Endogenous Trade Participation with Incomplete Exchange Rate Pass-Through

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Abstract

This paper investigates the implications of endogenous trade participation for international business cycles, trade flow dynamics and exchange rate pass-through when price adjustments are staggered across firms. I develop a two-country dynamic stochastic general equilibrium model wherein firms make state-dependent decisions on entry and exit in the export market and the frequency of price adjustment is time-dependent. Consistent with recent empirical findings at the firm and sectoral levels, producers of traded goods in this model differ in their productivities, trade status and prices. At the aggregate level, quantitative properties of the model successfully reproduce some important characteristics of international business cycle moments in data.

In contrast to previous findings in the literature, my model reveals that the inclusion of exporter entry and exit generates notable changes in trade dynamics following aggregate shocks, such as large, immediate responses of the number of exporters, export volumes and export price index. I trace this result to the micro-level price stickiness present in my model but absent in existing models of endogenous trade participation. Micro price rigidity reduces the extent to which changes in the prices of existing traded goods absorb changes in the value of exporting following a persistent macroeconomic shock, and instead permits substantial movements in the benefits of exporting and hence the extensive margin of trade. This suggests that market structure and pricing conventions may be critical in analyzing the role of endogenous trade participation for international business cycles. Moreover, I show that productivity heterogeneity rather than price rigidity plays a dominant role in firms' export decisions, and hence the additional realism of endogenous trade participation in the model does not mitigate incomplete exchange rate pass-through arising from nominal rigidity.

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1 Introduction

Over the past decade, the increasing availability of micro-level trade data has revealed important exporter characteristics previously overlooked in theoretical models of international trade. In particular, empirical studies document that, beyond changes in the volume of trade, countries also experience changes in the number of exporters and export varieties over time, and that exporters differ from non-exporters in several dimensions such as productivity and firm size.

There is also substantial empirical evidence in international trade that exchange rate movements lead to less than proportional changes in the prices of traded goods, with those changes occurring with a substantial delay. Recent studies have suggested a number of factors that may explain this empirical regularity, called incomplete exchange rate pass-through. Among them is an explanation based on micro-level price rigidities, the observation that individual prices do not adjust flexibly period by period.

This paper investigates the implications of endogenous trade participation for international business cycles, the dynamics of trade flows and exchange rate pass-through when price adjustments are staggered across firms. I develop a two-country dynamic stochastic general equilibrium model wherein (a) fixed entry and continuation costs in the export market induce endogenous, state-dependent entry and exit decisions among exporters, and (b) individual firms reset their nominal prices infrequently, with the timing of those adjustments governed by a time-dependent hazard that rises in the age of a price. Consistent with recent empirical findings at the firm and sectoral levels, producers of traded goods in my model exhibit persistent differences in their prices, productivities and trade status.

In my model, individual firms' decisions to enter or exit the export market are influenced by fixed costs of entry and continuation. I allow for idiosyncratic elements in these costs in order to ensure that an interior fraction of each firm type chooses to export in any given period. As mentioned above, I also assume a price-adjustment hazard that implies probabilities of price adjustment smoothly rising in the time since last adjustment. Thus, the pricing convention in my model is designed to be consistent with empirical evidence in the microeconomic data and recent work on models of state dependent pricing.[1]

I calibrate the parameters governing the microfoundations of my model to match a number of empirical observations regarding exporter characteristics and turnover. In particular, I target the fraction of firms that export, the entry rate into exporting, the continuation rate among existing exporters as well as the average difference in productivity among exporting versus non-exporting firms observed in the data on U.S. exporters. In addition to these exporter characteristics, my model is calibrated to reproduce the average frequency of price adjustment in disaggregated data and the aggregate imports-to-GDP ratio.

[1] Time-dependent price adjustment with an increasing hazard function is also used in Khan, King and Wolman (2003).

In any given period, each of the individual characteristics mentioned above influences whether and how much a firm exports. For example, all else equal, firms with higher current productivity are more likely to export since their larger expected values imply greater willingness to incur trade costs. For the same reason, firms are also more likely to export if their prices are closer to a forward-looking target price that is consistent with their productivity and the economy's aggregate state. Moreover, because the model's calibration implies that the average entry cost for newly exporting firms exceeds the average continuation cost, forward-looking exporters recognize that exit may be costly. Since re-entering the export market in the future will involve another payment of the entry cost, a large fraction of firms that export in one period choose to continue exporting in the next period, even when they face current losses from exporting. This generates persistence in firms' export decisions as observed in the data.

Like other open-economy models with microeconomic price rigidities, my model generates incomplete exchange rate pass-through in the aggregate.[2] However, the extent of pass-through in the presence of exporter entry and exit may not follow directly from the assumed frequency of firms' price adjustment and a common price chosen by price-adjusting firms, as it also depends on individual firms' decisions regarding their trade status. As mentioned above, all else equal, firms are more likely to export if their prices are closer to the optimal price consistent with the aggregate state of the economy. Incumbent exporters adjusting their prices weigh their current productivity against continuation costs, while those not adjusting prices take into account both productivity and price for their continuation decision. Therefore, in any given period, the number and types of firms choosing to export and their effective prices influence the aggregate speed of adjustment of prices to exchange rate movements, and may have implications for the degree of pass-through. This channel is shaped by the endogenous distribution of exporters over productivity, prices and export status.

Business cycle properties of macroeconomic variables in my model are consistent with some important characteristics of the data. In particular, my model generates countercyclical net exports and positive cross-country correlations of investment and labor, despite my extension to include large movements along the extensive margin of trade in a complete markets setting. In standard two-country models with complete asset markets, cross-country correlations of investment and employment are negative while the reverse is true in the data.[3] This 'international comovement puzzle' (Baxter, 1995) has led subsequent studies to propose a number of mechanisms to resolve it.[4] One way to resolve the puzzle, while retaining complete markets, is to allow some degree of microeconomic-level nominal price rigidity, as in the monetary economy with monopolistic competition and staggered price adjustment of Chari, Kehoe and McGrattan (2002). In principle, the

[2]See, for example, Goldberg and Hellerstein (2008) and Nakamura and Zerom (2010).

[3]See, for example, Backus, Kehoe, and Kydland (1993), Baxter and Crucini (1993) and Baxter (1995).

[4]For example, Kehoe and Perri (2002) show that the introduction of endogenous incomplete markets generates the desired positive cross-country correlations. Heathcote and Perri (2002) show that a model with financial autarky can account for observed cross-country investment and employment correlations.

inclusion of endogenous exporter entry and exit in a model with price rigidities allows the model an extra margin of flexibility in responding to macroeconomic shocks, and hence has the potential to offset the implications of micro-level price rigidities and thus reinstating the comovement problem. However, I find that the success of nominal rigidities in delivering comovement in a complete markets setting survives my extension of the model to accommodate greater realism with respect to the micro-level trade data. In fact, absent any endogenous exporter entry and exit, a relatively minimal degree of micro-level price rigidity generates strong comovement in my model, even more than what is observed in the cross-country data. When I allow for extensive margin export decisions, these positive correlations survive, but they are weakened, thereby improving the quantitative performance of the model.

Exploring aggregate fluctuations in my model and in comparison with a series of special-case variants, I arrive at two main results. First, in contrast to previous findings in the literature, I find that the inclusion of endogenous trade participation generates notable changes in the dynamics of international trade.[5] In particular, I find that the number of exporters exhibits large and procyclical responses to aggregate shocks, which in turn generates larger movements in the export price index, export volume and hence net exports relative to the case without exporter entry and exit. I trace this different finding to the micro-level price rigidity present in my model but absent in other such studies. Relative to existing endogenous trade participation models, my inclusion of microeconomic price rigidity reduces the extent to which changes in the prices of existing traded goods absorb changes in the value of exporting following a persistent macroeconomic shock. Because the departure from perfectly flexible prices prevents incumbent exporters from fully absorbing the effects of a shock, my model permits substantial movements in the benefits to exporter entry and continuation. As a result, it delivers large procyclical responses in the number of exporters following a persistent aggregate shock. This, in turn, delivers notable differences in the model's dynamic responses in exports and the aggregate export price index relative to a model without endogenous trade participation. This finding is in contrast to Alessandria and Choi (2007) who find that introducing sunk entry costs has little implications for net export dynamics and other international business cycle moments in their flexible-price environment. Their model, however, cannot explain some aspects of international business cycles including the observed cross-country correlations, and, as they note in their conclusion, this may be the reason why they find little impact of export decisions on net export dynamics. My finding suggests that market structure and pricing conventions may be critical in determining the role of endogenous trade participation for international business cycles.

