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Inflation, Debt, and Default[1]

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[PRELIMINARY]

Abstract

We document that the co-movement of inflation and domestic consumption growth fluctuates over time. We argue that the co-movement of inflation and domestic consumption growth affects debt pricing and debt dynamics. In particular, a positive co-movement of inflation and consumption makes returns on government bonds negatively correlated with domestic consumption: this lowers risk premia on nominal domestic debt while making the debt more risky for the government. We construct a simple model of nominal domestic government debt incorporating an explicit default decision and an exogenous inflation risk to assess the overall effect of the inflation process on risk premia, probability of explicit default and equilibrium borrowing costs. Consistent with the data, we find that borrowing costs fall as the covariance of inflation and consumption growth increases.

Keywords: Inflation, Default, Domestic Debt
JEL classification codes: F34, G12

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1 Introduction

This paper studies the connection between inflation, nominal debt pricing and debt dynamics. We start from the observation that the conditional co-movement of inflation with domestic consumption growth fluctuates over time. If inflation co-varies with domestic consumption growth, then returns on domestic nominal debt are high (low) when consumption growth is low (high). This feature makes domestic nominal bonds less risky, from a domestic investor perspective, and thus, if government debt is mostly held domestically as in many developed countries, they should trade at a, *ceteris paribus*, lower interest rate.

In the first part of the paper we show that, for advanced economies, in countries/periods in which the covariance of inflation with domestic consumption growth is high, real interest rates on government bonds tend to be low, suggesting that this covariance is an important factor in the pricing of government debt. Having established that conditional covariance is empirically connected to bond pricing, we fully explore its impact on debt dynamics by developing a simple structural model of debt and default with stochastic inflation.

Our model extends existing models of sovereign debt in two directions. First, we introduce domestic, risk averse, lenders, as opposed to foreign, risk neutral as usually assumed in the literature. This distinction is important since a large amount of public debt is held domestically in advanced economies. Second, we introduce exogenous stochastic inflation so that government bond rates reflect both inflation and default risk. These two features allow us to explicitly analyze the connection between stochastic discount factors of the domestic agents (lenders **and** borrowers), debt pricing and default probabilities, and to analyze how this relation changes as the co-movement between inflation and consumption growth varies.

Consistent with the data, we find that borrowing costs fall as the covariance of inflation and consumption growth increases. This reflects the reduced risk associated with the nominal debt from the perspective of a domestic lender. Despite the reduced borrowing costs, debt levels do not necessarily rise as the covariance of inflation and consumption growth increases. In fact, precautionary motives increase as the covariance of inflation and consumption growth rises. For the government, debt becomes less attractive the more inflation tends to occur in good times. This channel can offset the low rates given by the lenders. Through the interplay of these channels, default probabilities also change as the co-movement of inflation and consumption changes.

Our paper is related to several strands of literature. On the empirical side our findings are related to studies on the importance of the inflation risk premium and its variation, as, for example, Boudoukh (1993) or Ang, Bekaert and Wei (2008). On the theoretical side, the backbone of our set-up is a debt default model with incomplete markets as in Eaton and Gersowitz (1983) or Arellano (2008). While these papers focus on foreign debt, Reinhart and Rogoff (2011) suggest that the connection between default, domestic debt and inflation is an important one. D'Erasmo and Mendoza (2013) tackle on the issue of default on domestic debt but do not include inflation. Our general question is also analyzed in recent work that studies how joining a monetary union can affect the probability of a self-fulfilling crisis in a debt default model (see Aguiar et al. 2013 and Corsetti and Dedola 2013). In contrast to those papers, our focus is not on self-fulfilling crises but rather on the impact that inflation can have on fundamentally driven default crises, so we view our work as complementary to theirs. The paper is structured as follows. In section 2 we discuss our empirical findings, section 3 develops a model of domestic debt default, and section 4 presents our main results on the impact of stochastic inflation. Section 5 concludes.

2 Empirical Motivation

In this section, we present findings regarding conditional covariance of inflation and consumption growth (i.e. the cyclicality of inflation), which captures the dimension of monetary policy we find relevant for debt pricing and debt dynamics. Our dataset includes quarterly observations on real consumption (private plus public) growth, inflation measured as the growth in the GDP deflator, interest rates on government bonds and government debt to GDP ratios for 21 advanced OECD economies from 1970Q1 and 2012Q4.

First, we show that, after controlling for a variety of factors, conditional covariance of inflation and consumption growth is connected to the real rates faced by governments borrowing on domestic markets. In particular, high covariance is associated with low real rates.[2] These findings together suggest that the inflation process can have a significant effect on the

[2]We also find evidence that this conditional covariance falls significantly for countries which join a monetary union.

pricing of domestic debt and thereby debt and default dynamics.

