Macroprudential Regulation Versus Mopping Up After the Crash*

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October 2012

Abstract

This paper compares ex-ante policy measures (such as macroprudential restrictions on leverage) and ex-post policy interventions (such as bailouts) to respond to systemic risk, i.e. to the danger that an economy may experience financial feedback loops in which falling asset prices, declining net worth and tightening financial constraints reinforce each other. Ex-post policy measures are better targeted, since they are taken only once a crisis has materialized, but they aggravate the over-investment problem ex ante and introduce a time consistency problem. Ex-ante policy measures are more blunt, but they can both mitigate the pecuniary externalities that arise during crises and resolve the time consistency problem in ex-post policies. We find that except in limit cases, it is optimal to respond to systemic risk by using a mix of both ex-ante measures and ex-post interventions, and characterize the optimal policy mix. Furthermore, limiting bailouts to the revenue accumulated in a bailout fund reduces welfare.

JEL Codes: E44, H23
Keywords: financial crises, systemic risk, financial amplification macroprudential regulation, bailouts

1 Introduction

A growing literature has analyzed the role of macroprudential regulation in models of financial crises that are based on financial amplification, in which the economy

*The authors would like to thank the *Fondation Banque de France* for financial support. We would like to thank Philippe Bacchetta, Arnoud Boot, Emmanuel Farhi, Thomas Hintermaier, Alberto Martin, Jeremy Stein, Javier Suarez, Lars Svensson and Iván Werning as well as participants of the NBER 2012 Summer Institute, the Banco de Portugal Conference on Financial Intermediation, the International Conference on Macroeconomics and Monetary Policy at NES/HSE and of seminars at the Banque de France, Bocconi, CEU and Konstanz for helpful comments and discussions. We acknowledge excellent research assistance provided by Jonathan Kreamer and Elif Ture.

experiences a feedback loop between declining asset prices and tightening financial constraints. As pointed out by Gromb and Vayanos (2002), Lorenzoni (2008) and Jeanne and Korinek (2010b), financial amplification effects involve pecuniary externalities because atomistic agents do not internalize that their individual actions lead to relative price movements that reinforce shocks in the aggregate. This argument has been used by policymakers to make the case for so-called *macroprudential regulation* (see e.g. Borio, 2003; Bank of England, 2009; Blanchard et al., 2010).

However, there has been an intense debate about the relative desirability of prudential measures that attempt to curb financial risk-taking ex ante, before crises materialize, and policy measures that are taken ex post, once a crisis has hit. In the realm of fiscal policy, such measures include bailouts, transfers and subsidies such as investment tax credits. In the realm of monetary policy, they correspond to monetary easing in response to financial crises. In this context, the so-called "Greenspan doctrine" (see Greenspan, 2002, 2011; Blinder and Reis, 2005) suggests that ex-ante interventions to prevent booms are too blunt compared to "mopping up" measures after a financial crisis has materialized.

This paper studies the desirability of these two types of policy interventions in a stylized three-period model of financial amplification and crisis that follows the spirit of Kiyotaki and Moore (1997). Entrepreneurs borrow and invest in capital in the initial period, they experience a productivity shock and reinvest in the intermediate period, and they repay their debts and consume the remainder in period 2. However, they are subject to financial constraints, which may limit how much they can reinvest. If the constraint forces them to reduce their reinvestment, the value of their capital assets and therefore their collateral declines, and they have to cut back futher on reinvestment, giving rise to financial amplification. We use this setup to study the desirability of ex-ante macroprudential policy interventions, which are taken in the initial period before binding financial constraints occur, as well as ex-post mopping up measures, which are taken in the intermediate period if an adverse shock triggers binding financial constraints.

To study optimal macroprudential interventions, we solve the problem of a constrained social planner who obeys the same financial constraint as decentralized agents, but—unlike competitive agents—internalizes the effects of her actions on aggregate asset prices. As shown in the earlier literature, such a planner induces private agents to reduce borrowing in the initial period so as to mitigate financial amplification effects when the constraint is binding. This policy measure has a natural interpretation in the theory of the second-best: the planner's intervention in unconstrained times introduces a second-order cost, but the relaxation of binding constraints in the future results in a first-order benefit.

Next we turn our attention to ex-post policy measures: we assume a planner who has superior borrowing capacity compared to private agents and who can relax the financial constraints of private agents by providing them with a bailout transfer. The planner pays for the transfer by imposing distortionary taxes. This policy measure captures what we view as the essential characteristic of policy measures to mitigate financial amplification: it relaxes binding financial constraints but induces a distortion into the economy because of the need to raise taxes. We interpret this measure as a typical way of "mopping up" after the crash.[1] We show that a planner finds it optimal to engage in such bailout transfers when financial constraints are binding, since the benefits from relaxing a binding constraint are first-order, whereas the costs from introducing a tax distortion are second-order. We also find that such a bailout is financed purely by raising debt that is repaid in the future when borrowing constraints are loose again—there is no benefit to providing a bailout that is financed with taxes that are raised in the constrained period.

Ex-post policy measures lead to a time consistency problem: they relax binding constraints and provide private agents with greater incentives to borrow and invest more in the initial period. Ex-ante, the planner would like to commit to smaller bailouts than what is optimal under discretion so as to mitigate the overinvestment problem. However, ex-post, once the economy has entered a period with binding constraints, the planner would like to provide the optimal discretionary bailout.

Finally, we introduce a setup in which the planner has access to both ex-ante macroprudential regulation and ex-post bailout tranfers and study the optimal policy mix. We show that a planner finds it optimal to reduce borrowing ex-ante via macroprudential regulation and provide bailouts ex-post whenever there are states of nature in which borrowing constraints are binding. The optimal policy mix consists of a combination of both measures such that the marginal cost of each intervention equals its expected marginal benefit.[2] Bailouts are better targeted, since they are taken only once an adverse state of nature has materialized, whereas macroprudential regulation is more blunt, since it is imposed in the expectation that a crisis may occur in the future.

However, optimal macroprudential regulation resolves the time consistency problem associated with bailouts since the planner can use the macroprudential policy tool to provide the optimal incentives for borrowing and investment and has no more reason to commit to lower bailouts to reduce excessive borrowing. If the two policy instruments of macroprudential regulation and bailouts were given to two separate agencies, the optimal policy mix could be implemented by instructing the bailout agency to provide the optimal discretionary bailouts and instructing the macroprudential agency to resolve the time consistency problem. Macroprudential regulation is in a way a substitute for commitment, since the optimal policy mix could also be implemented by a planner who can credibly commit to a policy that is conditional

[1] Greenspan (2002) used the term "mopping up after the crash" to refer to the use of monetary policy to support the economy after a financial crisis has occurred. Our model does not have money, but some of the ex post measures that we consider—in particular, subsidies that reduce the real interest rate—have similar economic effects on constrained borrowers as a monetary stimulus.

[2] This is consistent with the findings of the general theory of the second best (Lipsey and Lancaster, 1956): it is generally desirable to intervene along all available dimensions when engaging in second-best policies.

on the level of borrowing of entrepreneurs as well as the state of nature and that includes carrots and sticks, i.e. both bailouts for compliant borrowers and penalties for excessive borrowers.

We investigate the desirability of accumulating a bailout fund and find that welfare is generally reduced if bailouts are limited to such a fund. Furthermore, even if the planner can supplement the bailout fund with additional tax revenue, there are no welfare benefits to accumulating such a fund. The planner has no comparative advantage in holding savings relative to private entrepreneurs; therefore there are no efficiency benefits to accumulating a bailout fund. However, distributions from the bailout fund distort the incentives of entrepreneurs, which would call for an increase in macroprudential regulation to offset the distortion. We conclude that bailout funds are generally undesirable from an efficiency perspective.[3]

Finally, we study alternative ways of providing bailouts to constrained entrepreneurs, including investment tax credits and subsidies to borrowing, which may be interpreted for example as interest rate cuts or crisis lending. We find that the different policy measures are equivalent from an ex-post perspective, since what matters is only the transfer of liquidity to mitigate the constraints. However, from an ex-ante perspective, investment tax credits and borrowing subsidies provide superior incentives since they reward enterpreneurs who keep more borrowing capacity and therefore mitigating the incentives for excessive borrowing.

Literature The macroprudential policies that we study address a pecuniary externality under incomplete markets.[4] If financial constraints are binding in the economy, there is a wedge between the marginal valuation of funds by borrowers and lenders. A relative price movement that redistributes resources between the two can therefore achieve a Pareto improvement, as shown by Stiglitz (1983) and Geanakoplos and Polemarchakis (1986). Gromb and Vayanos (2002), Lorenzoni (2008), Korinek (2007, 2010) and Farhi et al. (2009) have applied this to models of financial constraints. Financial amplification effects and the associated pecuniary externalities have been viewed to be at the center stage of many recent financial crises: see e.g. Brunnermeier (2009) or Adrian and Shin (2010) for a discussion of the role of financial amplification effects in the Global Financial Crisis of 2008/09, or Krugman (1999) and Mendoza (2002) for their role in the emerging market crises of the past two decades.

Whereas competitive agents take prices as given, a planner internalizes that she can affect prices in the economy, for example by reducing the borrowing of individual agents. Caballero and Krishnamurthy (2004) and Lorenzoni (2008) show that there is generally excessive borrowing and investment in such a setting, and Caballero

[3] There may be redistributive motives for accumulating a bailout fund, but this is outside the scope of our paper.

[4] Under complete markets, the welfare theorems imply that pecuniary externalities do not matter.

and Krishnamurthy (2003) and Korinek (2010) find that agents will not engage in sufficient insurance against adverse shocks that trigger financial amplification if such insurance is costly. Jeanne and Korinek (2010a) argue that total borrowing should be reduced if uncontingent debt is the only financial instrument. Gersbach and Rochet (2012) show that pecuniary externalities lead to excessive sectoral reallocations of credit. All these papers have in common that they focus on ex-ante or macroprudential measures to reduce the risk of experiencing financial amplification effects, whereas we focus on the optimal mix and the relationship between ex-ante and ex-post policy measures.

Farhi and Tirole (2012) analyze the problems that arise from policies to mitigate financial crises due to collective moral hazard, but in a setting in which there are no fire sales and pecuniary externalities. A number of papers, including Acharya and Yorulmazer (2008) and Philippon and Schnabl (2012), compare the efficiency of different ex-post policy measures. Acharya and Yorulmazer use a model of liquidity constraints and cash-in-the-market pricing to compare the incentive effects of bailouts to subsidies for take-overs of failed banks. Philippon and Schnabl study the optimal way of recapitalizing banks in a model of debt overhang, in which an asymmetric information problem between banks and the government is solved via a mechanism design setup. The contribution of our paper, by contrast, is to study the optimal policy mix between ex-ante and ex-post policy measures in a model of financial amplification and to focus on the interplay between the two.[5]

2 Model

2.1 Assumptions

We consider an economy with three time periods $t = 0, 1, 2$. Period 0 is the investment period in which the productive capital good is produced. The consumption good is produced with capital and labor in period 1 and also in period 2.

There are two classes of atomistic agents in the economy: entrepreneurs and workers. The entrepreneurs operate the productive capital and hire the workers in periods 1 and 2. The entrepreneurs do not have enough funds of their own to finance the desired level of capital in period 0 and so must borrow from the workers.

The utility of the representative worker in period 0 is given by,

$$U^w = E_0 \left(c_0 + c_1 + c_2 - \omega \ell_1 - \omega \ell_2 \right),$$

where $c_t \geq 0$ and ℓ_t are respectively the worker's level of consumption and labor supply in period t. The utility of the representative entrepreneur is the same but

[5] In the quantitative DSGE literature, Jeanne and Korinek (2010b, 2011), Bianchi (2011) and Bianchi and Mendoza (2011) present similar findings for ex-ante interventions in models of financial feedback loops, whereas Benigno et al. (2010) and Bianchi (2012) focus on ex-post interventions.

without the terms reflecting the disutility of labor (since to simplify, we assume that entrepreneurs do not supply labor in this economy).