The presence of an extensive margin of trade in my model also offers new insight into incomplete exchange rate pass-through in an environment with exporter entry and exit. There is compelling evidence in the empirical trade literature that transmission of exchange rate movements

[5] See, for example, Alessandria and Choi (2007) and Ruhl (2008).

to import prices is often delayed and less than complete. Campa and Goldberg (2005) report that exchange rate pass-through into U.S. import prices is approximately 23 percent in the short run and 42 percent over the long run. Using the dock price data for the U.S. imports at the product-level, Gopinath, Itskhoki and Rigobon (2010) report that, conditional on a price change, the short-run pass-through of the average good is only 25 percent. Theoretically, in the presence of price rigidities, when prices are set in the importer's currency, short-run exchange rate pass-through is zero at the good-level until a producer adjusts its price. My model allows for this possibility; however, with endogenous trade participation, because exporters with older prices are more likely to exit than those with newer prices, one might expect that the frequency of price adjustment among exporters would be higher, thereby leading to more pass-through. I find that firms' export decisions are primarily influenced by their productivity and trade cost differences rather than price-age differences, and hence incomplete exchange rate pass-through arising from price rigidity survives the additional realism of exporter entry and exit in my model.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. In Section 3, I describe the model. Section 4 summarizes calibration and introduces three variants of my full model: a sticky-price model without entry and exit, a flexible-price model with entry and exit, and a flexible-price model without entry and exit. Results are presented in Section 5. Section 6 concludes.

2 Related literature

Earlier studies on the implications of export costs and entry decisions for the dynamics of net exports include Baldwin (1988), Baldwin and Krugman (1989) and Dixit (1989a, 1989b). These authors developed partial equilibrium models including export decisions with sunk costs, and found that the presence of sunk entry costs can lead exporters to remain in a foreign market following an appreciation of their currency that makes their products relatively more expensive abroad. As such, they attribute the delayed improvement of the U.S. trade balance following the real exchange rate decline in the 1980s to the sunk cost aspect of trade. Among these studies, Dixit (1989a) focuses on the responses of imports and prices to an exogenously specified exchange rate process, and shows that exchange rate pass-through to import prices is close to one in the phases where foreign firms enter or exit, and near zero otherwise. I contribute to this line of his work by building a general equilibrium model with incomplete pass-through and endogenous export market entry and exit. Beyond offering a richer treatment of the extensive margin, my extension to a general equilibrium setting permits the analyses of the responses of trade flows and prices and their endogenous feedback on exchange rate fluctuations following macroeconomic shocks.

As mentioned in section 1, my model is consistent with a number of important exporter characteristics and firm-level production patterns documented in recent empirical studies using micro-level trade data. First, it has been documented that not all firms engage in international trade. For example, Bernard *et al.* (2003) report that only 21 percent of U.S. manufacturing plants exported in 1992. Second, decisions on export status are persistent. Bernard and Jensen (2004) show that, over the period from 1984 to 1992, on average each year 87.4 percent of exporters continued exporting in the following year, while 86.1 percent of nonexporters did not export in the following year. Finally, exporters tend to be larger in firm size, more profitable, and more productive. For example, Bernard and Jensen (1999) find that U.S. exporters are 12-18 percent more productive relative to nonexporters.[6]

Such micro-level trade distinctions as noted above cannot be captured by standard representative-firm models of international trade, as in Krugman (1980), where either all or no firms participate in international trade, and variation in the intensive margin alone must fully explain fluctuations in trade volume. This deficiency of the standard paradigm has given rise to the recent development of theoretical frameworks wherein heterogeneous firms individually choose whether and how much to export. Bernard, Eaton, Jensen and Kortum (2003) and Melitz (2003) develop heterogeneous-firm models in which the presence of export costs restricts participation in international trade to only relatively more productive firms in the economy. Melitz's (2003) model allows the range of available products to vary over time with firms specializing in production of differentiated goods. Chaney (2008) shows that a model of international trade with productivity heterogeneity and fixed export costs predicts that the impact of trade barriers on trade flows is dampened by the elasticity of substitution, instead of being magnified as shown by Krugman (1980). The relevance of firm-level fixed export costs has been documented by Aitken et al. (1997), Bernard and Jensen (2004), Besedes and Prusa (2006) and Das *et al.* (2007).

Given my focus on endogenous trade participation and international business cycles in a quantitative DSGE environment, my work here is perhaps most closely related to Ghironi and Melitz (2005) and Alessandria and Choi (2007). Ghironi and Melitz (2005) develop a model with heterogeneous productivity and endogenous export market entry. In their model, the presence of export costs induces less productive firms to avoid exporting. Further, they obtain persistent deviations from purchasing power parity that would not exist without firm heterogeneity and exporter entry. The export-market structure in my model differs from theirs in two main ways. First, I introduce additional micro-level decisions such as endogenous exit from the export market to study richer dynamics of trade participation. Second, I abandon the assumption of perfectly flexible pricing. Because my model allows micro-level price rigidities, it is distinguished in its ability to capture observed incomplete exchange rate pass-through.

Alessandria and Choi (2007) evaluate export hysteresis using a model with a sunk entry cost to a foreign market and a period-by-period continuation cost that together induce endogenous entry and exit in the export market. In their flexible-price setting, they find that export decisions have

[6] Such phenomena have been documented in a number of countries. See, for example, Bernard and Jensen (1999) and Bernard *et al.* (2003) for the U.S., Aw *et al.* (2001) for Taiwan, and Clerides *et al.* (1998) for Columbia, Mexico and Morocco. For a survey, see Tybout (2003).

negligible effects on the aggregate dynamics of the model. As mentioned above, my conclusions on the role of time-varying export participation are notably different. I show that, in a setting with nominal rigidity at the firm level, endogenous trade participation does affect international business cycles, in particular the trade and price dynamics as mentioned in section 1.

Ruhl (2008) examines a flexible-price model with an entry cost of exporting and heterogeneous firms, and uses a steady-state analysis to show that firms do not change export status in response to transitory aggregate shocks, because price adjustments alone can absorb the effects of the shocks. In contrast to his investigation, the inclusion of micro-level price rigidities in my model implies that aggregate shocks generate substantial short-run movements in the extensive margin of trade since price stickiness prevents changes in the intensive margin from fully absorbing the effects of shocks.

Finally, there is a substantial body of empirical work that documents that exchange rate movements lead to less than proportional changes in traded goods prices, and much of the price response occurs with a substantial delay.[7] Recent theoretical studies have suggested a number of channels in explaining such incomplete pass-through. Dornbusch (1987), Knetter (1989), Bergin and Feenstra (2001) and Atkeson and Burstein (2008) consider oligopolistic markets and explain the degree of pass-through with the response of prices to changes in production costs that depend on the curvature of demand and the market structure. Another widely-studied explanation emphasizes the importance of local costs as in, for example, Sanyal and Jones (1982), Burstein, Neves and Rebelo (2003) and Corsetti and Dedola (2004). Because local costs generate a wedge between prices and import costs that is unresponsive to exchange rate fluctuations, they mitigate the impact of changes in the price of an imported factor of production on total marginal costs. On the other hand, Giovannini (1988), Kasa (1992), Devereux and Engel (2002) and Bacchetta and van Wincoop (2003) argue that price rigidity and other dynamic factors in price adjustment contribute to incomplete transmission of exchange rate movements to prices. My work here emphasizes this price-rigidity explanation for incomplete pass-through. Given its inclusion of productivity heterogeneity, endogenous trade participation decisions and a rising time-dependent hazard governing the timing of individual firms' price adjustments, my model provides a realistic framework in which to evaluate this channel.

3 Model

There are two symmetric countries, home and foreign. In each country, there is a unit mass of monopolistically competitive firms each producing a differentiated intermediate product. All firms produce and costlessly sell their goods in their own country. However, exporting their goods abroad involves two types of costs, an entry cost and a continuation cost, and firms choose whether

[7] See, for example, Engel (1999), Parsley and Wei (2001), Marazzi *et al.* (2005) and Campa and Goldberg (2006).

or not to pay these costs and export. First, to enter the export market and participate there for one period, firms must pay a sunk entry cost and a randomly drawn operating cost that are summarized together as one random entry cost variable and drawn from a common distribution. Next, any incumbent firm in the export market must pay an operating cost to remain and this continuation cost is also drawn by incumbents from a common distribution. Both the entry cost and the continuation cost are denominated in units of labor from the firm's country of origin. These trade costs imply that not all firms choose to export in a given period and that the set of active exporters changes over time. Among any group of incumbent exporters sharing in common the same price and productivity level, those firms with lower continuation costs are more likely to continue exporting, while those with high continuation costs are less likely. A similar observation holds regarding entry.