The first object we construct with this data is a panel for our main object of interest, i.e. the conditional co-movement between inflation and consumption growth. To do, so we follow Boudoukh (1993) and first formulate a vector autoregression (VAR) model for inflation and consumption growth. The basic VAR is:

$$\begin{bmatrix} \pi_{it} \\ g_{it} \end{bmatrix} = A_i \begin{bmatrix} \pi_{it-1} \\ g_{it-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{\pi it} \\ \varepsilon_{git} \end{bmatrix}.$$

where π_{it} and g_{it} are inflation and consumption growth in country i in period t, A_i is a country-specific 2-by-2 matrix and $\varepsilon_{\pi it}$ and ε_{git} are innovations in the two time series. We then estimate the VAR using standard OLS and construct time series for residuals $\varepsilon_{\pi it}$ and ε_{git} for each country.

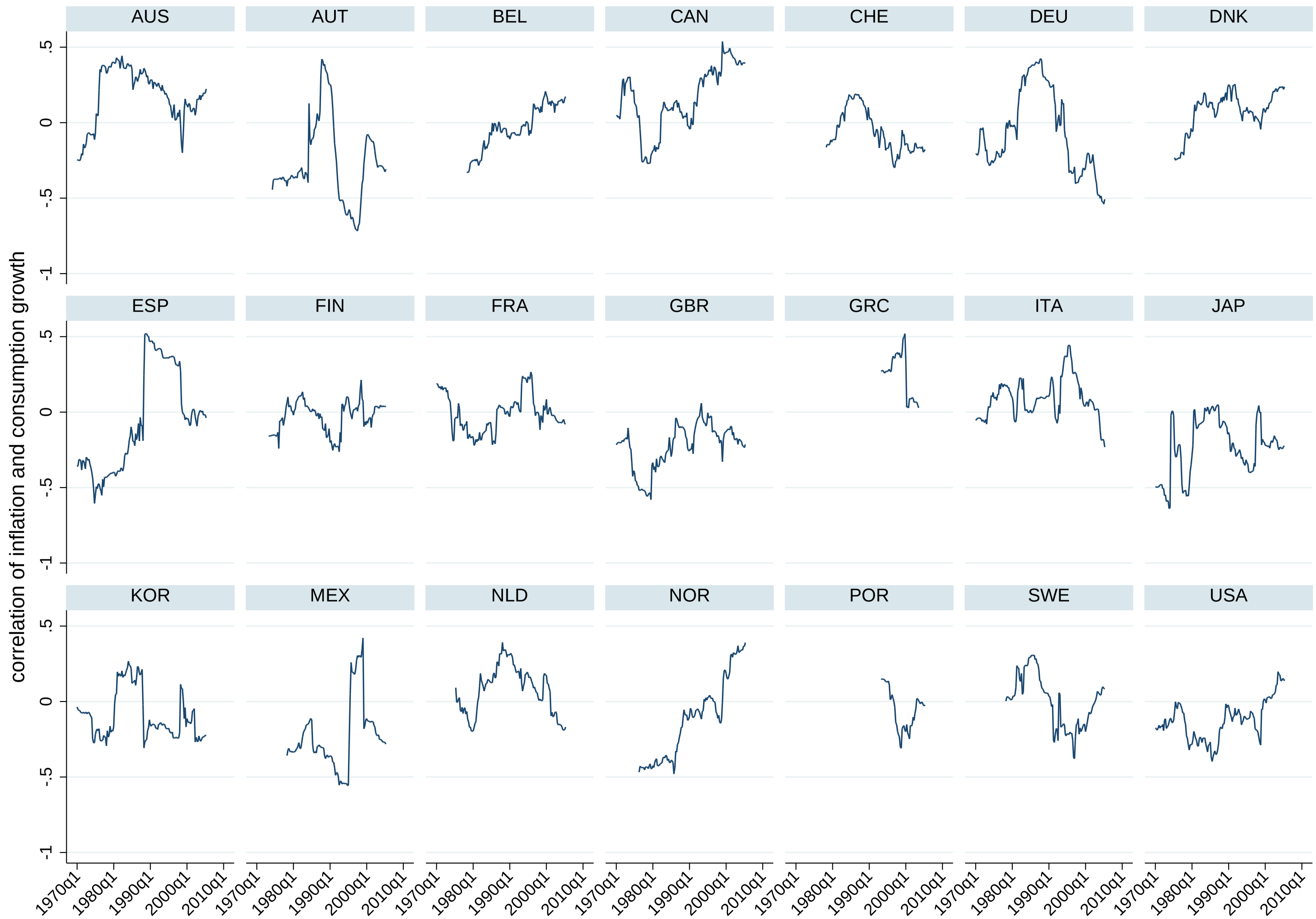
We then measure the conditional co-movement between these two series by measuring the correlation and the co-variance between the two innovations in overlapping country-windows, which in our benchmark results are comprised of 20 quarters. In figure 1 we report the path for this conditional correlation for a subset countries in our sample. Figure 1 shows that countries experience, over time, substantial variation in conditional co-movement.[3]

In table 1, we conduct more systematic analyses of this co-movement and in particular assess how it relates to interest rates faced by governments. First, we construct a measure of real interest rate faced by governments in our sample. To do so, we use the Datastream Benchmark bond (10 years) redemption yield for all the countries in our sample, and we subtracted from it expected inflation, computed using forecast based on the VAR estimated above. In table 1, we then regress this measure of the real interest rate on two measures of the conditional co-movement of inflation with consumption growth. All specifications include a full set of country and times fixed effects.