Output is produced by the entrepreneurs using the Cobb-Douglas production function,

$$y_t = (A_t k_t)^{\alpha} \ell_t^{1-\alpha},$$

where A_t is the level of capital-augmenting productivity in period $t = 1, 2$. Productivity in period 1 will be taken to be stochastic and exogenous, and it is the only source of uncertainty. The productivity of an entrepreneur in period 2 is increasing with an investment expenditure x made by the entrepreneur in period 1,

$$A_2 = A(x), A' > 0.$$

where we assume that the function $A(\cdot)$ is increasing and concave, i.e., the returns on x are decreasing. The expenditure x can be interpreted for example as human capital or know-how that is complementary with the productive capital produced in period 0. It could also be interpreted as additional physical capital; but importantly, the productivity increase brought by the expenditure x is individual-specific and inalienable. The expenditure x raises the productivity of the entrepreneur who made the expenditure but does not raise the productivity of his capital k if it is used by others.[6]

Since workers have linear disutility, the real wage must be equal to ω in a perfectly competitive labor market. It follows that an entrepreneur operating a quantity k_t of capital makes a profit

$$\max_{\ell_t} (A_t k_t)^{\alpha} \ell_t^{1-\alpha} - \omega \ell_t = \kappa A_t k_t,$$

with $\kappa \equiv \alpha \left[(1-\alpha)/\omega\right]^{(1-\alpha)/\alpha}$.

Productive capital is produced in period 0 with consumption good. The capital good can be produced only in period 0 and the aggregate stock of physical capital is constant in periods 1 and 2. The workers are endowed with a certain quantity of consumption good in period 0, y_0, but the entrepreneurs have no endowment. The entrepreneurs have the technology to transform consumption good into capital good, and must borrow from the workers in period 0 to produce the capital that they will use in periods 1 and 2. We assume for now that entrepreneurs finance their investments by issuing one-period debt in period 0 (alternatives will be discussed).

The budget constraints of entrepreneurs and workers are collected in Table 1.

[6] Comparing our specification to Kiyotaki and Moore (1997), investment in k in our setup corresponds to investment in land in theirs; investment in x in our setup corresponds to investment in trees in theirs. Just as they assume that trees are lost when land is transferred, we assume that the investment x is lost when an entrepreneur defaults and her capital k is seized as collateral. In both setups, the assumption that an investment that is complementary to collateral cannot be transferred ensures that collateral prices depend on aggregate variables, not individual-specific investment. This is an important ingredient to obtain the price dynamics that lead to financial amplification.

Table 1. Budget constraints

Period	Entrepreneurs	Workers
$t = 0$	$c_0 + I(k) = d_0 k$	$c_0 + b_0 = y_0$
$t = 1$	$xk + c_1 + d_0 k = \kappa A_1 k + d_1 k$	$c_1 + b_1 = \omega \ell_1 + b_0$
$t = 2$	$c_2 + d_1 k = \kappa A_2 k$	$c_2 = \omega \ell_2 + b_1$

Some parts of the budget constraints require further explanation. First, in period 0, $I(k)$ is the quantity of consumption good that the representative entrepreneur needs to produce a quantity k of productive capital. It takes a positive level of consumption good to produce a positive level of capital ($I(0) = 0$) and function $I(\cdot)$ is of course increasing. We will further assume that there are decreasing returns in the production of capital, i.e., function $I(\cdot)$ is strictly convex.

Second, the level of borrowing by the representative entrepreneur and the level of lending by the representative worker in period t are respectively denoted by $d_t k$ and b_t. The variable d_t is the entrepreneurs' level of debt per unit of capital, or debt ratio, which is an approximate indicator for leverage. In equilibrium one must have $d_t k = n b_t$ where n is the number of workers per entrepreneur. The equilibrium interest rate on debt is equal to zero because there is no default risk and the lenders (the workers) are risk-neutral and do not discount the future. Finally, note that that the productivity-enhancing expenditure x is scaled by k: a larger level of capital raises the expenditure that is required to reach a certain level of productivity.[7]

2.2 First-Best Equilibrium

First, let us characterize the first-best symmetric equilibrium without collateral constraint. It is easy to see that the workers do not receive any surplus from working or lending (since their utility is linear in consumption and labor), so that their welfare is equal to their initial endowment, $U^w = y_0$. Using the budget constraints, it is also easy to see that the welfare of the representative entrepreneur is equal to the expected profit on the capital net of the productivity-enhancing expenditure minus the cost of producing capital,

$$\max_{k,x} E_0 \left[\kappa A_1 + \kappa A(x) - x\right] k - I(k).$$

[7] To relate our production technology to more traditional production functions, denote by K and X the total resources spent on investment in capital k and on the productivity-enhancing investment x and define $I^{-1}(\cdot)$ as the inverse of the investment cost function $I(k)$. Then we could write period 2 production as $F(K, X) = F(I(k), kx) = A\left(X/I^{-1}(K)\right) I^{-1}(K)$. For appropriate choices of $A(\cdot)$ and $I(\cdot)$, we can replicate a number of different production functions. The benefit of our specific notation is that it greatly simplifies the ensuing analysis.

The first-order conditions for welfare maximization on period 0 capital and period 1 investment are,

$$I'(k) = E_0\left[\kappa\left(A_1 + A_2\right) - x\right], \tag{1}$$

$$\text{and} \quad \kappa A'(x) = 1. \tag{2}$$

The second equation determines x independently of the level of capital k or of the realization of A_1. The first-best levels of the variables are denoted with a superscript FB, i.e. k^{FB} and x^{FB}.

2.3 Financial Constraint

We assume that there is a collateral constraint coming from the fact that entrepreneurs can renegotiate their debt at the time of repayment. The constraint is the same as in Kiyotaki and Moore (1997) or Lorenzoni (2008). At the beginning of period $t = 1, 2$, the entrepreneur can make a take-it-or-leave-it offer to repay a lower amount than the debt coming due. If the creditors reject this offer, they can seize a fraction ϕ of the entrepreneur's productive capital and then sell it at price p_t. The creditors, thus, will accept the entrepreneur's offer as long as the offered repayment is not smaller than $\phi k p_t$, the amount that they will obtain by foreclosing on the capital.

We assume for now that debt is default-free, i.e., it is never renegotiated in equilibrium (this assumption will be relaxed). This implies the following constraint,

$$d_t \leq \phi \min_t p_{t+1}, \tag{3}$$

where $\min_t p_{t+1}$ is the minimum possible price at which the capital of defaulting entrepreneurs can be sold in $t+1$ as estimated in period t.

Capital, if it is seized by the creditors, is auctioned off to the non-defaulting entrepreneurs. Expressions for the equilibrium levels of p_1 and p_2 will be derived in the next section.

3 Laissez-Faire Equilibrium

We solve for the equilibrium in the absence of government intervention via backward induction, starting with the last period. It will be important in our derivations to differentiate between variables related to an individual atomistic entrepreneur and variables related to the *representative* entrepreneur. Thus, we index individual entrepreneurs by i and denote with a superscript i the variables related to that entrepreneur. We denote without superscript the variables for the representative entrepreneur.

Period 2 Entrepreneur i starts period 2 with capital k^i and debt $d_1^i k^i$. If this entrepreneur came to default, his capital would be sold to the other entrepreneurs at a price that is equal to the return on capital for the representative entrepreneur,

$$p_2 = \kappa A_2 = \kappa A(x).$$

We write A_2 and x without superscript because it is the productivity of the representative entrepreneur to whom the defaulting entrepreneur's capital is sold in the auction.[8] In equilibrium, there is no default, and all the entrepreneurs repay their debts to the workers.

Period 1 All the uncertainty is resolved in period 1. The next-period price of capital is known and the collateral constraint can be written as

$$d_1^i \leq \phi p_2. \tag{4}$$

Because of this constraint, entrepreneur i may not be able to finance the optimal level of productivity-enhancing expenditure, x^{FB}. Using the period-1 budget constraint of entrepreneur i, the non-negativity constraint on consumption, $c_1^i \geq 0$, and $p_2 = \kappa A(x)$, the collateral constraint (4) per unit of capital can be written,

$$x^i + d_0^i \leq \kappa \left[A_1 + \phi A(x) \right]. \tag{5}$$

Thus, if the level of productivity, A_1, is low relative to the entrepreneur's debt ratio, d_0^i, it may be impossible to finance x^{FB}.

In a symmetric equilibrium we have $x^i = x$ and both sides of constraint (5) are increasing with x. To avoid the complications associated with multiple equilibria, we assume that the slope of the right-hand side is lower than 1.

Assumption 1 $\forall x$, $\kappa \phi A'(x) < 1$.

An important implication of equation (5) is that the impact of a negative productivity shock is amplified by the collateral constraint. Suppose that the level of period-1 productivity is sufficiently low that the financial constraint on entrepreneurs is binding. Assume that productivity is further reduced by a small amount $dA_1 < 0$. The first-round impact is to reduce the productivity-enhancing expenditure x by κdA_1, but the lower expenditure then reduces the next-period price of capital, which further tightens the constraint by $\phi \kappa A'(x) dA_1$. After the successive rounds of tightening have taken place (within the period), the net impact is given by,

$$dx = \frac{\kappa}{1 - \phi \kappa A'(x)} dA_1.$$

[8] In order to ensure that the entrepreneurs have resources to buy more capital at the beginning of period 2, we can assume that they receive an exogenous endowment (which could be infinitesimally small).

The denominator in this expression captures the effects of financial amplification. Individual entrepreneurs take prices as given and do not internalize the impact of financial amplification—which provides the justification for macroprudential intervention in this model.

If we denote the period 1 liquid net worth of the entrepreneur per unit of capital by $n^i = \kappa A_1 - d_0^i$, then we can express the optimization problem of the entrepreneur in period 1 as maximizing the payoff per unit of capital

$$\max_{x^i} \kappa A(x^i) - x^i + \lambda^i \left[n^i + \phi \kappa A_2 - x^i \right], \tag{6}$$

where A_2 is taken as exogenous. This implies the first-order condition

$$\lambda^i = \kappa A' \left(x^i \right) - 1. \tag{7}$$

The period-1 price of capital, p_1, is derived in the appendix.

Period 0 Without loss of generality we set the entrepreneur's consumption in periods 0 and 1 to zero, $c_0 = c_1 = 0$. (If there is a possibility that the constraint is binding in period 1, then this is the optimal choice in order to minimize borrowing; otherwise it is one of a continuum of allocations of consumption $c_0 + c_1 + c_2$ over time.) It follows that the debt ratio is a simple function of the level of capital,

$$d_0^i = d(k^i) \equiv \frac{I(k^i)}{k^i}.$$

The debt ratio function $d(k)$ is increasing with the level of capital because $I(\cdot)$ is a convex function and $I(0) = 0$. Furthermore, we assume that the period-0 borrowing constraint is loose, i.e. that $d_0 < \phi \min_0 p_1$ for the optimal d_0. The conditions on the exogenous parameters under which this is true are derived in the appendix.

We can then write the entrepreneur's period-1 welfare as

$$v\left(k^i\right) = \max_{x^i} \left\{ \left[\kappa A_1 + \kappa A(x^i) - x^i \right] k^i - I(k^i) + \lambda^i \left[\kappa A_1 + \phi \kappa A_2 - x^i - d(k^i) \right] k^i \right\}. \tag{8}$$

In period 0 the entrepreneur chooses the level of capital k^i that maximizes his expected welfare $E\left[v(k^i)\right]$. In the following, we will drop the superscript i to abbreviate notations.

Intuitively, the fact that the productivity-enhancing expenditure is reduced below the first-best level because of the financial constraint should lower the return that the entrepreneur expects on his capital, and so his investment in period 0. This intuition is stated formally in the following proposition, where we denote with a superscript LF the level of the endogenous variables in the laissez-faire equilibrium.