In any given period, intermediate good producers are able to reset their prices with some probability which varies depending on the number of periods since their last price adjustment. In addition, firm-specific productivity is discrete and follows a Markov chain. Therefore, intermediate good producers are heterogeneous in terms of export costs, export status, prices, and productivity.

In each country, competitive final good producers combine intermediate goods produced in their own country and those imported from abroad to produce the final good used for consumption and capital investment. Each country is inhabited by a continuum of identical households who own the intermediate good producers of their country and purchase final goods produced there. They also have access to a complete set of one-period, state-contingent nominal bonds denominated in the home currency.

The following subsections describe the model from the perspective of the home country. Analogous conditions hold for the foreign country. Foreign counterparts to home-country variables are indicated by an asterisk.

3.1 Final good producers

In each country, each intermediate-good firm i produces a differentiated intermediate good i. Competitive final good producers in the home country combine home intermediate goods and the intermediate goods imported from the foreign country to produce final goods, D_t:

$$D_t = \left\{ \omega \left[\int_0^1 y_t^H(i)^{\frac{\gamma-1}{\gamma}} di \right]^{\frac{\gamma}{\gamma-1}\frac{\rho-1}{\rho}} + (1-\omega) \left[\int_{i \in \Theta_t} y_t^F(i)^{\frac{\gamma-1}{\gamma}} di \right]^{\frac{\gamma}{\gamma-1}\frac{\rho-1}{\rho}} \right\}^{\frac{\rho}{\rho-1}}, \tag{1}$$

where γ is the elasticity of substitution between goods produced in the same country, ρ is the elasticity of substitution between home and foreign intermediate goods commonly referred to as the Armington elasticity, ω is the weight of domestically-produced intermediate goods often termed the home bias parameter, and Θ_t is the endogenous, time-varying set of foreign-produced intermediate goods available in the home country in period t. The home final good is purchased by domestic households and used for consumption and investment: $D_t = C_t + I_t$.

Final good producers take as given the price of the final good P_t, the price of the ith intermediate good produced in the home country $P_t^D(i)$ and the price of the ith intermediate good imported from the foreign country $P_t^{X*}(i)$, and choose inputs of home-produced intermediate goods $y_t^H(i)$ and home imports from the foreign country $y_t^F(i)$ to maximize profits:

$$\max_{y_t^H(i), y_t^F(i)} P_t D_t - \int_0^1 P_t^D(i) y_t^H(i) di - \int_{i \in \Theta_t} P_t^{X*}(i) y_t^F(i) di, \tag{2}$$

subject to the production technology (1). Exporters are assumed to follow local currency pricing (i.e., prices are denominated in the currency of the destination country), and hence $P_t^{X*}(i)$ is denominated in the home currency. The solution to this problem gives the final-good producers' demand for home-produced intermediate goods $y_t^H(i)$ and their demand for foreign-produced intermediate goods $y_t^F(i)$, respectively:

$$y_t^H(i) = \omega^\rho \left(\frac{P_t^D(i)}{P_t^D} \right)^{-\gamma} \left(\frac{P_t^D}{P_t} \right)^{-\rho} D_t, \tag{3}$$

$$y_t^F(i) = (1-\omega)^\rho \left(\frac{P_t^{X*}(i)}{P_t^{X*}} \right)^{-\gamma} \left(\frac{P_t^{X*}}{P_t} \right)^{-\rho} D_t, \tag{4}$$

where

$$P_t^D \equiv \left[\int_0^1 P_t^D(i)^{1-\gamma} di \right]^{\frac{1}{1-\gamma}}, \tag{5}$$

$$P_t^{X*} \equiv \left[\int_{i \in \Theta_t} P_t^{X*}(i)^{1-\gamma} di \right]^{\frac{1}{1-\gamma}}. \tag{6}$$

Since the final goods market is competitive, the zero-profit condition yields the price level of the final good (the home consumer price index):

$$P_t \equiv \left[\omega^\rho \left(P_t^D \right)^{1-\rho} + (1-\omega)^\rho \left(P_t^{X*} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}}. \tag{7}$$

Similarly, the problem of the foreign final-good producers yields their demand for foreign intermediate goods, $y_t^{F*}(i) = \omega^\rho \left(\frac{P_t^{D*}(i)}{P_t^{D*}} \right)^{-\gamma} \left(\frac{P_t^{D*}}{P_t^*} \right)^{-\rho} D_t^*$, and their demand for home intermediate goods, $y_t^{H*}(i) = (1-\omega)^\rho \left(\frac{P_t^X(i)}{P_t^X} \right)^{-\gamma} \left(\frac{P_t^X}{P_t^*} \right)^{-\rho} D_t^*$.

3.2 Intermediate good producers

Each intermediate good firm produces its output through a Cobb-Douglas production technology,

$$y = z_c A K^{\nu} L^{1-\nu}, \tag{8}$$

where z_c represents the firm's current individual productivity realization, A is aggregate productivity, L is the amount of labor hired, and K is the amount of capital rented. Given the downward-sloping demand function the firm faces, its domestic demand $y^H_{j,t}(z_s)$, is determined by its nominal price in that market, $P^D_{j,t}(z_s)$, where the subscripts j and t together reflect the aggregate state of the economy when the firm's domestic price was last set j periods ago, and z_s represents the productivity draw it had in that date. Similarly, foreign demand for the firm's good $y^{H*}_{j',t}(z_{s'})$, is determined by the export price it last set $t-j'$ periods ago given the aggregate state in that date, alongside its own productivity $z_{s'}$. Thus, the firm's total production in equilibrium can be written as:

$$y^H_{j,t}(z_s) + \tau y^{H*}_{j',t}(z_{s'}) = z_c A_t K_t\left(z_c, P^D_{j,t}(z_s), P^X_{j',t}(z_{s'})\right)^{\nu} L_t\left(z_c, P^D_{j,t}(z_s), P^X_{j',t}(z_{s'})\right)^{1-\nu}, \tag{9}$$

where $\tau \geq 1$ is an iceberg export cost, z_c for $c = 1, \cdots, n_z$ is firm-specific productivity in the current period, and $L_t(\cdot)$ and $K_t(\cdot)$ denote the firm's labor and capital inputs. The subscript j (j') denotes the number of periods since the domestic (export) price was last set, and throughout I assume that the maximum duration of a price is J.[8]

Capital is owned by households and rented to firms at the rental rate r_t. Household labor is supplied to firms at real wage rate w_t. Since the firm's production technology has constant returns to scale, total inputs of labor and capital can be linearly decomposed between those used to produce goods for domestic sales (L^D_t and K^D_t) and those used to produce exports (L^X_t and K^X_t): $L_t\left(z_c, P^D_{j,t}(z_s), P^X_{j',t}(z_{s'})\right) = L^D_t\left(z_c, P^D_{j,t}(z_s)\right) + L^X_t\left(z_c, P^X_{j',t}(z_{s'})\right)$ and $K_t\left(z_c, P^D_{j,t}(z_s), P^X_{j',t}(z_{s'})\right) = K^D_t\left(z_c, P^D_{j,t}(z_s)\right) + K^X_t\left(z_c, P^X_{j',t}(z_{s'})\right)$. This implies that total production cost can also be divided into the cost for domestic production

$$w_t L^D_t\left(z_c, P^D_{j,t}(z_s)\right) + r_t K^D_t\left(z_c, P^D_{j,t}(z_s)\right) = \frac{w_t^{1-\nu} r_t^{\nu}}{(1-\nu)^{1-\nu}\nu^{\nu}} \frac{1}{z_c A_t} y^H_{j,t}(z_s), \tag{10}$$

and the cost for export production

$$w_t L^X_t\left(z_c, P^X_{j,t}(z_s)\right) + r_t K^X_t\left(z_c, P^X_{j,t}(z_s)\right) = \frac{w_t^{1-\nu} r_t^{\nu}}{(1-\nu)^{1-\nu}\nu^{\nu}} \frac{1}{z_c A_t} \tau y^{H*}_{j,t}(z_s). \tag{11}$$

Notice that this implies the firm's marginal costs of production are independent of its scale of operation.