The key result from the table 1 is that in countries/periods with higher conditional co-movement between inflation and consumption growth governments face lower interest rates. This finding is robust to the inclusion of the average inflation, consumption growth, the variance of inflation, and the variance of inflation of consumption growth (columns 1 and

[3]The inclusion of Mexico and Korea is illustrative and does not affect the results presented here.

Figure 1: Conditional Correlation Between Inflation and Consumption Growth



3). This association also is robust to the inclusion of the level of government debt as an additional regressor (columns 2 and 3). Not surprisingly, more debt is weakly associated with higher interest rates, but the relation between inflation cyclicality and interest rates remains.

Table 1: Inflation co-movement and real interest rates

	Real yield on government debt		
	(1)	(2)	(3)
Inflation co-movement: $\text{cov}(\varepsilon_\pi, \varepsilon_{g_c})$	-1.379**	-2.007***	-2.066***
	(0.576)	(0.504)	(0.636)
Variance of inflation: $\text{var}(\varepsilon_\pi)$	0.418	0.721***	0.211
	(0.313)	(0.222)	(0.256)
Variance of consumption: $\text{var}(\varepsilon_{g_c})$	-0.229	-0.470**	-0.381**
	(0.152)	(0.219)	(0.148)
Inflation: π	1.979***		2.392***
	(0.302)		(0.338)
Consumption growth: g_c	-1.017		-1.041
	(0.630)		(1.091)
Public debt (percent of GDP)		0.00281	0.0104
		(0.0107)	(0.00765)
adj. R^2	0.897	0.871	0.917
N	2394	2049	2049

Standard errors in parentheses

All regressions include country and year fixed effects

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

3 Model

We extend the standard model of sovereign default of Eaton and Gersovitz (1981) and Arellano (2008) in two dimensions: exogenous *inflation* and risk averse *domestic lenders*.

3.1 Households

We consider a closed economy that is populated by a continuum of two types of households: poor, hand-to-mouth households who are impatient, and patient lenders. Both types of households have preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta_i^t u(c_{it}) \tag{1}$$

where $0 < \beta_h < \beta_\ell < 1$ and $c_{ht}, c_{\ell t}$ are the discount factors and consumption at time t of the hand-to-mouth households and lenders respectively. The households' period utility function is given by

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

The hand-to-mouth households receive a stochastic stream of non-storable consumption good y, which follows a Markov process. Lenders receive αy with $\alpha > 1$. For simplicity, we assume that the inflation follows an exogenous process $\pi(y, y')$.[4]

3.2 Government

The government has access to debt markets in which it issues one-period non-contingent bonds to the domestic lenders. Bonds are risky because debt contracts are not enforceable which may lead to government default, and also because they may lose value due to exogenous inflation. We assume that the government maximizes the welfare of only the poor households, possibly due to political economy motives not explicitly modeled here. All proceeds from the government's debt operations are rebated to the poor households in a lump sum fashion.

[4] This specification allows us to model the covariance channel without carrying the inflation as an additional state variable.

Given the option to default, $V^o(B,y)$ satisfies

$$V^o(B,y) = \max_{c,d} \left\{ V^c(B,y), V^d(y) \right\} \tag{2}$$

where B is incoming government assets, V^c is the value of not defaulting, and V^d is the value of default. When the government defaults, the economy is in temporary financial autarky and income may fall. The value of default is then given by

$$V^d(y) = u(y^{def}) + \beta_h \mathbf{E}_{y' \mid y} \left[\theta V^o(0,y') + (1-\theta) V^d(y') \right] \tag{3}$$

where θ is the probability that the government will regain access to credit markets, and

$$y^{def} = \begin{cases} \hat{y} & \text{if } y > \hat{y} \\ y & \text{if } y \leq \hat{y}. \end{cases} \tag{4}$$

The value, conditional on not defaulting, is given by

$$V^c(y) = \max_{B'} \left\{ u\left(y - q(B,y,B')B' + B\right) + \beta_h \mathbf{E}_{y' \mid y} \left[V^o \left(\frac{B'}{1+\pi(y,y')}, y' \right) \right] \right\} \tag{5}$$

where $q(B,y,B')$ is the bond price, π is inflation from this period to the next.