Proposition 1 *If the period-*1 *constraint is binding with a nonzero probability under laissez-faire (*$E\left[\lambda^{LF}\right] > 0$*), then entrepreneurs borrow and invest less than the unconstrained first-best level in period* 0,

$$k^{LF} < k^{FB}.$$

Proof. We show that $I'(k^{LF}) < I'(k^{FB})$, which will prove the proposition since $I'(\cdot)$ is convex. The first-order condition for the entrepreneur's problem is

$$E\left[v'(k)\right] = 0. \tag{9}$$

Using equation (8) and the envelope theorem and observing that $\lambda\left[\kappa A_1 + \phi\kappa A_2 - x - d(k)\right]k = 0$ in equilibrium, this implies

$$I'(k^{LF}) = E\left[\kappa A_1 + \kappa A(x^{LF}) - x^{LF}\right] - E\left[\lambda^{LF}\right]k^{LF}d'(k^{LF}). \tag{10}$$

In the special case where there is no collateral constraint this equation becomes the first-order condition for the first-best level of capital

$$I'\left(k^{FB}\right) = E_0\left[\kappa A_1 + \kappa A(x^{FB}) - x^{FB}\right]. \tag{11}$$

Comparing equations (10) and (11) shows that $I'(k^{LF}) < I'\left(k^{FB}\right)$ for two reasons. First, the fact that x^{LF} sometimes falls below x^{FB} reduces the first term on the r.h.s. of (10) below the r.h.s. of (11). The constraint reduces the average productivity-enhancing expenditure and so the return on capital. Second, the second term on the r.h.s. of (10) is negative because $E\left[\lambda^{LF}\right] > 0$ and $d'(k^{LF}) > 0$. This reflects another cost of increasing capital: it raises the debt ratio d_0 and so tightens the constraint on the productivity-enhancing expenditure x. ■

4 Ex-Ante Macroprudential Regulation

We analyze the scope for macroprudential regulation by solving the problem of a constrained social planner who determines the period-0 decisions on borrowing and investment but leaves the remaining decisions in periods 1 and 2 to be determined by private agents. We assume that the social planner maximizes social welfare defined as the sum of the utilities of all agents in the economy (entrepreneurs and workers). The difference between private agents and the planner is that the latter internalizes the general equilibrium effects that occur during financial amplification. In equilibrium, workers are always paid wages and interest rates that reflect their marginal disutility from labor and from lending. This implies that the welfare of workers is constant at y_0 so that a social planner who maximizes entrepreneurial welfare also maximizes social welfare. Any increase in social welfare is therefore a Pareto improvement.

We solve the problem via backward induction. The planner's expression for entrepreneurial welfare in a symmetric equilibrium of period 1 is

$$w(k) = \max_x \{[\kappa A_1 + \kappa A(x) - x]\, k - I(k) + \lambda\, [\kappa A_1 + \kappa\phi A(x) - x - d(k)]\, k\}\,. \quad (12)$$

This is the same optimization problem as for the entrepreneurs under laissez faire in equation (8) except that the planner internalizes that $p_2 = \kappa A(x)$ in the borrowing constraint. The associated first-order condition is

$$\tilde{\lambda} = \frac{\kappa A'(x) - 1}{1 - \phi\kappa A'(x)}, \quad (13)$$

where we use a tilde to refer to the equilibrium values of shadow prices as perceived by the planner. The period-1 welfare of entrepreneurs remains unchanged at $v(k)$ conditional on the levels of capital k and debt $d(k)$.

The denominator of expression (13) captures that one additional dollar in period 1 leads to $1/(1 - \phi\kappa A'(x))$ additional dollars of investment in general equilibrium. Comparing with equation (7), we observe that $\tilde{\lambda} > \lambda$ when the financial constraint is binding for a given pair (k, A_1), i.e., the planner perceives the cost of binding constraints as higher than private agents in a given allocation. Because of this, we would expect that the social planner tries to reduce the economy's vulnerability to a credit crunch by reducing period-0 debt and investment, i.e. to engage in *macroprudential* policies. This is stated more formally in the following proposition, where we denote with a superscript MP the levels of the endogenous variables in an equilibrium in which the social planner engages in macroprudential policies.

Proposition 2 (Macroprudential Regulation) *Assume that the period-*1 *financial constraint is binding with positive probability in the laissez-faire equilibrium* ($E[\lambda^{LF}] > 0$)*. The optimal ex-ante macroprudential policy then satisfies the following properties:*

(i) the planner lowers borrowing and investment below the laissez-faire level:

$$k^{MP} < k^{LF},$$

(ii) the planner's chosen level of capital can be implemented by imposing a Pigouvian tax on borrowing or investment

$$\tau_0^{MP} > 0,$$

(iii) the planner mitigates but does not fully alleviate binding borrowing constraints,

$$E[\lambda^{LF}] > E[\lambda^{MP}] > 0.$$

Proof. Using equations (8), (12), the envelope theorem and equations (7) and (13), the optimality conditions of entrepreneurs and the planner are respectively

$$Ev'(k) = E\left[\kappa A_1 + \kappa A(x) - x\right] - I'(k) - kE\left[\kappa A'(x) - 1\right] d'(k) = 0, \quad (14)$$

$$Ew'(k) = E\left[\kappa A_1 + \kappa A(x) - x\right] - I'(k) - kE\left[\frac{\kappa A'(x) - 1}{1 - \phi\kappa A'(x)}\right] d'(k) = 0. \quad (15)$$

Both $Ev'(k)$ and $Ew'(k)$ are decreasing with k. The equilibrium levels of capital under laissez-faire and the social planner respectively satisfy $Ev'(k^{LF}) = 0$ and $Ew'(k^{MP}) = 0$. Observe that when the economy is constrained, the level of x is determined by the constraint and thus is the same whether or not there is a social planner (given k). Hence, for any given k, whenever $E[\lambda] = E[\kappa A'(x) - 1] > 0$, comparing (14) and (15) shows that $Ew'(k) < Ev'(k)$, i.e., the social planner has a strictly lower marginal valuation of capital than individual entrepreneurs. If the laissez-faire equilibrium satisfies $E\left[\lambda^{LF}\right] = E[\kappa A'(x) - 1] > 0$, then $Ev'(k^{LF}) = 0 > Ew'(k^{LF})$ and the planner finds it optimal to reduce capital investment and borrowing to a lower level $k^{MP} < k^{LF}$. This proves point (i) of the Proposition.

To see how the planner's equilibrium can be implemented via Pigouvian taxation, consider a tax $\tau_0 > 0$ on period-0 investment that is rebated to entrepreneurs in lump-sum fashion so as to be wealth-neutral. This modifies the period-0 budget constraint of entrepreneurs to

$$c_0 + (1 + \tau_0)I(k) = d_0 k + T,$$

where the rebate satisfies $T = \tau_0 I(k)$.[9] The tax modifies the optimality condition of entrepreneurs (14) by pre-multiplying the marginal cost of investment and adding a term to the perceived cost of binding constraints,

$$Ev'(k) = E\left[\kappa A_1 + \kappa A(x) - x\right] - (1 + \tau_0) I'(k) - E[\lambda]\left[kd'(k) + \tau_0 I'(k)\right] = 0. \quad (16)$$

(A tax on borrowing would introduce an equivalent wedge.) The optimal tax rate τ_0 is then chosen such that $Ev'\left(k^{MP}\right) = 0$ so that the decentralized equilibrium replicates the social planner's equilibrium. Substituting equations (15) and (16) and using the expressions (13) and (7) for $\tilde{\lambda}^{MP}$ and λ^{MP} we obtain

$$\tau_0^{MP} = \frac{E\left[\tilde{\lambda}^{MP} - \lambda^{MP}\right]}{1 + E\left[\lambda^{MP}\right]} \cdot \frac{d'\left(k^{MP}\right) k^{MP}}{I'\left(k^{MP}\right)}. \quad (17)$$

The level of the optimal tax τ_0 is strictly positive if there is a positive probability that the financial constraint is binding and that $\tilde{\lambda}^{MP} > \lambda^{MP}$. This proves point (ii) of the Proposition.

[9] One interpretation of the rebate is that the policy is introduced not literally as a tax, but instead as a quantity regulation, which implies that entrepreneurs keep the surplus that results from restricting borrowing and investment.

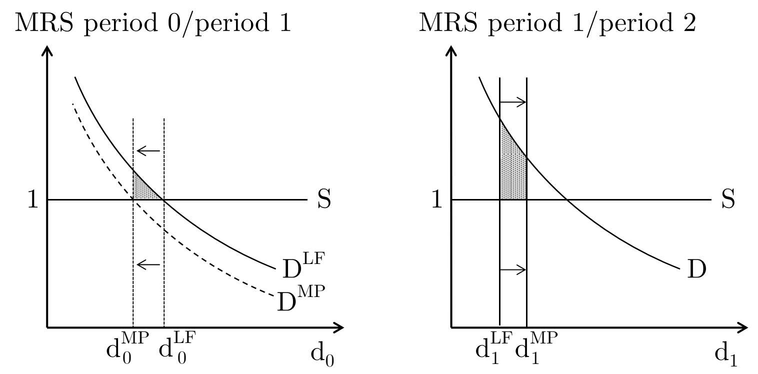

Figure 1: Macroprudential policy as a second-best intervention

If $E\left[\lambda^{LF}\right] > 0$, then there will still be a strictly positive expected cost of binding constraints $E\left[\lambda^{MP}\right] > 0$ even after the planner's intervention. Otherwise equation (15) would imply $k^{MP} > k^{LF}$, a contradiction with point (i) of the Proposition. The lower level of capital investment $k^{MP} < k^{LF}$ implies a lower debt ratio $d_0 = d(k)$. If the constraint is binding for a given pair (k^{LF}, A_1), then it is therefore looser for the pair (k^{MP}, A_1) and the expenditure x satisfies $x^{MP} > x^{LF}$. Thus, $\lambda^{MP} < \lambda^{LF}$ for the realizations of A_1 such that the financial constraint is binding under laissez-faire, and since these realizations have a non-zero probability, one has $E\left[\lambda^{MP}\right] < E\left[\lambda^{LF}\right]$. This proves (iii). ■

If the financial constraints bind with a zero probability ($E\left[\lambda^{LF}\right] = 0$), period-0 investment and welfare are equal to the first-best levels and there is no justification for macroprudential intervention. If the constraints bind with a nonzero probability, the social planner recognizes that there is a trade-off between period-0 investment k and period-1 re-investment x. She invests less in period 0 than in the laissez-faire equilibrium and so increases the investment gap relative to the first best, but keeps additional borrowing capacity and raises investment in period 1.

As explained in the literature on pecuniary externalities in financial amplification, the laissez-faire equilibrium is inefficient because both the risk and severity of a credit crunch are endogenous to aggregate debt, but private entrepreneurs take aggregate debt as given (see e.g. Jeanne and Korinek, 2010ab). The planner's intervention increases welfare because reducing borrowing d_0 in period 0 below the laissez-faire level introduces a second-order distortion (i.e. a distortion that is negligible for small τ_0), but achieves a first-order benefit by relaxing the binding constraint in period 1. These welfare effects are illustrated by the shaded areas in figure 1.

In the Proposition above, there is a single policy instrument and a strictly monotonic relationship between the macroprudential policy τ_0 and the outcome k. This allows us to obtain the clear result that $k^{MP} < k^{LF}$, i.e., borrowing and investment are always lower under the macroprudential policy. As we will see below in section 6, this may no longer be the case when there are multiple policy instruments involved.

5 Ex Post Bailout Measures

In this section, we study another approach to mitigating the financial friction, in which the planner implements a transfer payment (bailout) to relax the credit constraint on entrepreneurs *ex post*. We assume that each constrained entrepreneur i receives a transfer (subsidy) sk^i in period 1. The transfer is financed by taxes τ_1 and τ_2 on labor in periods 1 and 2.[10] Such a generic tax-and-transfer measure captures what we view as the essential characteristic of policies to mitigate financial amplification effects ex-post: it relaxes financial constraints at the expense of introducing a distortion in the economy (here, a tax distortion). We will discuss a number of alternative common policy measures that fall into this category in section 7. Observe that all the discussed policy measures are aimed not only at alleviating financial constraints at the individual level, but also at alleviating financial amplification (systemic crises) at the aggregate level by relaxing credit constraints across all entrepreneurs. There is thus both an “individual” and a “collective” or “systemic” element to bailouts.