[8] As will be described in greater detail below, individual prices evolve according to $P^D_{j,t+1}(z_c) = P^D_{j-1,t}(z_c)$ and $P^X_{j,t+1}(z_c) = P^X_{j-1,t}(z_c)$ for $c = 1, \cdots, n_z$ and $j = 1, \cdots, J-1$.

All intermediate-good producers sell in the domestic market and earn real profit $d_t^D\left(z_c, P_{j,t}^D(z_s)\right)$,

$$d_t^D\left(z_c, P_{j,t}^D(z_s)\right) = \frac{P_{j,t}^D(z_s)}{P_t} y_{j,t}^H(z_s) - w_t L_t^D\left(z_c, P_{j,t}^D(z_s)\right) - r_t K_t^D\left(z_c, P_{j,t}^D(z_s)\right). \tag{12}$$

In addition to domestic sales, firms can choose to export to the foreign country if they pay the relevant costs of exporting. If a firm did not export last period and chooses to enter the export market in the current period, it must pay a sunk entry cost to gain access to the foreign market. All exporters, including entrants, must pay trade costs that are randomly drawn in each period. Both entry costs and continuation costs for the export market are paid as wages to additional workers that exporters must hire. The profit from exporting, excluding the applicable trade costs that must be paid to participate, is:

$$d_t^X\left(z_c, P_{j,t}^X(z_s)\right) = Q_t \frac{P_{j,t}^X(z_s)}{P_t^*} y_{j,t}^{H*}(z_s) - w_t L_t^X\left(z_c, P_{j,t}^X(z_s)\right) - r_t K_t^X\left(z_c, P_{j,t}^X(z_s)\right), \tag{13}$$

where $Q_t \equiv e_t \frac{P_t^*}{P_t}$ is the real exchange rate and e_t is the nominal exchange rate, (the home-currency price of a unit of the foreign currency).

A firm's total real profit, $d_t(\cdot)$, consists of profits from domestic sales, $d_{j,t}^D(\cdot)$, and profits from exports, $d_{j',t}^X(\cdot)$, if the firm chooses to pay the relevant participation costs to operate in the export market, a question we consider in section 3.3 below. Since production of intermediate goods has constant returns to scale and the demand for these goods involves a constant elasticity of substitution, we can separately consider an intermediate producer's decisions regarding domestic sales and exports.

3.2.1 Domestic prices

First, consider the problem of a home intermediate good producer involving its domestic sales. The value of a firm that has current productivity level z_c and is currently adjusting its price is,

$$\begin{aligned} V_{0,t}^D(z_c) &= \max_{P_{0,t}^D(z_c)} d_t^D(z_c, P_{0,t}^D(z_c)) \\ &+ \beta \mathbf{E}_t \frac{\lambda_{t+1}}{\lambda_t}\left[\alpha_1 \sum_{\tilde{c}=1}^{n_z} \pi_{c\tilde{c}} V_{0,t+1}^D(z_{\tilde{c}}) + (1-\alpha_1) \sum_{\tilde{c}=1}^{n_z} \pi_{c\tilde{c}} V_{1,t+1}^D\left(z_{\tilde{c}}, P_{0,t}^D(z_c)\right)\right], \end{aligned} \tag{14}$$

for $c = 1, \cdots, n_z$, where β is the household discount factor, λ_t is the date-t household marginal utility of consumption, α_j is the probability of price adjustment in the current period given that the firm last adjusted its price j periods ago, and $\pi_{c\tilde{c}}$ denotes the probability of switching from $z = z_c$ to $z' = z_{\tilde{c}}$. Those firms able to reset their price in the current period choose $P_{0,t}^D(z_c)$ so as to maximize (14).

The value of a firm that has current productivity z_c and a price set j periods ago under individual productivity z_s, $P^D_{j,t}(z_s)$, that is not currently adjusting its price is:

$$\begin{aligned} V^D_{j,t}\left(z_c, P^D_{j,t}(z_s)\right) &= d^D_t(z_c, P^D_{j,t}(z_s)) \\ &+\beta \mathbf{E}_t \frac{\lambda_{t+1}}{\lambda_t}\left[\alpha_{j+1}\sum_{\tilde{c}=1}^{n_z}\pi_{c\tilde{c}}V^D_{0,t+1}(z_{\tilde{c}}) + (1-\alpha_{j+1})\sum_{\tilde{c}=1}^{n_z}\pi_{c\tilde{c}}V^D_{j+1,t+1}\left(z_{\tilde{c}}, P^D_{j,t}(z_s)\right)\right], \end{aligned} \tag{15}$$

for $c = 1, \cdots, n_z$, $s = 1, \cdots, n_z$, and $j = 1, \cdots, J-2$. All firms adjust their prices with probability 1 within J periods: $\alpha_J = 1$. Thus, in the case where $j = J-1$, the expression above collapses to

$$V^D_{J-1,t}\left(z_c, P^D_{J-1,t}(z_s)\right) = d^D_t(z_c, P^D_{J-1,t}(z_s)) + \beta \mathbf{E}_t \frac{\lambda_{t+1}}{\lambda_t}\sum_{\tilde{c}=1}^{n_z}\pi_{c\tilde{c}}V^D_{0,t+1}(z_{\tilde{c}}). \tag{16}$$

3.2.2 Export prices

Let $V^E_t(z_c, \eta)$ be the export-related value of a firm that was not an exporter last period and has current productivity z_c and current export entry cost η drawn from a time-invariant distribution $G^E(\eta)$. If this firm decides to enter the export market in the current period, it sets a price upon entry. Thus, the potential entrant's export value can be expressed as:

$$\begin{aligned} V^E_t(z_c, \eta) = \max\Bigg\{ &\max_{P^X_{0,t}(z_c)}\Bigg[d^X_t\left(z_c, P^X_{0,t}(z_c)\right) - \eta w_t \\ &+\beta \mathbf{E}_t \frac{\lambda_{t+1}}{\lambda_t}\sum_{\tilde{c}=1}^{n_z}\pi_{c\tilde{c}}H_{1,t+1}\left(z_{\tilde{c}}, P^X_{0,t}(z_c), \xi_{t+1}\right)\Bigg], \quad \beta \mathbf{E}_t \frac{\lambda_{t+1}}{\lambda_t}\sum_{\tilde{c}=1}^{n_z}\pi_{c\tilde{c}}V^E_{t+1}(z_{\tilde{c}}, \eta_{t+1})\Bigg\}, \end{aligned} \tag{17}$$

for $c = 1, \cdots, n_z$. The first term inside the binary max operator is the value of entering the export market in the current period, and the second term is the value of not entering this period and being a potential entrant again next period.

Next, I introduce the Bellman equations of incumbent exporters. Let $V_{0,t}(z_c, \xi)$ be the export-related value of an incumbent exporter that has current productivity z_c and current export continuation cost ξ drawn from a time-invariant distribution $G(\eta)$, and is able to reset its price this period. Let $V_{j,t}\left(z_c, P^X_{j,t}(z_s), \xi\right)$ be the export value of an incumbent that has current productivity z_c, current export cost draw ξ, and a non-adjusted export price $P^X_{j,t}(z_s)$ that was set j periods ago under productivity z_s. Prior to learning whether it will reset its export price in the current period, the export-value of an incumbent with current productivity z_c and a price from the previous period $P^X_{j,t}(z_s)$ is:

$$H_{j,t}\left(z_c, P^X_{j,t}(z_s), \xi\right) = \alpha_j \cdot V_{0,t}(z_c, \xi) + (1-\alpha_j)\cdot V_{j,t}\left(z_c, P^X_{j,t}(z_s), \xi\right), \tag{18}$$

for $c = 1, \cdots, n_z$, $s = 1, \cdots, n_z$, and $j = 1, \cdots, J-1$, and

$$H_{J,t}(z_c, \xi) = V_{0,t}(z_c, \xi), \tag{19}$$

for $c = 1, \cdots, n_z$ since $\alpha_J = 1$

The value conditional on price reset is

$$V_{0,t}(z_c, \xi) = \max \Bigg\{ \max_{P^X_{0,t}(z_c)} \Bigg[d^X_t \left(z_c, P^X_{0,t}(z_c) \right) - \xi w_t$$
$$+\beta \mathbf{E}_t \frac{\lambda_{t+1}}{\lambda_t} \sum_{\tilde{c}=1}^{n_z} \pi_{c\tilde{c}} H_{1,t+1} \left(z_{\tilde{c}}, P^X_{0,t}(z_c), \xi_{t+1} \right) \Bigg], \;\; \beta \mathbf{E}_t \frac{\lambda_{t+1}}{\lambda_t} \sum_{\tilde{c}=1}^{n_z} \pi_{c\tilde{c}} V^E_{t+1}(z_{\tilde{c}}, \eta_{t+1}) \Bigg\}, \tag{20}$$

for $c = 1, \cdots, n_z$.