3.3 Lenders

Let μ be the measure of lenders in the economy. Lenders take as given the policy functions for government assets, $B(\cdot)$, and default, $d(\cdot)$. Let $s = 0, 1$ denote the government having access to credit markets. The default policy and probability of regaining access to credit markets determine the evolution of s. The lender's value function is then given by

$$\begin{aligned} W(b;y,s,B) = \quad & \max_{b'} \quad \left\{ u(c^\ell) + \beta_\ell \mathbf{E}_{y',s' \mid y,s} \left[W\left(\frac{b'}{1+\pi(y,y')}; y', s', \frac{B'}{1+\pi(y,y')} \right) \right] \right\} \\ & \text{s.t.} \quad c^\ell = \begin{cases} \alpha y + b - q(B,y,B'(B,y))b' & \text{if } s = 0 \\ \alpha y^{def} & \text{if } s = 1 \end{cases} \end{aligned} \tag{6}$$

3.4 Recursive Equilibrium

Definition The *recursive equilibrium* for this economy is defined as a set of policy functions for (i) lender assets $b^*(\cdot)$ and consumption $c_\ell^*(\cdot)$, (ii) government assets $B^*(\cdot)$ and default $d^*(\cdot)$, and (iii) a price function $q(B, y, B')$ such that:

1. Taking as given government policies and bond price, the representative lender's policy functions solve the optimization problem in (6).

2. Taking as given the bond pricing function, the government's policy functions solve the optimization problem in (2), (3), and (5).

3. The bond market clears,

$$\mu b^* + B^* = 0. \tag{7}$$

3.5 Characterization

In this environment, the bond price satisfies

$$q(B,y,B') = \beta_\ell \mathbf{E}_{y' \mid y} \left[\frac{1 - d^*\left(\frac{B'}{1+\pi(y,y')}, y'\right)}{1+\pi(y,y')} \frac{u'\left(c_\ell^*\left(\frac{-B'}{\mu(1+\pi(y,y'))}; y', 0, B^*\left(\frac{B'}{1+\pi(y,y')}, y'\right)\right)\right)}{u'\left(c_\ell^*\left(\frac{-B}{\mu}; y, 0, B'\right)\right)} \right] \tag{8}$$

The bond price can be rewritten as:

$$\begin{aligned} q(B,y,B') \;\; = \;\; & \beta_\ell \mathbf{E}_{y' \mid y} \left[\frac{1 - d^*\left(\frac{B'}{1+\pi(y,y')}, y'\right)}{1+\pi(y,y')} \right] \mathbf{E}_{y' \mid y} \left[\frac{u'\left(c_\ell^{*'}\right)}{u'(c_\ell^*)} \right] \\ & + \beta_\ell \mathbf{cov}_{y' \mid y} \left[\frac{1 - d^*\left(\frac{B'}{1+\pi(y,y')}, y'\right)}{1+\pi(y,y')}, \frac{u'\left(c_\ell^{*'}\right)}{u'(c_\ell^*)} \mid y \right] \end{aligned} \tag{9}$$

The first term shows that the probability of default and inflation increase borrowing costs (standard effects). We focus on the co-movement of inflation and consumption growth: the second term shows that pro-cyclical inflation *reduces* borrowing costs (new channel).[5] On the other hand, for the government, the covariance term means that the debt becomes more risky as the covariance of inflation and consumption growth increases.

4 Quantitative Analysis

In this section, we use a calibrated version of the model to investigate the role of the inflation process on debt and default dynamics. First, in the model with no default, we assess the impact of different inflation processes on borrowing costs. Then, using the full model, we evaluate the impact of different inflation processes on borrowing costs, debt and default dynamics.

4.1 Functional Forms and Parameters

Endowments y follow:

$$\log y' = \rho \log y + \epsilon \tag{10}$$

where $\epsilon \sim N(0, \sigma_y^2)$. We set $\rho = 0.95$ and $\sigma_y^2 = 0.02$.

The process for inflation is given by:

$$\pi(y, y') \equiv \bar{\pi} + \frac{\eta}{v_y}\left[\log\left(\frac{y'}{y}\right) - \mu_y\right] \tag{11}$$

where $\mu_y \equiv \mathbf{E}\left[\log\left(y'/y\right) \mid y\right]$ and $v_y \equiv \mathbf{var}\left[\log\left(y'/y\right) \mid y\right]$. This process for inflation satisfies $E(\pi \mid y) = \bar{\pi}$ and $\mathbf{cov}\left[\log\left(y'/y\right), \pi \mid y\right] = \eta$.

We set $\bar{\pi} = 0$ and compare the case of pro-cyclical inflation $\eta = 0.001$ and countercyclical inflation $\eta = -0.001$. A summary of our parameters can be found in Table 2.[6]

[5] In addition, countercyclical default *increases* borrowing costs.

[6] In practice, we use a truncated process: $\widehat{\pi}(y, y') = \max\{-1, \pi(y, y')\}$ to put a lower bound on unreasonably large deflations arising for rare transitions.