If the period-1 transfer is financed with period-2 tax receipts, it requires that the planner issues public debt that is purchased by workers in period 1 and, thus, that the planner's borrowing capacity is superior to that of private agents. This is a common assumption in the literature, and it is generally justified by fact that the planner has the power to tax (Holmstrom and Tirole, 1998). The assumption is also plausible since debt-financed bailouts are commonly observed during financial crises. In such situations, we can interpret the planner's actions as lending his borrowing capacity to entrepreneurs at the expense of introducing a tax distortion in the economy.

The within-period optimization problem of entrepreneurs is affected by labor taxation as follows. In periods $t = 1$ and 2, the wage to workers net of taxation must still be equal to ω. After we impose a tax τ_t, entrepreneurs must therefore pay a gross wage $(1 + \tau_t)\omega$. The period profit of entrepreneurs is given by

$$\pi_t = \max_{\ell_t}(A_t k_t)^\alpha \ell_t^{1-\alpha} - (1 + \tau_t)\omega\ell_t = \kappa(\tau_t)A_t k_t,$$

where $\kappa(\tau) = \alpha\left[\frac{(1-\alpha)}{(1+\tau)\omega}\right]^{(1-\alpha)/\alpha}$ is the return on an effective unit of capital. We observe that labor taxation is distortionary and reduces the return $\kappa(\tau)$ per effective

[10] We assume that the planner is not able to raise lump-sum taxes. This is a reasonable assumption in practice, as raising fiscal revenue generally involves distortions. It is also the starting point of the literature on optimal Ramsey taxation. Appendix A.2 shows that lump-sum taxes and transfers would enable the social planner to replicate the first-best equilibrium because she can relax the credit constraint without inducing any distortions.

unit of capital Ak. However, the bailout has an a priori ambiguous impact on the period-2 price of capital $p_2 = \kappa(\tau_2) A(x)$, since it allows entrepreneurs to increase the productivity-enhancing expenditure x.

The subsidy is equal to the present value of the tax receipts per unit of capital, that is

$$sk = \tau_1 \omega \ell_1 + \tau_2 \omega \ell_2 = \tau_1 \varepsilon(\tau_1) A_1 k + \tau_2 \varepsilon(\tau_2) A_2 k, \tag{18}$$

where $\varepsilon(\tau)$ denotes the labor compensation per effective unit of capital, $\omega\ell/Ak$,

$$\varepsilon(\tau) = \omega \left[\frac{1-\alpha}{(1+\tau)\omega}\right]^{1/\alpha}. \tag{19}$$

In a first step, we assume a time-consistent social planner who designs the bailout ex post (in period 1) to maximize domestic welfare subject to the collateral constraint. The equilibrium bailout policy can be characterized by three functions $s(n)$, $\tau_1(n)$ and $\tau_2(n)$ that map the entrepreneur's period-1 liquid net worth per unit of capital $n = \kappa A_1 - d(k)$ into the rate of subsidy and the ex-post tax rates. The properties of the equilibrium bailout policies are summarized in the following proposition, where we denote with a superscript BL the levels of the endogenous variables under the equilibrium time-consistent bailout policy.

Proposition 3 (Bailouts) *The equilibrium bailout policy under discretion satisfies the following properties:*

(i) there is a bailout if and only if the financial constraint is binding in the laissez-faire equilibrium:

$$\lambda^{LF}(n) > 0 \Longleftrightarrow s^{BL}(n) > 0.$$

(ii) the bailout is financed by issuing public debt and taxing labor in period 2*, whereas the period-*1 *tax on labor income is set to zero*

$$\tau_1^{BL}(n) = 0,$$

(iii) the bailout mitigates the constraint but does not fully alleviate it,

$$\lambda^{LF}(n) > 0 \Longrightarrow \lambda^{LF}(n) > \lambda^{BL}(n) > 0,$$

(iv) if the financial constraints bind with a nonzero probability ($E\left[\lambda^{LF}\right] > 0$*), the expectation of bailouts increases period-*0 *investment above the laissez-faire level,*

$$k^{BL} > k^{LF}.$$

Proof. The period-1 welfare of an entrepreneur who takes the subsidy rate, the tax rates and the collateral price as given, is

$$\begin{aligned} v^{BL}(k^i) \quad &= \quad \max_{x^i} \left[\kappa(\tau_1)A_1 + \kappa(\tau_2)A(x^i) + s - x^i\right] k^i - I(k^i) \\ &\quad +\lambda^i \left[\kappa(\tau_1)A_1 + \phi p_2 + s - x^i - d(k^i)\right] k^i. \end{aligned} \tag{20}$$

Substituting for s and p_2, a planner who enters period 1 facing a set of state variables (k, A_1) solves

$$\begin{aligned} w^{BL}(k) &= \max_{x,\tau_1,\tau_2} [\eta(\tau_1) A_1 + \eta(\tau_2) A(x) - x] k - I(k) \\ &\quad + \tilde{\lambda} \{\eta(\tau_1) A_1 + [\phi\kappa(\tau_2) + \tau_2\varepsilon(\tau_2)] A(x) - x - d(k)\} k, \end{aligned} \tag{21}$$

where we denote by $\eta(\tau) = \kappa(\tau) + \tau\varepsilon(\tau)$ the social net return on capital, i.e., the entrepreneur's return plus tax revenue per unit of capital. We observe that $\eta'(\tau) = \tau\varepsilon'(\tau) < 0$ for $\tau > 0$, i.e., the social net return on capital is decreasing with the level of taxation.

The planner's optimality condition on τ_1 is

$$\tau_1\varepsilon'(\tau_1) A_1(k + \tilde{\lambda}) = 0,$$

which implies that $\tau_1 = 0$. This proves point (ii) of the Proposition.

Using $\varepsilon'(\tau) = -\frac{1}{\alpha}\varepsilon(\tau)/(1+\tau)$ and $\kappa'(\tau) = -\varepsilon(\tau)$, the optimality condition for τ_2 can be written

$$\frac{\tau_2}{1+\tau_2} = \alpha(1-\phi)\frac{\tilde{\lambda}}{1+\tilde{\lambda}}. \tag{22}$$

The shadow costs λ and $\tilde{\lambda}$ are respectively given by

$$\lambda = \kappa(\tau_2)A'(x) - 1, \tag{23}$$

$$\tilde{\lambda} = \frac{\kappa(\tau_2)A'(x) - 1}{1 - [\phi\kappa(\tau_2) + \tau_2\varepsilon(\tau_2)] A'(x)}. \tag{24}$$

Observe that as in the laissez faire equilibrium above, the period-1 liquid net worth n, which determines the tightness of the constraint in (21), is a sufficient statistic for the optimal tax rate $\tau_2(n)$ and the bailout $s(n) = \tau_2(n)\varepsilon(\tau_2(n)) A(x)$.

The social planner still values liquidity more than entrepreneurs in a constrained equilibrium, i.e., $\tilde{\lambda} > \lambda$ if $\lambda > 0$. It follows that if $\lambda^{LF}(n) > 0$, one must have a strictly positive tax rate τ_2 to finance a bailout in the amount of $s = \tau_2\varepsilon(\tau_2) A(x)$. If not, (i.e., if τ_2 were equal to zero), then there would be no bailout, implying $\lambda = \lambda^{LF}(n) > 0$ and $\tilde{\lambda}$, being larger than λ, would be strictly positive, which would contradict equation (22). Conversely, if $\lambda^{LF}(n) = 0$, the laissez-faire equilibrium is unconstrained and the social planner does not increase welfare by implementing a bailout. This proves points (i) of the Proposition.

Furthermore, equation (22) also implies that the constraint is still binding under the optimal bailout measure; otherwise $\tilde{\lambda}$ would be equal to zero and so would τ_2. This shows $\lambda^{LF}(n) > 0 \implies \lambda^{BL}(n) > 0$. To show that $\lambda^{BL}(n)$ is smaller than $\lambda^{LF}(n)$, observe that the planner chooses (τ_2, x) in optimization problem (21) so as to maximize the period-2 net return $\eta(\tau_2) A(x) - x$ per unit of capital subject to the borrowing constraint. This implies that the return at the planner's optimum, is greater than the return in the absence of intervention, $\eta\left(\tau_2^{BL}\right) A\left(x^{BL}\right) - x^{BL} >$

$\kappa A\left(x^{LF}\right) - x^{LF}$, which in turn is possible only if the bailout raises the expenditure, $x^{BL} > x^{LF}$. Then for $\tau_2 > 0$ we have $\kappa(\tau_2)A'(x^{BL}) < \kappa(0)A'(x^{LF})$, which with (23) implies $\lambda^{BL} < \lambda^{LF}$. This proves point (iii) of the Proposition.

Finally, we show that the expectation of bailouts raises investment. Taking the derivative of (20) and using the envelope theorem and $\tau_1 = 0$, an individual entrepreneur perceives the marginal benefit of investing in capital as

$$v_k^{BL} = \kappa A_1 + \eta\left(\tau_2\right) A\left(x^i\right) - x^i - I'\left(k^i\right) - \left[\kappa(\tau_2)A'(x^i) - 1\right] d'\left(k^i\right). \tag{25}$$

Increasing τ_2 from 0 to the optimal level set by the social planner (given by (22)) raises v_k^{BL} for two reasons. Firstly, as noted above, the return at the planner's optimum is greater than the return in the absence of intervention, $\eta\left(\tau_2^{BL}\right) A\left(x^{BL}\right) - x^{BL} > \eta(0)A\left(x^{LF}\right) - x^{LF}$; secondly, $x^{BL} > x^{LF}$ implies that $\kappa\left(0\right) A'\left(x^{LF}\right) > \kappa\left(\tau_2^{BL}\right) A'\left(x^{BL}\right)$. Combining these two observations with equation (25), we observe that for any realization of A_1 with binding constraints, v_k^{BL} is increased above the laissez faire level by the bailout. This implies that entrepreneurs choose a higher level of period 0 capital investment $k^{BL} > k^{LF}$, proving point (iv) of the Proposition.

Lastly, note that we have not taken into account the period-1 implementability constraint that private entrepreneurs are willing to invest the chosen level of x, i.e., $\kappa\left(\tau_2\right) A'\left(x\right) \geq 1$. But taking this constraint into account does not change our results. In general, the planner raises the tax either until either her optimal tax rate τ_2 determined by equation (22) is reached or the implementability constraint becomes binding so that $\kappa\left(\tau_2\right) A'\left(x\right) = 1$. In both cases, the chosen tax rate is strictly positive $\tau_2 > 0$. ■

The intuition behind points (i) and (ii) is the following. A bailout can raise welfare to the extent that it relaxes the credit constraint. There is no benefit for the planner to impose a tax in period 1 and transfer the receipts to entrepreneurs, since such a policy would both introduce a distortion into the resource allocation of the economy and tighten the financial constraint. On the other hand, by borrowing to make a transfer in period 1, the planner lends her own superior borrowing capacity to entrepreneurs. This yields a first-order welfare benefit since it relaxes a binding borrowing constraint, but comes at a second-order welfare cost in period 2 by reducing the ratio ℓ_2/Ak. The welfare effects are illustrated by the shaded areas in figure 2. According to the theory of the second best, it is always desirable to engage in some bailout when the financial constraint is binding, but not to fully undo the constraint, as noted in point (iii). The reason is that if the constraint were fully alleviated, the last bit of such a policy would have only second-order welfare benefits but would come at a first-order welfare cost. As for point (iv), the intuition is that the bailouts raise the return on capital ex post and so enhance the incentives to invest in capital k ex ante.