The values conditional on no price reset are,

$$V_{j,t} \left(z_c, P^X_{j,t}(z_s), \xi \right) = \max \Bigg\{ d^X_t \left(z_c, P^X_{j,t}(z_s) \right) - \xi w_t$$
$$+\beta \mathbf{E}_t \frac{\lambda_{t+1}}{\lambda_t} \sum_{\tilde{c}=1}^{n_z} \pi_{c\tilde{c}} H_{j+1,t+1} \left(z_{\tilde{c}}, P^X_{j,t}(z_s), \xi_{t+1} \right), \;\; \beta \mathbf{E}_t \frac{\lambda_{t+1}}{\lambda_t} \sum_{\tilde{c}=1}^{n_z} \pi_{c\tilde{c}} V^E_{t+1}(z_{\tilde{c}}, \eta_{t+1}) \Bigg\}, \tag{21}$$

for $c = 1, \cdots, n_z, s = 1, \cdots, n_z$, and $j = 1, \cdots, J-2$, and

$$V_{J-1,t} \left(z_c, P^X_{J-1,t}(z_s), \xi \right) = \max \Bigg\{ d^X_t \left(z_c, P^X_{J-1,t}(z_s) \right) - \xi w_t$$
$$+\beta \mathbf{E}_t \frac{\lambda_{t+1}}{\lambda_t} \sum_{\tilde{c}=1}^{n_z} \pi_{c\tilde{c}} H_{J,t+1} \left(z_{\tilde{c}}, \xi_{t+1} \right), \;\; \beta \mathbf{E}_t \frac{\lambda_{t+1}}{\lambda_t} \sum_{\tilde{c}=1}^{n_z} \pi_{c\tilde{c}} V^E_{t+1}(z_{\tilde{c}}, \eta_{t+1}) \Bigg\}, \tag{22}$$

for $c = 1, \cdots, n_z$ and $s = 1, \cdots, n_z$.

Incumbents with current productivity z_c that are resetting prices in the current period choose $P^X_{0,t}(z_c)$ so as to maximize (20). Entrants with current productivity z_c choose $P^X_{0,t}(z_c)$ that solves (17). Since the optimal price does not depend on the cost of entering or continuing in the export market, the optimal price chosen by an entrant with productivity z_c is the same as the one chosen by any price-resetting incumbent with the same current firm-specific productivity level, z_c.

3.3 Thresholds governing entry and exit

Since the net value of exporting is monotonically decreasing in the fixed participation cost, we can define a maximum participation cost that a firm is willing to pay in order to enter or remain in the foreign market. For any group of firms sharing a common last-period export status, current price, and current productivity, there is a common threshold cost representing the maximum cost any firm in the group is willing to pay to operate in the export market. Of course, threshold costs differ across groups of firms with differing individual states; moreover, these thresholds vary with the aggregate state of the economy over time.

We begin by characterizing the export choice of firms that did not export last period. Let $P_{0,t}^X(z_c)$ be the optimal price chosen by entrants with current productivity z_c. Let $\eta_t^E(z_c)$ be the maximum export cost last period's non-exporters with current productivity z_c are willing to pay in order to enter the export market this period. This threshold cost equates the value of entering the export market this period (the first element of the binary max operator in equation (17)) to the value of not entering this period (the second element of the binary max operator in equation (17)):

$$\beta \mathbf{E}_t \frac{\lambda_{t+1}}{\lambda_t} \sum_{\tilde{c}=1}^{n_z} \pi_{c\tilde{c}} V_{t+1}^E(z_{\tilde{c}}, \eta_{t+1}) = d_t^X(z_c, P_{0,t}^X(z_c)) - \eta_t^E(z_c) w_t + \beta \mathbf{E}_t \frac{\lambda_{t+1}}{\lambda_t} \sum_{\tilde{c}=1}^{n_z} \pi_{c\tilde{c}} H_{1,t+1}\left(z_{\tilde{c}}, P_{0,t}^X(z_c), \xi_{t+1}\right), \quad \text{for } c = 1, \cdots, n_z. \tag{23}$$

Next, we describe the participation decisions of incumbent exporting firms that are able to adjust their nominal price this period. Using (20) and letting $P_{0,t}^X(z_c)$ denote the optimal price chosen by the price-resetting incumbent exporters with current productivity z_c, the maximum export cost, $\xi_t^0(z_c)$, these firms are willing to pay in order to continue exporting this period is:

$$\beta \mathbf{E}_t \frac{\lambda_{t+1}}{\lambda_t} \sum_{\tilde{c}=1}^{n_z} \pi_{c\tilde{c}} V_{t+1}^E(z_{\tilde{c}}, \eta_{t+1}) = d_t^X(z_c, P_{0,t}^X(z_c)) - \xi_t^0(z_c) w_t + \beta \mathbf{E}_t \frac{\lambda_{t+1}}{\lambda_t} \sum_{\tilde{c}=1}^{n_z} \pi_{c\tilde{c}} H_{1,t+1}\left(z_{\tilde{c}}, P_{0,t}^X(z_c), \xi_{t+1}\right), \quad \text{for } c = 1, \cdots, n_z. \tag{24}$$

Finally, using (21) and (22), we can define the maximum export cost, $\xi_t^j(z_c, z_s)$, that non-price-resetting incumbent exporters are willing to pay in order to continue exporting this period:

$$\beta \mathbf{E}_t \frac{\lambda_{t+1}}{\lambda_t} \sum_{\tilde{c}=1}^{n_z} \pi_{c\tilde{c}} V_{t+1}^E(z_{\tilde{c}}, \eta_{t+1}) = d_t^X(z_c, P_{j,t}^X(z_s)) - \xi_t^j(z_c, z_s) w_t + \beta \mathbf{E}_t \frac{\lambda_{t+1}}{\lambda_t} \sum_{\tilde{c}=1}^{n_z} \pi_{c\tilde{c}} H_{j+1,t+1}\left(z_{\tilde{c}}, P_{j,t}^X(z_s), \xi_{t+1}\right) \tag{25}$$

for $c = 1, \cdots, n_z$, $s = 1, \cdots, n_z$, and $j = 1, \cdots, J-2$, and

$$\beta \mathbf{E}_t \frac{\lambda_{t+1}}{\lambda_t} \sum_{\tilde{c}=1}^{n_z} \pi_{c\tilde{c}} V_{t+1}^E(z_{\tilde{c}}, \eta_{t+1}) = d_t^X(z_c, P_{J-1,t}^X(z_s)) - \xi_t^{J-1}(z_c, z_s) w_t + \beta \mathbf{E}_t \frac{\lambda_{t+1}}{\lambda_t} \sum_{\tilde{c}=1}^{n_z} \pi_{c\tilde{c}} H_{J,t+1}\left(z_{\tilde{c}}, \xi_{t+1}\right) \tag{26}$$

for $c = 1, \cdots, n_z$ and $s = 1, \cdots, n_z$.

Using the threshold participation costs derived above, alongside the time-invariant distributions from which the costs η and ξ are drawn, we can determine firms' probabilities of entering and exiting the export market, prior to the realizations of these costs. For entrants, the probability of entering the export market is $\zeta_t^E(z_c) = G^E\left(\eta_t^E(z_c)\right)$ for $c = 1, \cdots, n_z$. For price-adjusting incumbents, the probability of remaining in the export market is $\zeta_t^0(z_c) = G\left(\xi_t^0(z_c)\right)$ for $c = 1, \cdots, n_z$. For non-adjusting incumbents, the probability of remaining in the export market is $\zeta_t^j(z_c, z_s) = G\left(\xi_t^j(z_c, z_s)\right)$ for $c = 1, \cdots, n_z$, $s = 1, \cdots, n_z$, and $j = 1, \cdots, J-1$. Note that, given the law of large numbers, these probabilities also reflect the fractions of each firm type that enter, and continue operation, in export activities.