Table 2: Parameters

Discount factors	$\beta_h = 0.953$ $\beta_\ell = 0.983$
Risk aversion	$\gamma = 2$
Endowment process	$\rho = 0.95$ $\sigma_y^2 = 0.02$
Inflation process	$\bar{\pi} = 0$ $\eta \in \{\pm 0.0005, \pm 0.0010\}$
Lender relative endowment	$\alpha \in \{10, 100\}$
Probability of re-entry	$\theta = 0.282$
Population	$\mu = 1$

4.2 Model with No Default

In the version of the model where we shut down the default margin, borrowing costs are significantly lower with procyclical inflation as shown in Table 3.[7]

Table 3: Difference in Borrowing Costs

	$r_{-\eta} - r_\eta$ (in percent)	
	$\eta = 0.0005$	$\eta = 0.0010$
$\alpha = 10$	0.83	1.56
$\alpha = 100$	0.85	2.22

[7]This difference in borrowing costs is also larger the higher the risk aversion.

4.3 Debt Dynamics with Default

In the full model with default, we find that pro-cyclical inflation has two main effects on debt dynamics. The effects of the inflation co-movement are reported in Table 4. On the one hand, pro-cyclical inflation lowers equilibrium interest rates, inducing governments to go deeper into debt. On the other hand, this increases risk for the borrower, inducing government to reduce debt. The first effect is stronger when the government has less debt, while the second effect is stronger when the government has more debt. Overall, default probabilities are lower when the inflation is more *pro-cyclical* - that is when the covariance of inflation and consumption growth increases.

Table 4: Debt and Default

	Positive co-movement ($\eta = +0.0010$)	Negative co-movement ($\eta = -0.0010$)
Default rate (percent)	2.52	3.04
Spreads (percent)	2.81	3.52
Debt (percent)	4.29	5.48

Precautionary Motives

Figure 2 shows that in response to the same sequence of shocks, precautionary motives from pro cyclical inflation increase with debt: the difference in debt accumulation between the two economies get larger as the debt levels increase. Figure 3 shows the evolution of borrowing costs for the same sequence of shocks. We find that borrowing costs are uniformly higher in the economy with countercyclical inflation.

An Example

These effects of the inflation process on the evolution of interest rates, debt levels, and default probabilities can also be understood using a non-constant sequence of output shocks. Figure 4 shows debt dynamics in response to the same sequence of variable shocks across an