In the extreme, the incentive effects of bailouts on capital investment k may lead to multiple equilibria. The period-0 optimality condition of private entrepreneurs on

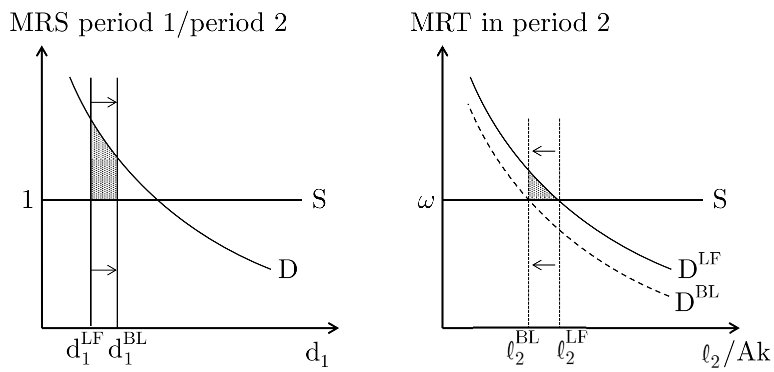

Figure 2: Bailouts as second-best interventions

capital investment defines k as an increasing function of expected bailouts τ_2, and the period-1 optimality condition of the planner on the optimal bailout measure defines τ_2 as an increasing function of capital investment k, since a greater capital stock implies more debt and tighter financial constraints.[11] If the two functions intersect more than once, there are multiple equilibria with smaller or larger bailouts: if entrepreneurs expect small bailouts, they will be prudent and invest less, which in turn makes it optimal for the planner to provide only small bailouts; if entrepreneurs expect large bailouts, they will invest more, experience tighter constraints, and the planner will find it ex-post optimal to provide large bailouts.[12] For the remainder of our paper, we assume that the equilibrium is unique or that private agents always manage to coordinate on the better equilibrium.

Bailouts increase welfare ex post (in period 1), but their impact on ex-ante (period-0) welfare is in general ambiguous since they increase investment in k, which magnifies the overinvestment problem identified in Proposition 2. If the planner can commit to a bailout policy $s(n)$ that is contingent on the net worth of entrepreneurs, she would like to do so.[13]

Proposition 4 (Bailouts Under Commitment) *If the planner can commit to a bailout policy $s(n)$, she would choose a lower level of bailouts than under discretion. This implies that the planner faces a time consistency problem in designing her optimal bailout policy.*

[11] The period-0 optimality condition for capital investment is $Ev_k^{BL} = 0$ as defined in (25). The optimality condition for bailouts is (22).

[12] We refer to Farhi and Tirole (2012), who term this phenomenon "collective moral hazard," for a rigorous discussion of multiple equilibria under bailouts.

[13] Our result is reminiscent of (but not quite the same as) many similar results in the literature on financial safety nets in which discretionary bailouts induce excessive risk-taking ex ante. The difference is that the excessive risk-taking, in our model, involves a systemic component and exists even in the absence of bailouts because of pecuniary externalities. For an excellent general analysis of time consistency problems in models of fire-sale externalities see Davila (2011).

Proof. See appendix. ■

If the ex ante welfare impact of bailouts is negative, it may even be optimal for the social planner to commit to do no bailouts whatsoever than to allow for discretionary bailouts. In our numerical illustration below, we will discuss the conditions under which this case may arise.

6 Optimal Policy Mix

We have now laid down the groundwork to address the paper's core question: to integrate ex-ante macroprudential regulation and ex-post bailouts in a common framework and compare the benefits and costs as well as the interplay of the two policies. In this section we assume a social planner who can use the full set of instruments considered in the previous two sections: the macroprudential tax on period-0 borrowing, τ_0, as well as a period-1 bailout s that is financed by a tax τ_2 on labor in period 2. As before, taxation in period 1 will not be used and we accordingly omit the tax τ_1 from the problem. We start by describing the optimal policy mix under discretion; then we will show that our solution coincides with the optimal policy mix under commitment.

An important element of our analysis is that the ex-ante policy in the optimal policy mix can be described in terms of setting the instrument τ_0 or in terms of setting the outcome k. Depending on the results that we analyze, it is useful to focus on one or the other. For example, we obtain sharp results on the sign of the optimal policy instrument τ_0 in the following Proposition, but the implications for the direction of change in the outcome k is ambiguous. By contrast, we obtain a clean characterization of the complementarity of capital investment k and bailouts s, but we show that the complementarity or substitutability of the ex-ante policy instrument τ_0 and the bailout policy s is generally ambiguous.

Proposition 5 (Optimal Policy Mix) *Assume that the period-1 financial constraint is binding with positive probability in the laissez-faire equilibrium ($E\left[\lambda^{LF}\right] > 0$). The optimal policy mix under discretion then satisfies the following properties:*

(i) the planner imposes a positive Pigouvian tax on borrowing or investment,

$$\tau_0^{MIX} > 0,$$

(ii) the planner provides the optimal discretionary bailout whenever the financial constraint is binding

$$\lambda^{MIX}(n) > 0 \Longleftrightarrow s^{MIX}(n) > 0.$$

Proof. We proceed by backward induction. The proof of point (ii) of the Proposition is identical to the proof of point (i) of Proposition 3, and we find that the magnitude of the optimal bailout policy is identical $s^{MIX}(n) = s^{BL}(n)$.

To show point (i), we follow similar steps as in the proof of Proposition 2. Observe that the ex-ante optimization problem of the planner under discretion (superscript d) is

$$\max_k Ew^{MIX,d}(k),$$

where $w^{MIX,d}(k) = w^{BL}(k)$ as given in equation (21), and similarly $v^{MIX,d}(k) = v^{BL}(k)$. The optimality conditions of entrepreneurs and the planner are, respectively

$$Ev_k^{MIX,d} = E[\kappa A_1 + \eta(\tau_2)A(x) - x] - I'(k) - E[\lambda]d'(k) = 0, \tag{26}$$

$$Ew_k^{MIX,d} = E[\kappa A_1 + \eta(\tau_2)A(x) - x] - I'(k) - E\left[\tilde{\lambda}\right]d'(k) = 0, \tag{27}$$

where λ and $\tilde{\lambda}$ are given by equations (23) and (24).

If financial constraints are binding with positive probability in the laissez-faire equilibrium $E\left[\lambda^{LF}\right] > 0$ then, by Proposition 3 $E\left[\lambda^{MIX}\right] > 0$. Furthermore, since $\tilde{\lambda}^{MIX} > \lambda^{MIX}$ in that case, the marginal return on capital is strictly smaller for the social planner than for entrepreneurs, $Ew_k^{MIX,d} < Ev_k^{MIX,d}$. The social planner, as a result, chooses a strictly lower level of capital investment $k^{MIX} < k^{BL}$. Following the steps of Proposition 2 in deriving equation (17), this capital level can be implemented by setting the macroprudential tax τ_0 to

$$\tau_0^{MIX} = \frac{E\left[\tilde{\lambda}^{MIX} - \lambda^{MIX}\right]}{1 + E\left[\lambda^{MIX}\right]} \cdot \frac{d'\left(k^{MIX}\right)k^{MIX}}{I'\left(k^{MIX}\right)}. \tag{28}$$

If there is a nonzero probability that the financial constraint is binding, then $E\left[\tilde{\lambda}^{MIX}\right] > E\left[\lambda^{MIX}\right] > 0$ and this expression is positive. ■

The optimal policy mix, thus, gives a role to both macroprudential policy and bailouts. From the point of view of the theory of the second best, each instrument involves a first-order gain but a second-order loss at the margin. It is possible to reduce the total second-order cost by spreading it across the two policy instruments.

The finding that there is still a role for macroprudential policy is not surprising since there was such a role without bailouts, and bailouts tend to magnify the overinvestment problem. However, macroprudential measures are taken in the *expectation* that a systemic crisis state with binding financial constraints *may* occur in the ensuing period. This is captured by the terms $E[\lambda^{MIX}]$ and $E\left[\tilde{\lambda}^{MIX}\right]$ in the expression for the optimal tax rate τ_0^{MIX}. If the economy enters a good state of nature in the following period, then macroprudential measures have introduced a distortion without any corresponding ex-post benefit. It is in the nature of any prudential intervention that its costs are incurred with certainty whereas its benefits materialize with probability less than one. In this sense, macroprudential regulation is a "blunter" policy instrument than ex-post interventions.

Conversely, macroprudential policy does not obviate the need for bailouts since bailouts are more state-contingent. They are implemented conditional on the *realization* of a systemic crisis to alleviate financial constraints ex post. Their magnitude can be precisely targeted at the tighteness of binding constraints $\tilde{\lambda}$ in a given state of nature, as reflected by expression (22) for the optimal bailout tax τ_2. In the limit, this allows us to make the following observation:

Corollary 6 *If the probability of binding constraints as captured by $E[\lambda]$ goes to zero, the planner ceases to use macroprudential regulation $\tau_0 \to 0$. However, if a state with a strictly binding constraint $\lambda > 0$ occurs, the planner will engage in a strictly positive bailout $s > 0$.*

Macroprudential regulation is a function of both the probability of experiencing binding constraints and the tightness of such constraints. It is only useful if the probability of experiencing a financial crisis is bounded away from zero.

Implications for Capital Investment k Let us next translate the implications of the optimal policy mix for capital investment k. We proceed in two steps. First, we show that capital investment and bailouts are complementary. Secondly, we use this finding to derive the implications of implementing the optimal policy mix for capital investment.

Lemma 7 *Capital investment k and bailouts s are complements as long as $s \leq s^{MIX}(n)$, i.e., more capital investment increases the desirability of bailouts and greater bailouts increase the desirability of capital investment.*

Proof. Looking at expression (27), we observe that increasing the bailout s for given k reduces the tightness of financial constraints and lowers $\tilde{\lambda}$. If the bailout is not suboptimally large, i.e. for $s \leq s^{MIX}$, it also raises the expected period 2 return on capital $[\eta(\tau_2) A(x) - x]$. This implies that the cross derivative satisfies

$$Ew_{ks}^{MIX} > 0,$$

which implies that the two are complements. ■

We observe that the complementarity of k and s holds for any allocation in which $s \leq s^{MIX}(n)$, including for the laissez-faire equilibrium LF and the purely macroprudential equilibrium MP. If the bailout is inefficiently large $(s > s^{MIX}(k, A_1))$ then it may reduce the social returns to capital investment $Ew_k^{MIX}(k)$ and the substitutability breaks down. An alternative proof would be to observe that increasing k raises the debt ratio $d(k)$. For a given realization of the productivity shock A_1, the financial constraint on entrepreneurs is therefore tighter so that λ and $\tilde{\lambda}$ increase. This raises the benefits of bailouts and the optimal size of bailouts.

Proposition 8 (Effects of Optimal Policy Mix) *(i) The optimal policy mix lowers borrowing and investment below the level that arises in the decentralized equilibrium with bailouts only,*

$$k^{MIX} < k^{BL}.$$

(ii) The change in borrowing and investment compared to the laissez-faire equilibrium is ambiguous, $k^{MIX} \gtrless k^{LF}$.

Proof. For a given bailout policy $s^{MIX}(n)$, point (i) of the Proposition is analogous to point (i) of Proposition 2 and can be proven in the same manner.