3.4 Evolution of the distribution of firms

We first examine the distribution of domestically producing firms. Let $\theta_{j,t}(z_c, z_s)$ be the mass of firms starting period t with current productivity z_c and a price set j periods ago when its productivity was z_s, $P_{j,t}^D(z_s)$. Since there is a unit mass of firms in each country, the total number of domestic firms sums to 1,

$$\sum_{j=1}^{J}\sum_{c=1}^{n_z}\sum_{s=1}^{n_z}\theta_{j,t}(z_c, z_s) = 1. \tag{27}$$

The evolution of $\theta_{j,t}(z_c, z_s)$ can be written as

$$\theta_{j+1,t+1}(z_{\tilde{c}}, z_{\tilde{s}}) = (1-\alpha_j)\sum_{c=1}^{n_z}\pi_{c\tilde{c}}\theta_{j,t}(z_c, z_{\tilde{s}}), \tag{28}$$

for $j = 1, \cdots, J-1$, $\tilde{s} = 1, \cdots, n_z$, and $\tilde{c} = 1, \cdots, n_z$. The total number of firms starting period $t+1$ with productivity $z_{\tilde{c}}$ that set a new price in time t, $P_{1,t+1}^D(z_{\tilde{s}})$, is

$$\theta_{1,t+1}(z_{\tilde{c}}, z_{\tilde{s}}) = \pi_{\tilde{s}\tilde{c}}\sum_{j=1}^{J}\sum_{s=1}^{n_z}\alpha_j\theta_{j,t}(z_{\tilde{s}}, z_s), \tag{29}$$

for $\tilde{s} = 1, \cdots, n_z$, and $\tilde{c} = 1, \cdots, n_z$.

We now characterize the distribution of exporters. Let $\psi_{j,t}(z_c, z_s)$ be the mass of incumbent exporters starting period t with current productivity z_c and an export price set j periods ago when its productivity was z_s, $P_{j,t}^X(z_s)$. Let $N_t^E(z_c)$ be the mass of new exporters with z_c in period t,

$$N_t^E(z_c) = \zeta_t^E(z_c)\left[\sum_{j=1}^{J}\sum_{s=1}^{n_z}\theta_{j,t}(z_c, z_s) - \sum_{j=1}^{J}\sum_{s=1}^{n_z}\psi_{j,t}(z_c, z_s)\right], \quad \text{for } c = 1, \cdots, n_z. \tag{30}$$

The evolution of $\psi_{j,t}(z_c, z_s)$ can be written as

$$\psi_{j+1,t+1}(z_{\tilde{c}}, z_{\tilde{s}}) = (1-\alpha_j)\sum_{c=1}^{n_z}\zeta_t^j(z_c, z_{\tilde{s}})\cdot\pi_{c\tilde{c}}\cdot\psi_{j,t}(z_c, z_{\tilde{s}}), \tag{31}$$

for $j = 1, \cdots, J-1$, $\tilde{s} = 1, \cdots, n_z$, and $\tilde{c} = 1, \cdots, n_z$. The mass of incumbents at the beginning of period $t+1$ that last set prices in period t is:

$$\psi_{1,t+1}(z_{\tilde{c}}, z_{\tilde{s}}) = \underbrace{\pi_{\tilde{s}\tilde{c}}\cdot\zeta_t^0(z_{\tilde{s}})\sum_{j=1}^{J}\sum_{s=1}^{n_z}\alpha_j\cdot\psi_{j,t}(z_{\tilde{s}}, z_s)}_{\text{price-adjusting incumbents surviving time } t} + \underbrace{\pi_{\tilde{s}\tilde{c}}\cdot N_t^E(z_{\tilde{s}})}_{\text{entrants at time } t}, \tag{32}$$

for $\tilde{c} = 1, \cdots, n_z$ and $\tilde{s} = 1, \cdots, n_z$. Finally, the total mass of firms exiting the export market in period t is $\psi_t - \left(\psi_{t+1} - N_t^E\right)$, where $\psi_t \equiv \sum_{c=1}^{n_z}\sum_{s=1}^{n_z}\sum_{j=1}^{J}\psi_{j,t}(z_c, z_s)$, and $N_t^E \equiv \sum_{c=1}^{n_z} N_t^E(z_c)$.

3.5 Price index

Using the distribution of domestic firms $\theta_{j,t}(z_c, z_s)$ and the distribution of incumbent exporters $\psi_{j,t}(z_c, z_s)$, we can aggregate individual prices to define price indices. The price index for domestically-produced intermediate goods (5) can be expressed as

$$P_t^D = \left[\sum_{j=1}^{J}\sum_{c=1}^{n_z}\sum_{s=1}^{n_z}\alpha_j\cdot\theta_{j,t}(z_c, z_s)\cdot P_{0,t}^D(z_c)^{1-\gamma} + \sum_{j=1}^{J-1}\sum_{c=1}^{n_z}\sum_{s=1}^{n_z}(1-\alpha_j)\cdot\theta_{j,t}(z_c, z_s)\cdot P_{j,t}^D(z_s)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}. \tag{33}$$

The price index for intermediate goods imported from the foreign country, the foreign analogue of (6), can be written as

$$\begin{aligned} P_t^{X*} = \Bigg[&\sum_{c=1}^{n_z} N_t^{E*}(z_c)P_{0,t}^{X*}(z_c)^{1-\gamma} + \sum_{j=1}^{J}\sum_{c=1}^{n_z}\sum_{s=1}^{n_z}\alpha_j\cdot\zeta_t^{0*}(z_c)\cdot\psi_{j,t}^*(z_c, z_s)\cdot P_{0,t}^{X*}(z_c)^{1-\gamma} \\ &+ \sum_{j=1}^{J-1}\sum_{c=1}^{n_z}\sum_{s=1}^{n_z}(1-\alpha_j)\cdot\zeta_t^{j*}(z_c, z_s)\cdot\psi_{j,t}^*(z_c, z_s)\cdot P_{j,t}^{X*}(z_s)^{1-\gamma}\Bigg]^{\frac{1}{1-\gamma}}. \end{aligned} \tag{34}$$

3.6 Households

In this economy, households in both countries have access to a financial market where they can purchase a complete set of state-contingent, one-period nominal bonds denominated in the home currency which pay one unit of the home currency (or $\frac{1}{e_{t+1}}$ units of the foreign currency if held by foreign households). Let $B(s^{t+1})$ denote home households' holdings of a bond purchased in period t and state s^t that will pay out in period $t+1$ if the state s_{t+1} occurs, and let $q(s^{t+1}|s^t)$ be the associated price of that nominal bond in units of the home currency in period t and state s^t.

A representative household in the home country chooses consumption C_t, labor L_t, investment I_t, bond holdings $B_{t+1}(s^{t+1})$, and money balances M_t, to maximize expected, discounted lifetime utility subject to a budget constraint and a law of motion for capital:

$$\max \mathbf{E}_t \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\sigma_c} C_t^{1-\sigma_c} + \chi_1 \frac{1}{1-\sigma_m} \left(\frac{M_t}{P_t} \right)^{1-\sigma_m} + \chi_2 \frac{1}{1-\sigma_L} (1-L_t)^{1-\sigma_L} \right], \tag{35}$$

subject to

$$\sum_{s^{t+1}} q(s^{t+1}|s^t) B(s^{t+1}) + P_t C_t + P_t I_t + M_t \leq B(s^t) + P_t d_t + P_t w_t L_t + P_t r_t K_t + M_{t-1} + T_t^M, \tag{36}$$

and

$$K_{t+1} = (1-\delta) K_t + I_t - \phi \left(\frac{I_t}{K_t} \right) K_t, \tag{37}$$

where T_t^M is a lump-sum nominal government transfer, d_t represents dividends returned by home-country firms, δ is the depreciation rate of capital, and $\phi(\cdot)$ is a convex capital adjustment cost function that satisfies $\phi(\delta) = 0$ and $\phi'(\delta) = 0$.

