (56). The allocations and prices that solve Problem B.1 are therefore identical to the ones that solve Problem 2.1 where the planner chooses allocations and prices directly subject to implementability constraints.