To see point (ii) of the proposition, observe that moving from the laissez faire equilibrium in which $\tau_0 = s(n) = 0$ to the optimal policy mix $\tau_0 > 0, s^{MIX}(n) \geq 0$ affects capital investment via two distinct channels: First, raising τ_0 for a given bailout policy $s(n)$ reduces capital investment. Secondly, raising $s(n)$ in constrained states for a given τ_0 increases investment in k, as we observed in the Lemma above, because it relaxes binding constraints and raises the period 2 return on capital $[\eta(\tau_2) A(x) - x]$. Taking the two policy changes together, borrowing and investment can either go up or down compared to the laissez faire equilibrium. ■

The literature on macroprudential policy has sometimes described the finding that entrepreneurs borrow and invest more than a social planner ($k^{MP} < k^{LF}$) as "overborrowing." In models that focus exclusively on ex-ante policy measures, this description of outcomes mirrors the optimal policy prescription that $\tau_0^{MP} > 0$. However, once we introduce multiple policy measures such as in the optimal policy mix, a simple comparison between k^{MIX} and k^{LF} no longer reflects the direction of the optimal policy τ_0. In our framework, it is always desirable to set $\tau_0 > 0$, but situations arise in which $k^{MIX} > k^{LF}$ since the bailout policy has an independent effect on k. We discuss an example of this ambiguity in our numerical illustration below.[14]

Looking at the effects of moving from the macroprudential policy regime to the optimal policy mix, we find that the effects on both capital investment k and the tax rate τ_0 are ambiguous. Introducing bailouts makes it more attractive to invest in capital k, which allows for a relaxation of macroprudential restrictions, but also raises the private incentives to invest, which calls for a tightening of macroprudential restrictions. The net effect is ambiguous, as we also show in our numerical illustration below.

6.1 Time Consistency of the Optimal Policy Mix

As we noted in section 5, the bailout policy suffers from a time-consistency problem. The time-consistent bailout policy is excessively generous because it does not

[14] Benigno et al. (2010) also illustrate such situations.

take into account its impact on the ex-ante accumulation of capital. One possible advantage of macroprudential policy is that it may help resolve the time consistency problem in the bailout policy by restricting investment in period 1. In fact, as we show below, the time consistency problem in the bailout policy is perfectly resolved by macroprudential policy in the optimal policy mix. Respectively denoting by $s^{MIXc}(n)$ and $s^{MIXd}(n)$ the optimal bailout policy under commitment (i.e., when it is chosen in period 0) and under discretion (when it is chosen in period 1), this result is stated formally in the following proposition.

Proposition 9 (Resolving Time Inconsistency) *The optimal policy mix resolves the time consistency problem introduced by bailouts, i.e., the optimal policy mix under commitment is identical to the optimal policy mix under discretion,*

$$s^{MIXc}(\cdot) = s^{MIXd}(\cdot).$$

Proof. Assume that a planner has chosen the optimal policy mix under discretion described by k^{MIX} and $s^{MIX}(n)$ as characterized in Proposition 5. If this policy is time inconsistent, then a planner under commitment would choose a different bailout policy $s^{MIX,c}(n) \neq s^{MIX}(n)$. Since $s^{MIX}(n)$ maximizes the period 1 payoff per unit of capital for a given n, the only reason could be to affect k^{MIX}. But if it is welfare-improving to deviate from k^{MIX}, then the discretionary planner would also choose a different k, contradicting our assumption that k^{MIX} was optimal. ■

The problem of time inconsistency under bailouts (Proposition 4) arises because the planner has one instrument, a bailout, but would like to affect two targets, the incentive to invest in period 0 and the tightness of constraints in period 1. The time consistency problem is resolved if the planner can target these two objectives independently.

At the optimal policy mix, there is no conflict between using macroprudential policy to solve the time consistency problems of bailouts and correcting the pecuniary externalities from financial amplification effects. This is because the only reason for the time consistency problem in the absence of macroprudential regulation was that lower bailouts could reduce the incentives for excessive period-0 investment, leading to a conflict between what is optimal ex ante and ex post. If we add the macroprudential tax τ_0, then the planner has an independent instrument to set the correct incentives for period-0 investment and the conflict is resolved. Macroprudential regulation kills two birds with one stone.[15]

One could assume that the bailout policy is chosen for a given τ_0 rather than a given k. This changes the analysis because now, changes in the bailout policy

[15] Naturally, our result requires that macroprudential policy can perfectly determine capital investment k. If there were an additional distortion in the economy, e.g. resource costs or imperfect targetig associated with regulation, then the planner could not implement her optimal k and a planner under commitment would like to use her bailout policy to nudge k closer to the optimal level. In this case, the time inconsistency problem would reappear.

have an impact on k. The optimal bailout policy, given τ_0, is no longer necessarily the same under discretion and under commitment. For $\tau_0 = \tau_0^{MIX}$, however, the optimal bailout policy remains $s^{MIX}(n)$ under both commitment and discretion. Assuming otherwise leads to a contradiction with the fact that k^{MIX}, $s^{MIX}(n)$ maximizes welfare under commitment. So one can characterize the optimal level of macroprudential taxation as the level of τ_0 such that the optimal bailout policy is time-consistent.

Corollary 10 (Allocation of Policy Objectives) *Assume that the bailout policy $s(n)$ and macroprudential regulation τ_0 are granted to two different agencies. Then the constrained optimal allocation can be achieved by giving the mandate of maximizing welfare ex post to the bailout agency and the mandate of removing the time-inconsistency in bailouts to the macroprudential agency.*

One interesting implication of our analysis above is that macroprudential regulation is in a certain way a substitute for commitment. We observed in Proposition 4 that the planner can improve upon the equilibrium under discretionary bailouts, but cannot deliver the optimal policy mix if she can commit to a bailout policy $s(n)$ that is contingent solely on n. An interesting question is whether bailout policy under commitment to a more refined set of state variables can replicate the optimal policy mix. We find that this is indeed the case:

Corollary 11 (Commitment as a Substitute to Macroprudential Policy) *If the planner can commit to a bailout policy $s(k, A_1)$ that is conditional on both k and A_1 and unrestricted in sign, then she can replicate the optimal policy mix described in Proposition 5.*

Proof. Assume that the planner commits to a bailout policy

$$s(k, A_1) = \begin{cases} s^{BL}(\kappa A_1 - d(k)) & \text{for} \quad k \leq k^{MIX} \\ -\underline{s} & \text{for} \quad k > k^{MIX} \end{cases}$$

Under full commitment and for a sufficiently large penalty $\underline{s}$, this replicates the optimal policy mix since it ensures that entrepreneurs will find it optimal to invest at most k^{MIX} and since the policy provides the optimal bailout $s^{MIX}(n)$ where $n = \kappa A_1 - d(k)$. ■

The corollary captures that a planner who can commit to "carrots and sticks," i.e., to rewarding prudent entrepreneurs with $k \leq k^{MIX}$ and to sufficiently punishing reckless entrepreneurs, can implement the optimal policy mix. The crucial feature is that the planner can condition her policy on the variable k that is the target of macroprudential policy. In this sense, a bailout policy under commitment is a substitute to macroprudential policy if (i) it is contingent on the outcome targeted by prudential policy and (ii) it has not only carrots but also sticks.

If we set $\underline{s} = 0$, then we remove the "sticks" and the planner has to rely exclusively on the "carrot" of bailouts. In that case, entrepreneurs can choose whether to operate under the bailout umbrella or not, and commitment works only under a limited set of cirumstances. In general, entrepreneurs will accept the conditions of the bailout umbrella if restricting their investment k is relatively cheap because the implied macroprudential tax τ_0 is low compared to the benefit of receiving a bailout. This will be the case if crises are rare but deep. Otherwise entrepreneurs will opt out from the bailout umbrella and implement the laissez-faire equilibrium.[16]

6.2 Bailout Fund

Since it is optimal to tax borrowing or investment in period 0 and to implement bailouts in period 1, one might be tempted to combine the two policy measures and use the proceeds of the period-1 prudential tax to finance the bailouts. This can be done by accumulating the prudential tax proceeds in a "bailout fund" that can be distributed in the future if entrepreneurs experience binding financial constraints.[17] It would seem much preferable to finance the bailouts with a tax that tends to correct the distortions induced by the expectation of bailouts than by a tax that introduces new distortions in the economy.

We analyze this policy proposal by considering a planner who saves the tax revenue $T = \tau_0 I(k)$ raised via macroprudential taxation in period 0 and uses it to bail out entrepreneurs in period 1 in order to relax their financial constraints. We continue to assume that this bailout is made in proportion to the capital holdings k of entrepreneurs so that each unit of asset receives an additional payment from the bailout fund of $f = T/k$. In order to keep our analysis as general as possible, we allow for an additional potential bailout that is financed by a period 2 tax and that is described by policy functions $s(n)$ and $\tau_2(n)$. For example, if we set $s(n) = \tau_2(n) = 0$, we replicate an equilibrium in which only bailouts from the bailout fund are provided. If we set $s(n) = s^{BL}(n)$, entrepreneurs obtain the optimal discretionary bailout described in section 5.

The period-1 problem of entrepreneurs under a bailout fund, denoted by the superscript BF, is[18]

$$v^{BF}(k, d_0; f) = \max_x \left[\kappa A_1 + \kappa(\tau_2(n)) A(x) - x - d_0 + f + s(n)\right] k + \\ + \lambda \left\{\kappa A_1 - x - d_0 + f + s(n) + \phi p_2\right\},$$

[16] The planner may even want to commit to bailouts that are larger than what is optimal under discretion in order to "bribe" entrepreneurs to invest less.

[17] This is for example common practice for most deposit insurance systems (see Garcia, 1999).

[18] For ease of notation, we assume that the transfer T/k is made to entrepreneurs no matter if their financial constraint is binding or not. Similar results are obtained if the planner rebates the tax revenue T in other ways when the financial constraint on entrepreneurs is loose, e.g. in lump-sum fashion.

and similarly for $w^{BF}(k, d_0; f)$ under the planner. The welfare properties of a bailout fund are described in the following proposition.

Proposition 12 (Bailout Fund) *(i) Limiting bailouts to the resources available from a bailout fund reduces welfare compared to the optimal policy mix.*

(ii) Introducing a bailout fund in addition to the optimal policy mix described above does not affect welfare, but requires a higher level of macroprudential taxation $\tau_0^{BF} > \tau_0^{MIX}$.

Proof. Using the period 0 budget constraint of entrepreneurs $d_0 k = (1+\tau_0) I(k)$ together with the planner's budget constraint $f = T/k = \tau_0 I(k)/k$, we find that the revenue accumulated in the bailout cancels out, i.e. $d_0 - f = I(k)/k = d(k)$ and therefore $w^{BF}(k, d_0; f) = w^{BF}(k, d(k); 0)$. This implies that the optimal level of capital investment remains unchanged from what it was in the absence of a bailout fund.

The private optimality condition for investment of entrepreneurs is

$$Ev_k^{BF} = E[\kappa A_1 + \kappa A(x) - x] + f - (1+\tau_0) I'(k) - kE[\lambda]\left[d'(k) + \tau_0 I'(k)/k\right] = 0,$$

and is increased because $v_k^{BF} = v_k^{MIX}(k) + f$. This captures that the bailout fund increases moral hazard because entrepreneurs expect to receive greater transfers.

Equating $Ev_k^{BF} = Ew_k$ therefore requires that we set the macroprudential tax to

$$\tau_0^{BF} = \frac{E\left[\tilde{\lambda} - \lambda\right] d'(k)\, k}{(1 + E[\lambda]) I'(k) - d(k)},$$

in order to implement precisely the same equilibrium as in the absence of the bailout fund. This expression differs from the optimal tax τ_0 in (17) and (28) by the term $-d(k)$ in the denominator. This term clearly increases the optimal macroprudential tax rate above the leve in the respective equilibrium without bailout fund.

To prove statement (i) in the Proposition, observe that setting $s(n) = \tau_2(n) = 0$ implies that the equilibrium under the bailout fund implements precisely the equilibrium under macroprudential regulation MP. Welfare in this equilibrium is below welfare under the optimal policy mix, since we have restricted the magnitude of bailouts.

Statement (ii) in the Proposition immediately follows from the observation that the transfer from the bailout fund cancels out and the equilibrium under the optimal policy mix is implemented. ■

The intuition for our results is that introducing a bailout fund does not yield any efficiency benefits — the planner has no comparative advantage in holding precautionary savings compared to entrepreneurs, as long as she can determine the correct level of savings via macroprudential regulation. As we observe in point (i), limiting bailouts to the resources available from the fund therefore replicates the macroprudential equilibrium, which exhibits lower welfare than the equilibrium under the optimal policy mix.