The representative household in the foreign country solves an analogous problem, and its budget constraint is,

$$\begin{aligned} \sum_{s^{t+1}} \frac{q(s^{t+1}|s^t) B^*(s^{t+1})}{e_t} + P_t^* C_t^* + P_t^* I_t^* + M_t^* \leq \frac{B^*(s^t)}{e_t} + P_t^* d_t^* + P_t^* w_t^* L_t^* \\ + P_t^* r_t^* K_t^* + M_{t-1}^* + T_t^{M*}. \end{aligned} \tag{38}$$

The first-order necessary conditions associated with optimal bond purchases among home and foreign households are, respectively,

$$q(s^{t+1}|s^t) = \beta \cdot prob(s^{t+1}|s^t) \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \right], \tag{39}$$

$$q(s^{t+1}|s^t) = \beta \cdot prob(s^{t+1}|s^t) \left[\frac{\lambda_{t+1}^*}{\lambda_t^*} \frac{P_t^*}{P_{t+1}^*} \frac{e_t}{e_{t+1}} \right]. \tag{40}$$

Since these conditions hold for all periods and all states of the world, we can use (39) and (40) to obtain

$$\frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} = \frac{\lambda_{t+1}^*}{\lambda_t^*} \frac{P_t^*}{P_{t+1}^*} \frac{e_t}{e_{t+1}}. \tag{41}$$

Iterating (41) and rearranging, we have

$$Q_t = e_0 \frac{\lambda_0}{\lambda_0^*} \frac{P_0^*}{P_0} \frac{\lambda_t^*}{\lambda_t}, \tag{42}$$

since $Q_t \equiv e_t \frac{P_t^*}{P_t}$. Thus, as in any two-country model with complete asset markets, we find that the real exchange rate is proportional to the relative marginal utility of consumption between the two countries.[9]

3.7 Money supply

The monetary authority in each country injects money into its economy through nominal transfers, $T_t^M = M_t^s - M_{t-1}^s$, where M_t^s is aggregate nominal money supply. The money supply evolves according to $M_t^s = \mu_t M_{t-1}^s$, where μ_t is the gross rate of money growth. I assume money growth follows an AR(1) process in logarithms: $\log \mu_{t+1} = \rho_\mu \log \mu_t + \varepsilon_{t+1}^\mu$. The money market clearing condition is $M_t = M_t^s$.

4 Functional forms and parameter values

The length of a period in the model corresponds to one quarter. The representative household's momentary utility is specified as

$$\log C_t + \chi_1 \log\left(\frac{M_t}{P_t}\right) + \chi_2(1 - L_t). \tag{43}$$

The household subjective discount factor is selected to imply a 4 percent average annual real interest rate. The weight on leisure χ_2 is set to imply that households work on average 1/3 of their time in the baseline model. Recall the CES function, (1), through which final goods are produced. To identify this function, I must specify values of ρ, γ and ω. Gallaway, McDaniel and Rivera (2003) estimate the elasticity of substitution between home and foreign goods (the Armington elasticity) using the U.S. disaggregated data between 1980 and 1988, and report that the value ranges from 0.14 to 3.49. In this paper, the elasticity ρ is set to 1.5, as in Backus, Kehoe and Kydland (1993), Chari, Kehoe and McGrattan (2001) and Alessandria and Choi (2007). Following Ghironi and Melitz (2005), I set the elasticity of substitution between goods produced in the same country γ to be 3.8. The capital depreciation rate δ is 0.025 to imply the annual investment-to-capital ratio of 10 percent. Hummels (2001) estimated that the freight rate varies from 4 to 13 percent of shipment value. I set the iceberg-trade cost τ equal to 1.05 so that 5 percent of exported goods is paid as the cost. I consider a capital adjustment cost function of the form $\phi(I/K) = \frac{\kappa}{2}\left(\frac{I_t}{K_t} - \delta\right)^2$. With this specification, the steady state total and marginal costs of capital adjustment are 0. In my baseline calibration, I set the marginal capital adjustment cost κ equal to 2 so that the correlation between GDP and investment is 0.94 as in the data. The price adjustment hazard is specified as $\alpha = [0.05, 0.09, 0.25, 0.49, 0.7, 1]$ in the baseline model. These values imply that firms that adjust

[9] In our calibration, we normalize $e_0 \frac{\lambda_0}{\lambda_0^*} \frac{P_0^*}{P_0} = 1$.

their prices in the current period expect their prices to last for 3.89 quarters.[10] In the baseline model, the average age of domestic prices over the steady state distribution of firms is 2.7 quarters. These values fall within the range of estimates (1.4-4.3 quarters) of micro-level price adjustments in the recent literature (see, for example, Bils and Klenow (2004) and Nakamura and Steinsson (2008)). Table 1 summarizes the baseline parameter values.

To reduce the number of additional parameters in the model, I assume that entry and continuation costs (η and ξ, respectively) are both drawn from uniform distributions. The firm-level productivity follows a discrete state Markov chain. The home bias parameter ω, the upper support of entry costs paid to enter the export market η_U, that of continuation costs to remain there k, and the processes of the firm-level productivity (ρ_z and σ_{ε_z}) are calibrated to match (i) the mass of exporters, (ii) the average rate of entry, (iii) the average rate of exit, (iv) the average productivity of exporters relative to that of non-exporters and (v) the imports-to-GDP ratio in the data. In the baseline model, the steady-state mass of exporters is 21 percent of all the firms in the economy, to be in line with the finding of Bernard, Eaton, Jensen and Kortum (2003) from data on U.S. manufacturers in 1992. For the entry and exit rates, Bernard and Jensen (2004) report that on average each year, 87.4 percent of the exporters continued exporting in the following year and 13.9 percent of nonexporters began exporting in the following year. These numbers translate to a 97 percent quarterly continuation rate and a 4 percent quarterly entry rate. In the model, the probability that incumbent exporters continue exporting next period is 87 percent while the probability of non-incumbent firms entering the export market is 4 percent.[11] Exporters are 13 percent more productive relative to non-exporters to be in line with the observed range of 12-18 percent (Bernard and Jensen, 1999). The steady-state ratio of imports to GDP is 0.12 as in the data (Drozd and Nosal, 2011). Table 2 reports the steady state characteristics of the baseline model and its variants.

In terms of aggregate productivity shocks, Backus, Kehoe and Kydland (1992) assume that productivity shocks in two countries follow a vector autoregressive (VAR) process of the form

$$\begin{bmatrix} A_t \\ A_t^* \end{bmatrix} = \begin{bmatrix} a_1 & a_2 \\ a_2 & a_1 \end{bmatrix} \begin{bmatrix} A_{t-1} \\ A_{t-1}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_t^A \\ \varepsilon_t^{A^*} \end{bmatrix}. \tag{44}$$

The innovations ε_t^A and $\varepsilon_t^{A^*}$ are serially independent, multivariate normal random variables with contemporaneous covariance matrix V, which allows for contemporaneous correlation between innovations in the two countries. Thus, the shocks are stochastically related through the off-diagonal element a_2 (spillover parameter) and the off-diagonal elements of the covariance matrix V. I adopt

[10] $\alpha_1 1 + (1-\alpha_1)\alpha_2 2 + (1-\alpha_1)(1-\alpha_2)\alpha_3 3 + \cdots = 3.89$

[11] The steady state continuation rate in my baseline model is lower than the observed rate. This is because, with the current calibration, the continuation rate of exporters with high productivity is already nearly 1, and hence any attempt to further increase the average continuation rate will result in these exporters having the continuation rate of 1, in which case the model cannot be solved by the linear method currently employed.

Backus, Kehoe and Kydland's estimates, setting $a_1 = 0.906$, $a_2 = 0.088$, $var(\varepsilon_t^A) = var(\varepsilon_t^{A^*}) = (0.007)^2$ and $corr\left(\varepsilon_t^A, \varepsilon_t^{A^*}\right) = 0.258$.

For the money growth process, I follow Chari, Kehoe and McGrattan (2002) and assume the following,

$$\begin{bmatrix} \mu_t \\ \mu_t^* \end{bmatrix} = \begin{bmatrix} 0.68 & 0 \\ 0 & 0.68 \end{bmatrix} \begin{bmatrix} \mu_{t-1} \\ \mu_{t-1}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{\mu} \\ \varepsilon_t^{\mu^*} \end{bmatrix}.$$

I assume that the steady state money growth rate is $\bar{\mu} = 1.01$ to imply a 4 percent annual inflation rate. I select the value of $var(\varepsilon^{\mu}) = var(\varepsilon^{\mu^*})$ so that the standard deviation of GDP is 1.42 in the baseline model, as in the U.S. data.

Except where explicitly noted otherwise, the resulting parameter set is held fixed across all model comparisons.