Figure 2: Debt Dynamics

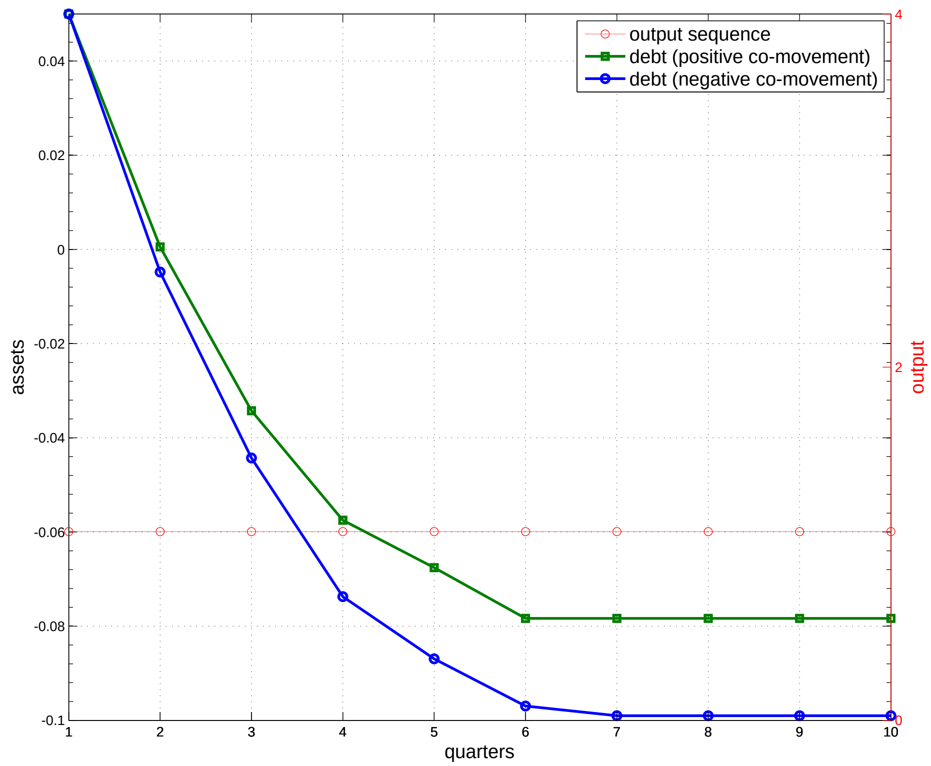
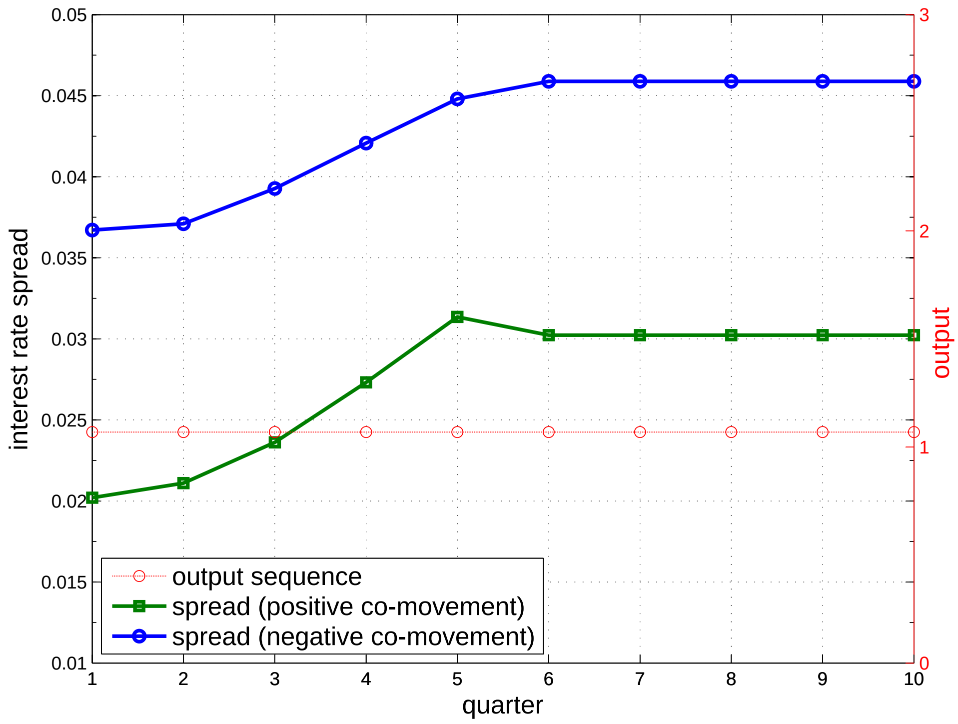
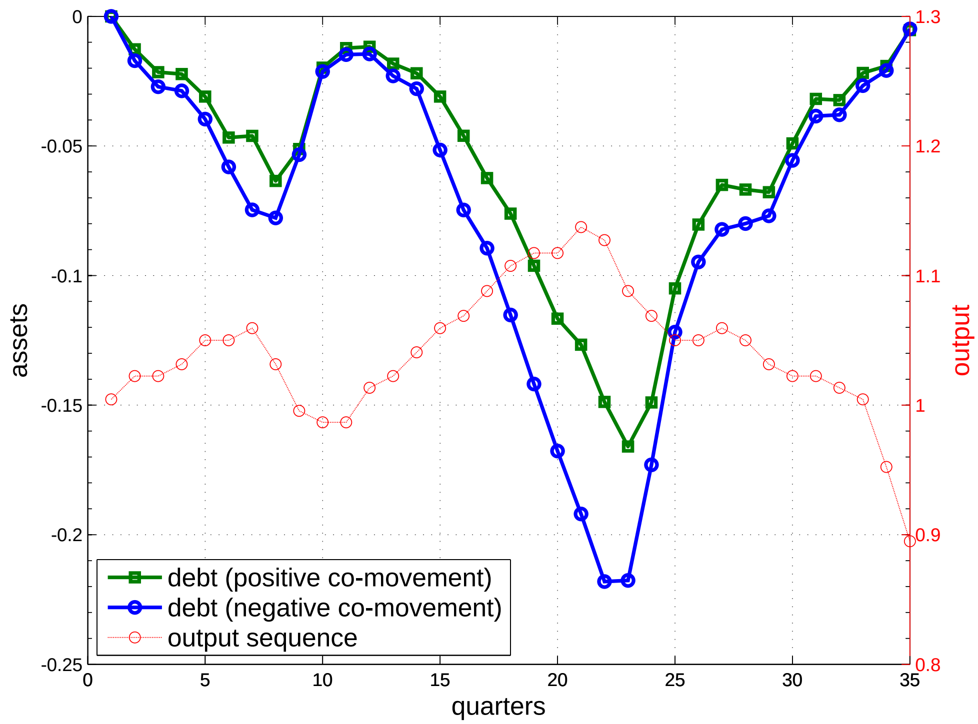


Figure 3: Interest Rate



economy with negative co-movement and an economy with positive co-movement. Figures 5 and 8 show the evolution of corresponding borrowing costs. Default probabilities are shown in Figure 6 and 7. These sample paths show that the inflation process has subtle effects on the joint dynamics of interest rates, debt, and default.