In addition, observe that the bailout fund distorts incentives, since it implies greater transfers in period 1. This calls for even higher levels of the macroprudential tax τ_0 in order to undo the distortion. As stated in point (ii), the equilibrium with a bailout fund and additional discretionary bailouts therefore implements the same equilibrium as the optimal policy mix, but with a higher macroprudential tax rate.

6.3 Numerical Illustration

We now present a simple numerical illustration of the results that we have obtained to provide additional intuition. For the sake of simplicity, we assume that the investment cost function $I(k)$ and the productivity $A(x)$ are given by

$$\begin{aligned} I(k) &= k^2 \\ \text{and} \quad A(x) &= \min(x, \bar{x}). \end{aligned}$$

The numerical values of the parameters chosen are given below in Table 1. Furthermore, we assume that A_1 follows a symmetrically truncated normal distribution with mean 1 and standard deviation σ. Further details on implementing the simulation are given in the Numerical Appendix B.

α	ω	ϕ	$\bar{x}$
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	1

Table 1: Parameter values for numerical illustration

In Figure 3 we vary the standard deviation σ and illustrate the effects on the equilibrium across the different policy regimes: laissez faire (LF), macro-prudential regulation (MP), bailouts (BL) and the optimal policy mix (MIX). As the standard deviation σ increases, the probability of binding constraints in the economy rises.

Panel 1 illustrates the effects on welfare under the different regimes. Under all four regimes, welfare is a strictly declining function of σ. The welfare losses are minimized under the optimal policy mix. For a low σ, using discretionary bailouts is superior to using macroprudential regulation – this is because the probability of binding constraints is low and bailouts allow for greater state contingency. For $\sigma > 0.47$, macroprudential regulation is superior to bailouts. In the Figure, welfare is always lowest under laissez faire. This is not necessarily always the case, since discretionary bailouts create moral hazard.[19]

In panel 2, we depict the initial capital investment k. Under macroprudential policy, capital k is always lower than under laissez faire. With bailouts, k is always

[19] In our simulations, we found that $W^{BL} < W^{LF}$ may occur if the probability of being constrained is close to 1 and if amplification effects are strong, i.e. $\phi\kappa$ is close to 1.

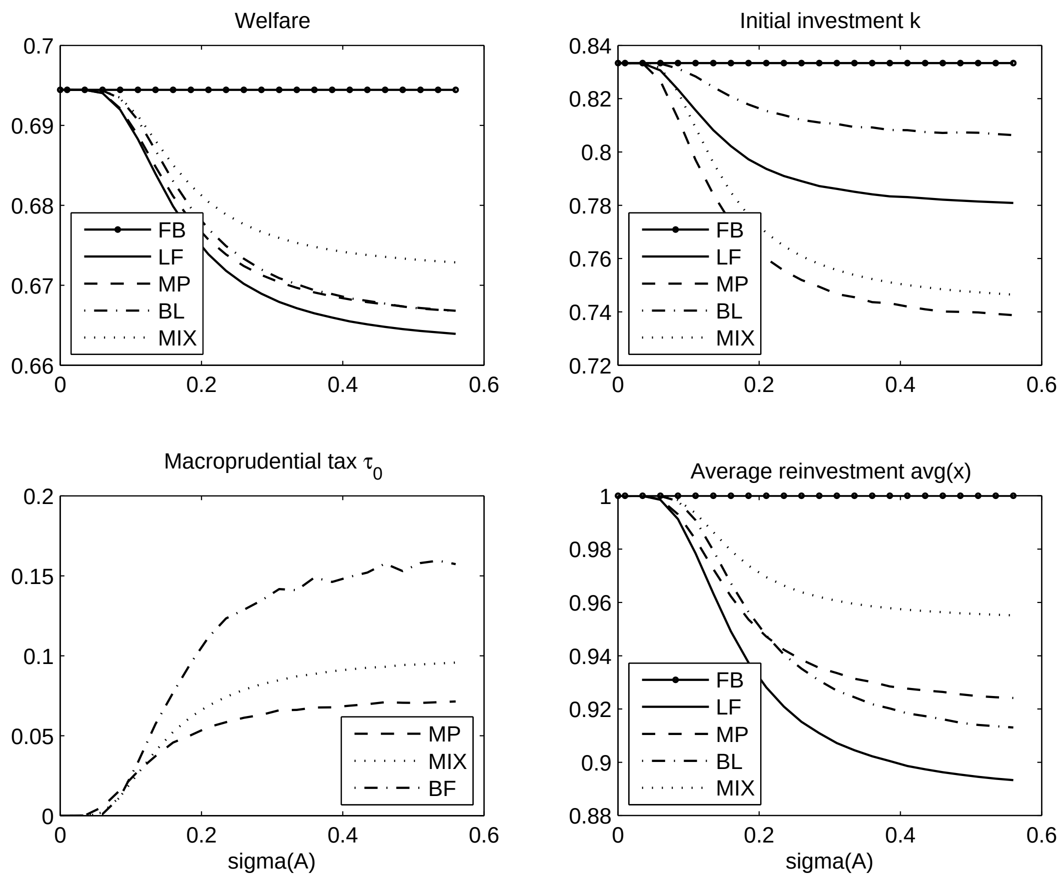

Figure 3: Numerical illustration of policy regimes

higher than under laissez faire. Under the optimal policy mix, capital k is higher than laissez faire as long as $\sigma < 0.07$ and lower than laissez faire if $\sigma > 0.08$. This is because for a low probability of being constrained, the planner relies more on bailouts and less on macroprudential regulation. Finally, observe that investment under the optimal policy mix is always greater than under macroprudential regulation alone.

Panel 3 describes the optimal macroprudential instrument τ_0. For low values of $\sigma < 0.12$, the planner imposes a lower τ_0 in the optimal policy mix than if only macroprudential regulation is available. This is because she relies mostly on bailouts to address binding financial constraints, and bailouts occur relatively rarely. For higher levels of σ, binding constraints and bailouts are increasingly common, and both factors induce the planner to raise τ_0 more heavily under the optimal policy mix. The regime under a bailout fund BL always requires greater macroprudential taxation than the optimal policy mix, but delivers the same real allocation.

Finally, panel 4 illustrates the average re-investment x across the different policy regimes. Reinvestment is greatest under the optimal policy mix and lowest under laissez faire. For low values of $\sigma < 0.21$, average reinvestment x is greater under bailouts; however, bailouts provide increasingly stronger incentives for additional investment in k, which increases the tightness of constraints. If σ is above 0.21, then reinvestment is actually greater under macro-prudential regulation.

More broadly speaking, the described economy faces a trade-off between how much to invest ex-ante in k and how much to reinvest ex-post in x when financial constraints are binding. Macroprudential regulation allows the planner to target the former and is most useful when constraints bind frequently. Bailouts allow her to target the latter, but distort investment in k: they are most useful when constraints bind rarely or when the distortion can be offset by macroprudential regulation.

7 Alternative Bailout Measures

This section compares how alternative bailout measures, including subsidies to borrowing and investment tax credits, affect the ex-ante incentives of entrepreneurs.[20] The common feature of all bailout measures is that they transfer resources to constrained entrepreneurs in period 1, which relaxes their constraint and leads to an amplified response of investment in that period. We maintain the assumption that the bailout transfers are financed by a distortionary tax on labor.

The effects of bailouts in period 1 do not depend on how they are provided, as long as the investment of entrepreneurs is determined by binding financial constraints. The only thing that matters for the equilibrium is the size of the transfer. Viewed from period 1 all our alternative bailout measures are therefore equivalent.

[20] We can think of subsidies to borrowing e.g. as being implemented through interest rate cuts. Furthermore, crises lending programs often also include an implicit subsidy to borrowing, as governments provide loans at an interest rate that is cheaper than the market rate.

From the perspective of period 0, however, alternative ways of providing bailouts have different effects on ex-ante incentives. We noted in section 5 that our benchmark measure, a bailout of s per unit of capital, increased the incentives for entrepreneurs to invest in period 0, which captures a form of "moral hazard" created by bailouts.[21] Unsurprisingly, the effects of lump-sum bailouts on ex-ante incentives are more benign. They achieve the positive liquidity effects of bailouts without a direct effect on the incentives for period-0 investment. In addition, the tax τ_2 that is imposed to recover the costs of the bailout reduces the incentives for period-0 investment and therefore mitigates the overborrowing problem that we identified in section 4 because it lowers the private returns to capital below the social returns $\kappa(\tau_2) < \eta(\tau_2)$. However, in practice, lump sum transfers are difficult to implement: agents who own more capital usually receive larger bailouts, as captured by the notion of "too big to fail."

Investment tax credits and subsidies to borrowing differ from standard bailouts because both of them channel subsidies specifically into the activity that the financial constraint restricts – into investment or borrowing. This raises the perceived cost of financial constraints for entrepreneurs and induces them to engage in more precautionary savings and to reduce period 0 investment compared to a bailout that is proportional to k.

Proposition 13 *Bailouts in the form of investment tax credits or borrowing subsidies mitigate overinvestment and reduce the need for macroprudential regulation compared to bailouts in the form of subsidies to capital.*

Proof. An entrepreneur who receives a tax credit s per unit of investment x in period 1 maximizes the objective

$$\begin{aligned} v^{ITC}(k^i; s, \tau_2) &= \max_{x^i} \left[\kappa A_1 + \kappa(\tau_2) A(x^i) - (1-s)\, x^i\right] k^i - I(k^i) \\ &\quad + \lambda^i \left[\kappa A_1 + \phi p_2 - (1-s)\, x^i - d(k^i)\right]. \end{aligned}$$

The optimality condition on investment x determines the shadow cost of the constraint perceived by the entrepreneur,

$$\frac{\lambda^{ITC}}{k^i} = \frac{\kappa(\tau_2)\, A'(x)}{1-s} - 1. \tag{29}$$

As in previous sections, the planner finds it optimal to provide a tax credit $s > 0$ when the financial constraint is binding. The denominator in the fraction then implies that entrepreneurs experience the shadow price under the investment tax

[21] Subsidies to period-1 production or period-1 employment have similar effects on ex-ante incentives since production and employment increase in k, but they introduce an additional distortion in the choice of labor ℓ_1, which pushes the productivity of capital below κA_1.

credit to be greater than under a regular bailout $\lambda^{ITC} > \lambda^{BL}$. Observe that the magnitude of the optimal bailout as captured by τ_2 continues to be the same in both cases and is determined by (22).

The marginal value of period-0 capital investment that results from this measure is

$$v_k^{ITC}(\cdot) = \kappa A_1 + \eta(\tau_2) A\left(x^i\right) - x^i - I'\left(k^i\right) - \frac{\kappa(\tau_2)A'(x^i) - 1}{1-s} d'\left(k^i\right).$$

For any given level of τ_2, a simple comparison with equation (25) reveals that $v_k^{ITC}(\cdot) < v_k^{BL}(\cdot)$, i.e. that the overborrowing/overinvestment problem is mitigated under the investment tax credit. Borrowing less in period 0 allows entrepreneurs to invest more and receive a higher subsidy in period 1.

A subsidy to borrowing (SB) targets the same wedge as an investment tax credit and leads to the same expression (29) for λ^{ITC}. The only difference between the two is that the planner's budget constraint implies that the wedge $s^{ITC} = \tau_2 \varepsilon(\tau_2) A(x)/x$ for the investment tax credit and $s^{SB} = \tau_2 \varepsilon(\tau_2) A(x)/d_1$ for an interest rate cut. Depending on $x^i \gtrless d_1^i$, the planner's tax revenue $\tau_2 \varepsilon(\tau_2) A(x) k$ is spread over a smaller or larger base, implying a greater percentage subsidy s for one case or the other. ■

Next we compare the investment incentives of entrepreneurs under an investment tax credit to those of a social planner. Using the planner's budget constraint $sx = \tau_2 \varepsilon(\tau_2) A(x)$, the shadow price of entrepreneurs under the investment tax credit can be written as

$$\frac{\lambda^{ITC}}{k^i} = \frac{\kappa(\tau_2) A'\left(x^i\right)}{1 - \tau_2 \varepsilon(\tau_2) A\left(x^i\right)/x^i} - 1.$$

The planner's shadow price on the financial constraint continues to be given by the expression for $\tilde{\lambda}^{BL}$ – it depends only on the liquidity effect of bailouts, not on the type of bailout provided. The marginal value of period-0 capital investment for entrepreneurs and the planner depends on their respective shadow prices,

$$\begin{aligned} Ev_k^{ITC}(\cdot) &= E\left[\kappa A_1 + \eta(\tau_2) A(x) - x\right] - I'(k) - \lambda d'(k), \\ Ew_k^{ITC}(\cdot) &= E\left[\kappa A_1 + \eta(\tau_2) A(x) - x\right] - I'(k) - kE\left[\frac{\eta(\tau_2) A'(x) - 1}{1 - \left[\phi\kappa(\tau_2) + \tau_2 \varepsilon(\tau_2)\right] A'(x)}\right] d'(k) \end{aligned}$$

where $\lambda = \lambda^{ITC}$ or $\tilde{\lambda}^{BL}$ for entrepreneurs and the planner – a higher shadow price on the constraint implies a lower marginal valuation of capital and therefore less investment. The overall effect depends on a comparison between the two,

$$\lambda^{ITC} = \frac{\kappa(\tau_2)A'(x)}{1 - \tau_2 \varepsilon(\tau_2) A(x)/x} - 1 \gtrless \frac{\eta(\tau_2) A'(x) - 1}{1 - \left[\phi\kappa(\tau_2) + \tau_2 \varepsilon(\tau_2)\right] A'(x)} = \tilde{\lambda}^{BL}$$

We observe that there are four differences between the two expressions. First, the planner values the full social return $\eta(\tau_2) A'(x)$ of x in the numerator whereas entrepreneurs only perceive the net-of-tax return $\kappa(\tau_2) A'(x)$. Second, the planner recognizes the amplification effects from the borrowing constraint as captured by the term $-\phi\kappa(\tau_2)$ in the denominator. These two effects tend to increase the planner's shadow price compared to that of entrepreneurs. Third, in the denominator, entrepreneurs observe a flat subsidy rate which depends on the average return of period 1 investment $A(x)/x$ whereas the planner's valuations depend on the marginal return $A'(x)$, which is lower due to the concavity of $A(x)$. Fourth, for entrepreneurs the term "-1" is not divided by the denominator since the tax credit introduces a wedge between the return to investment and the cost of investment, whereas the term is in the numerator of the expression of the planner since the cost of investment is also subject to amplification effects. These latter two effects tend to increase the shadow price of entrepreneurs compared to that of the planner.

Overall, we cannot determine which of the two expressions is larger. If the constraint is marginally binding, then $d\lambda^{ITC}/d\tau > d\tilde{\lambda}^{BL}/d\tau$ at $\tau_2 = 0$, but more generally situations may arise when $\tilde{\lambda}^{BL} > \lambda^{ITC}$, for example when ϕ is high so that the amplification effects internalized by the planner are high. Conceivably, the optimal macroprudential regulation could take the form of a subsidy (rather than a tax) on investment and borrowing in period 0.

In conclusion, we note that alternative forms of providing bailouts have superior incentive effects for capital investment k in period 0. In the absence of macroprudential regulations, they therefore create smaller distortions in period 0. If the planner has access to a macroprudential policy instrument in period 0, then they require smaller macroprudential policies to correct the distortions in incentives stemming from bailouts.

8 Conclusions

This paper develops a simple framework of optimal policies in an environment where collateral-dependent borrowing constraints lead to financial amplification. If policymakers have access to lump-sum transfers, they can restore the first-best equilibrium in which borrowing constraints are irrelevant. Otherwise, all policies fall into the category of second-best interventions, i.e. they achieve first-order welfare gains by mitigating binding borrowing constraints in the economy, but at the expense of introducing second-order distortions, i.e. distortions that are initially negligible but grow with the square of the policy intervention.

In accordance with the theory of the second-best (see Lipsey and Lancaster, 1956), it is optimal to use all second-best instruments available in such a setting. In particular, we show that it is optimal to both restrict borrowing ex-ante via macroprudential regulation and to relax borrowing constraints ex-post by providing bailouts. This implies that policymakers should both "lean against the wind" and "mop up after the crash."

In comparing the relative benefits and disadvantages, we find that bailouts are better targeted because they are conditional on an adverse state of nature having materialized, but they lead to problems of time consistency as they distort the ex-ante incentives of entrepreneurs to invest. Macroprudential regulation is more blunt since it is imposed in the anticipation that crises may occur in the future, but it can resolve the time-inconsistency of bailouts. These distinctions between the two policy measures reinforce the message that it is generally desirable to use both of them.

There are a number of issues that we have left for future research. We have crafted the model in this note such that the various policy measures under consideration do not lead to redistributions among different sectors in the economy. This allowed us to focus exclusively on the efficiency implications of different policies. However, in practice the level of financial regulation has first-order redistributive implications, especially if bailout are financed out of general tax revenues.

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A Appendix

A.1 Binding constraint in period 0

The capital of a defaulting entrepreneur can be sold in period 1 at a price equal to,

$$p_1 = \kappa A_1 - x + \phi\kappa A_2 + \frac{(1-\phi)\,\kappa A_2}{1+\lambda_1} = \kappa A_1 - x + \frac{1+\phi\lambda_1}{1+\lambda_1}\kappa A_2$$

Looking at the borrowing constraint that we defined above, the net cash flow in period 1 (including additional borrowing capacity) is $-p_1 + \kappa A_1 - x + \phi\kappa A_2$ and the additional cash flow (net of repayment) in period 2 is $(1-\phi)\,\kappa A_2$.

A.2 First-Best Solution

If a planner has the power to engage in lump-sum transfers, it is easy to see that she can always restore the first-best equilibrium. In the first-best equilibrium, entrepreneurs borrow $I\left(k^{FB}\right)$ in period 0 and up to $x^{FB}k^{FB}$ in period 1. The planner can replicate this allocation simply by transferring $I\left(k^{FB}\right)$ and $x^{FB}k^{FB}$ from workers to entrepreneurs in periods 0 and 1 respectively, and transferring the same amount back in the following period. As a result, entrepreneurs never experience binding borrowing constraints. Similarly, a planner who can raise revenue via lump-sum taxes and use it to subsidize the asset price in order to fully relax binding financial constraints can restore the first-best equilibrium.

A.3 Bailouts under Commitment

Assume a planner who can commit to a bailout policy $s^{BLc}(n)$ financed by a tax $\tau_2^{BLc}(n)$, which are both functions of the aggregate period 1 liquid net worth per unit of capital $n = \kappa A_1 - d(k)$. Such a planner will solve the following optimization problem, denoted by superscript BLc to capture bailouts under commitment:

$$\max_{k,x(n),\tau_2(n)} E\left\{w^{BLc}(k, x(n), \tau_2(n))\right\} \quad \text{s.t.} \quad Ev_k^{BLc}(k; \tau_2(n)) = 0 \quad [\xi]$$

where

$$w^{BLc}(k, x(n), \tau_2(n)) = \{\kappa A_1 + \eta(\tau_2) A(x(n)) - x(n)\} k - I(k) + \\ +\lambda \{[\kappa A_1 + [\phi\kappa(\tau_2) + \tau_2\varepsilon(\tau_2)] A(x(n)) - x(n)] k - I(k)\}$$

and $\quad v_k^{BLc} = \kappa A_1 + \eta(\tau_2(n)) A(x(n)) - x(n) - I'(k) - \left[\kappa(\tau_2(n))A'(x) - 1\right] d'(k)k$

Observe that individual entrepreneurs take aggregate net worth and therefore the tax rate and bailout as given.

The difference between this optimization problem and the time-consistent problem is that the planner sets $x(n)$, $\tau_2(n)$ and by implication $s(n)$ already in period 0 and internalizes how her decisions affect the ex-ante incentives of entrepreneurs to invest, as reflected by the implementability constraint $Ev_k^{BLc} = 0$. Unlike in our formulation of the discretionary planning problem, the function $w^{BLc}(\cdot)$ is not a Bellman equation that is maximized – it is just short-hand notation for the payoff of entrepreneurs, given the optimal policies chosen in period 0. To save on notation we will omit the argument n on x and τ_2 in the following. The Lagrangian associated with the planner's problem is

$$L = E\left\{w^{BLc}(k, x, \tau_2)\right\} - \xi E v_k^{BLc}(k, \tau_2)$$

The planner's optimal capital investment k is determined by the condition

$$Ew_k^{BLc} = \xi E v_{kk}^{BLc}$$

However, the implementability constraint captures that private agents determine k according to their first-order condition $Ev_k^{BLc} = 0$. As we observed earlier, $Ew_k < 0$ at the privately optimal level of k. The planner's optimality condition on k therefore pins down the shadow price

$$\xi = \frac{Ew_k}{Ev_{kk}} > 0.$$

The optimality conditions on the optimal tax τ_2 and period 1 investment x are

$$\left[(1+\lambda)\tau_2\varepsilon'(\tau_2) + \lambda(1-\phi)\varepsilon(\tau_2)\right] A(x) k = \xi v_{k\tau_2} \qquad (30)$$

$$\left\{\eta(\tau_2) A'(x) - 1 - \lambda\left[1 - (\phi\kappa(\tau_2) + \tau_2\varepsilon(\tau_2)) A'(x)\right]\right\} k = \xi v_{kx} \qquad (31)$$

For loose borrowing constraints, the two conditions can be simplified to yield $\tau_2\varepsilon(\tau_2) A(x) = 0$ and $\kappa A'(x) = 1$, implying that the planner does not intervene and the first-best level of investment x^{FB} is implemented.

If the borrowing constraint is binding, we observe that $v_{kx} > 0$ and $v_{k\tau_2} > 0$ for $\tau_2 < \tau_2^{BLd}$. The optimality condition on τ_2 implies

$$\lambda^{BLc} = \frac{\eta(\tau_2) A'(x) - 1 - \xi/kv_{kx}}{1 - [\phi\kappa(\tau_2) + \tau_2\varepsilon(\tau_2)] A'(x)} \qquad (32)$$

The planner under commitment perceives the shadow price of relaxing the constraint as lower than the shadow price under discretion because of the term $-\xi/kv_{kx} < 0$. This term captures that relaxing the constraint induces entrepreneurs to engage in more period 0 capital investment, which is already excessive.

The optimality condition on τ_2 becomes

$$\frac{1+\lambda}{\alpha}\frac{\tau_2}{1+\tau_2} = \lambda\,(1-\phi) - \frac{\xi v_{k\tau_2}}{\varepsilon\,(\tau_2)\,A\,(x)\,k}$$

Comparing this expression to equation (22), the optimal tax rate $\tau_2^{BLc}(n)$ in a bailout regime under commitment is below the optimal tax rate under discretion $\tau_2^{BLd}(n)$ for two reasons: first, the planner perceives a lower cost of binding constraints λ^{BLc}; second, the planner lowers the transfer in order to reduce the incentives for excessive borrowing and investment, as captured by the term on the right-hand side.

In short, the planner reduces the magnitude of the bailout measures that are ex-post efficient in order to provide better ex-ante incentives.

We observe that if the planner committed to a bailout policy that is a function $s\,(A_1)$ of the period 1 productivity shock A_1 rather than a function $s\,(n)$, the outcome would be identical since entrepreneurs take both A_1 and aggregate n as given and since both are sufficient statistics for the state of nature. By contrast, if the planner can commit to a bailout transfer policy that is conditional on k and that includes penalties on entrepreneurs who borrow excessively, then commitment may solve the time consistency problem. We discuss this in detail in section 6.