Figure 4: Debt Dynamics



5 Conclusion

The goal of this paper was to investigate how the inflation process affects borrowing costs, and debt and default dynamics. Empirically, we documented that the co-movement of inflation innovations and consumption growth innovations fluctuates over times across a large number of countries. Moreover, we find that increased co-movement of inflation and consumption growth is associated with lower borrowing costs. Theoretically, we showed that the inflation processes can be important in explaining the cross section of government debt and interest rates.

Figure 5: Interest Rates

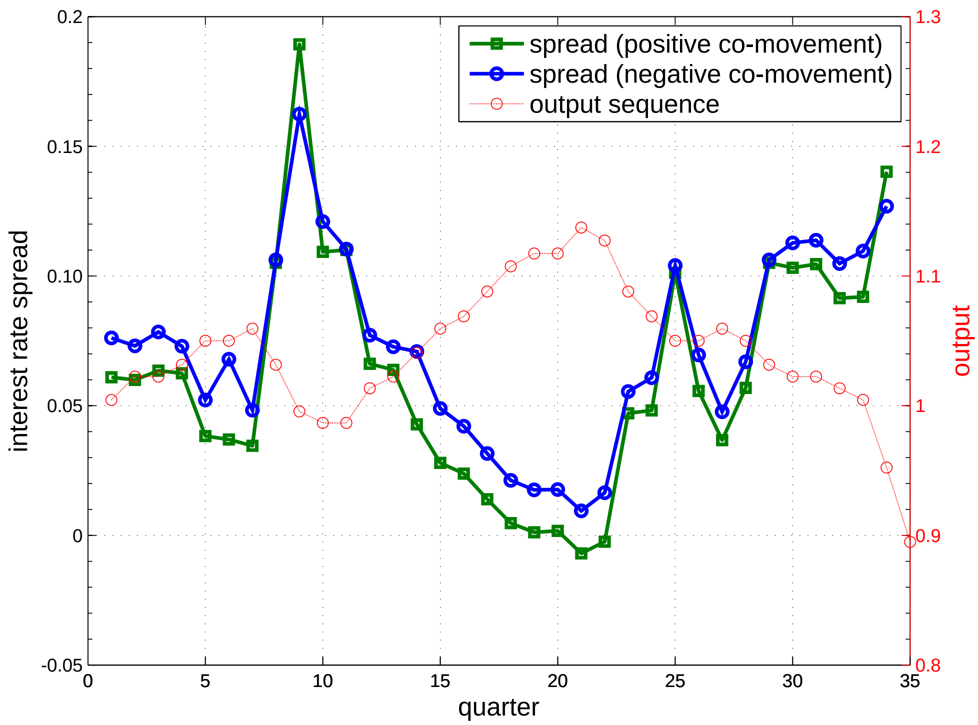


Figure 6: Default Probability

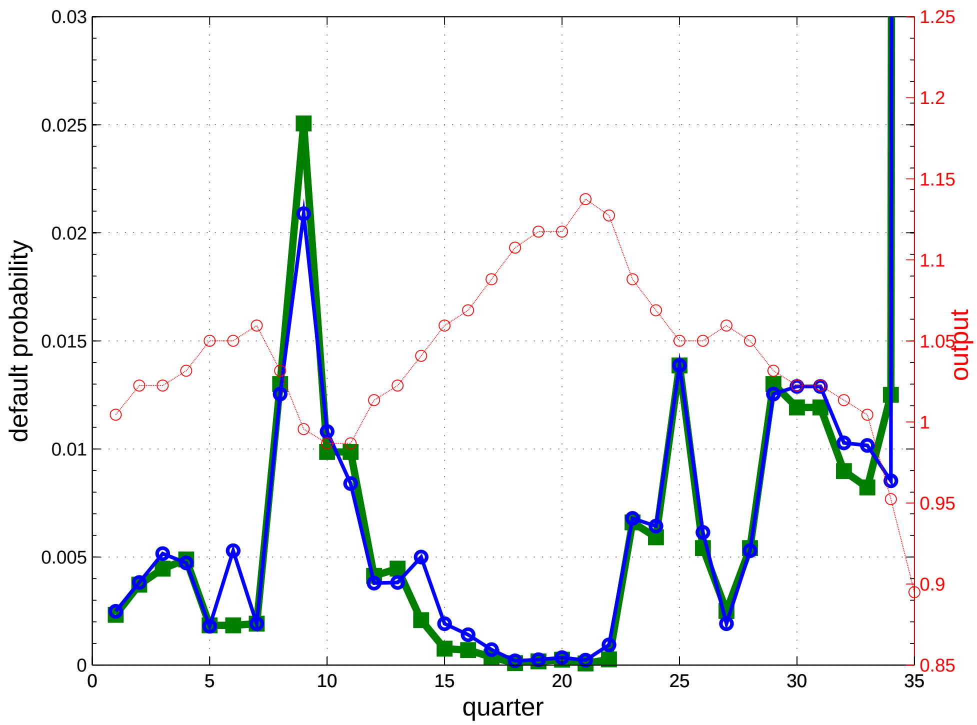


Figure 7: Default Probability Differences

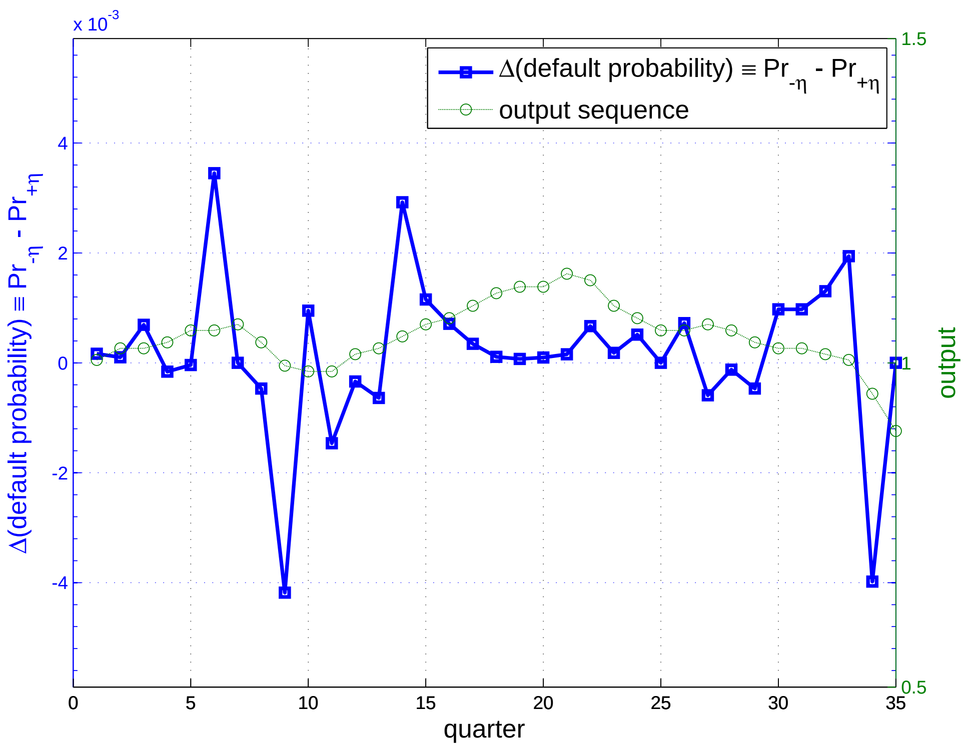
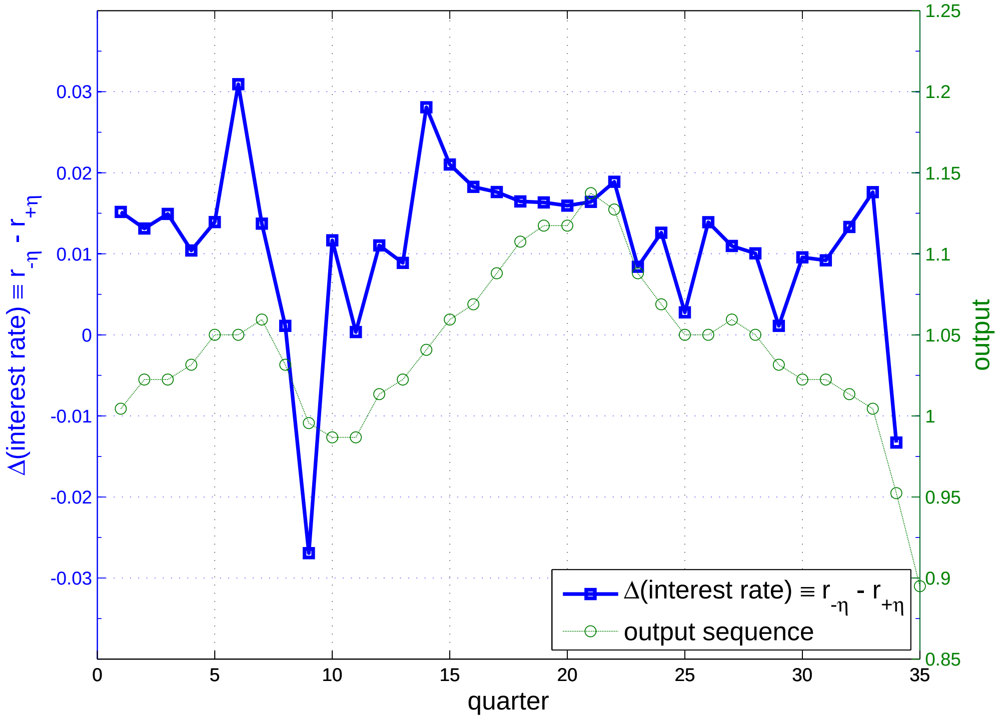


Figure 8: Interest Rate Differences



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