5 Results

5.1 Business cycle moments

Table 3 reports business cycle moments from my full model and its variants under the assumption that aggregate fluctuations are driven by shocks to aggregate productivity and money growth rates in home and foreign countries. I report the logged (except for the net exports-to-GDP ratio), Hodrick-Prescott filtered statistics from the U.S. and European aggregate data and the models. My model is able to reproduce some well-documented features of international business cycles such as counter-cyclical net exports and positive cross-country correlations of employment and investment. These properties suggest that my model is an adequate framework to evaluate the role of exporter entry and exit for international business cycles.

Backus, Kehoe and Kydland (1993), Baxter and Crucini (1993) and Baxter (1995) find that standard international business cycle models with complete international financial markets predict negative cross-country correlations of investment and employment. However, in the data these variables comove positively across countries (0.18 and 0.27, respectively). This same anomaly arises in versions of my model with flexible prices, with and without entry and exit (-0.32 and -0.31, respectively, in the model with entry and exit (column 3), and -0.32 and -0.40, respectively, in the model with a constant set of exporters (column 4)). Examining the columns labeled 'Sticky' (columns 1 and 2), note that the introduction of micro-level price rigidities resolves this anomaly even under complete financial markets, as consistent with earlier results obtained by Chari, Kehoe and McGrattan (2002) in a monetary model with monopolistic competition and sticky prices. In my sticky-price model without entry and exit, the cross-country correlation is 0.30 for investment and 0.54 for employment (column 2). I find that the success of nominal rigidities in delivering comovement in a complete markets setting survives my extension of the model to accommodate greater realism with respect to the micro-level trade data. The introduction of endogenous exporter entry and exit in fact improves the model's quantitative fit to the data, as these correlations are reduced to 0.17 and 0.46, respectively (column 1).

Examining a flexible-price model with fixed costs leading to endogenous export participation, Alessandria and Choi (2007) arrive at an invariance result. They find that introducing sunk entry costs does not alter the business cycle properties predicted by a model without such costs. I obtain this same invariance result when I compare my two flexible-price models (columns 3 and 4). However, I find that this is not the case for some macroeconomic aggregates in the models when price rigidities are introduced. In addition to the cross-country correlations discussed above, we find that the introduction of the entry/exit mechanism has more distinct effects on the impulse responses of the export price index and total exports following an aggregate productivity shock, as shown in the following subsection.

5.2 Responses to a productivity shock

The mechanism behind the positive cross-country correlations in my full model becomes clear with the examination of impulse responses following a positive persistent home aggregate productivity shock with spillover effects in the foreign country. In this subsection, we compare the economy's impulse responses in the models with entry and exit, under sticky- versus flexible-price settings. In order to first understand the international transmission mechanism in the presence of entry and exit, we begin by examining the impulse responses from the flexible-price model with entry and exit.

Figure 1 shows the impulse responses to a 1 percent positive aggregate productivity shock in the home country with a spillover of 0.088 to the foreign country, as used in the simulation exercises in the previous subsection. In response to the shock, the rise in home relative productivity shifts the production of intermediate goods toward the home country. Home output immediately rises while foreign output falls until the spillover effect sufficiently increases the foreign productivity in later periods. This results in an increase in productive inputs - labor and capital, hence investment - at home and an initial decrease in these variables abroad, generating their negative cross-country correlations contrary to the data.

Due to the cross-country risk-sharing implied by complete financial markets, consumption in both countries increases with the rise in home productivity. However, the rise in home consumption is larger in compensation for the increased labor supply by home households. Hence, the marginal utility of consumption in the home country falls more than in the foreign country, resulting in a real depreciation of the home currency (a rise in the real exchange rate).

Next, we consider the shock's implications for trade flows and the number of firms participating in export markets. Continuing our examination of figure 1, notice that the home country experiences an immediate fall in net exports as it increases its imports of foreign-produced intermediate goods to produce the final good for domestic consumption and investment. The outward shift in the demand for foreign exports raises the profitability for foreign exporters. Thus, we see an immediate increase in the number of new foreign intermediate-good producers entering the export market and an immediate fall in the number of foreign incumbent exporters exiting the market. Together, these changes raise the total number of foreign exporters by about 0.2 percent. On the other hand, in the early periods following the shock, the number of home exporters changes little until production in the foreign country starts to sufficiently increase due to the delayed spillover effect. With higher productivity, the price of intermediate goods falls in the home country while it rises in the foreign country.

Figure 2 shows the impulse responses of the same set of aggregate variables to the same 1 percent positive aggregate productivity shock in the home country, this time allowing for micro-level price rigidities. With the positive productivity shock, home intermediate good producers are now able to produce any given amount of output with less labor and capital inputs. What is different here relative to the flexible-price setting is that, since many intermediate goods firms are unable to reset their prices immediately after the shock, demand for their products changes little. As a result, domestic demand for productive inputs falls at the date of the shock, leading to a brief fall in home employment and investment. Thereafter, an increasing number of domestic firms begin lowering their prices in response to the shock. Thus, home demand for investment and employment steadily rise above the steady-state levels for several periods. While these rises are taking place in the home country, foreign employment and investment slowly increase and rise above the steady-state levels with the spillover effect. Thus, over the periods of their largest changes, the directions of change in investment and employment abroad match those in the home country. As a result, my model succeeds in generating positive international comovements that are absent in the flexible price model.

Another observation regarding figure 2 has to do with the real exchange rate movements. Staggered reductions in nominal prices over time mean that the initial rise in home consumption is dampened relative to the flexible price model, thereafter slowly climbing to the level that is immediately reached in the flexible price model. This generates a notably different real exchange rate path. The diminished changes to the marginal utility of consumption initially mean a much smaller initial rise (0.08 percent versus 0.56 percent) of the real exchange rate. Thereafter, as home consumption continues to rise, increasing differences in the marginal utility of consumption across countries yields a hump-shaped response of the real exchange rate that we do not observe in the flexible price model.

Although domestic consumption immediately rises, domestic output does not jump up immediately at the impact of this shock in my full model. This occurs because of the unanticipated decline in the demand for capital. Given the existing stock, investment falls for a period allowing the increased demand for consumption to be satisfied with essentially no change in final production.

Because the capital stock is pre-determined when the shock hits the economy, and because the rise in overall demand is sharply restrained by some firms' inability to reduce their nominal prices, the marginal value of capital falls. This in turn reduces the rental rate on capital, further lowering the marginal cost of production in the home country beyond the direct effect of the productivity shock. These effects of the pre-determined capital stock are temporary as forward-looking households adjust capital for subsequent periods. Thus, following the shock's impact, home output, employment and investment rise, and then monotonically revert to the trend in subsequent periods just as in the flexible price model.

Considering the shock's implications for trade in this model, notice that there are sharp qualitative differences relative to the responses in the model with flexible prices. First, restrained rises in home demand, alongside raised productivity and lower capital rental rates together raise the profitability of operating in the export market for home intermediate-good producers. Thus, we see a large increase in new exporters who absorb the extra existing capital, and a sharp decline in the number of incumbent exporters leaving the export market. These two forces generate an immediate, substantial (1.45 percent) rise in the total number of exporters in the home country where the productivity is higher, followed by a persistent slight growth in subsequent periods. This is in contrast to the flexible-price model previously discussed where the number of exporters in the home country experiences only a slight increase (0.04 percent) in the first few periods after the shock followed by a persistent decline in the number of home exporters in subsequent periods. In data, we observe that the number of exporters is highly procyclical. Figure 3 plots the number of U.S. exporters between 1996 and 2010 reported in the Profile of U.S. Importing and Exporting Companies by the U.S. Census Bureau. The figure also plots the U.S. real GDP over the same period, and indicates the U.S. recessions defined by the National Bureau of Economic Research. As can be seen in this figure, the number of exporters significantly dropped during both the 2001 recession and the recent Great Recession. The procyclical response of the number of exporters in my full model is consistent with this observation shown in figure 3.

An important implication of endogenous trade participation in the presence of price rigidities can be seen by examining the dynamics of trade flows and export prices in comparison to their behavior in the model without endogenous trade participation. Figure 4 compares the dynamic responses of select trade-related variables of the home country with entry and exit in the export market to those in the model without entry and exit, both under price rigidities. Here, again, I examine a 1 percent positive productivity shock in the home country, allowing the same spillover effects to the foreign country as were present in the simulation exercises. As a comparison, I present in figure 5 the flexible-price counterpart to figure 4.

As discussed above, we see in figure 4 that with endogenous trade participation the number of exporters increases sharply in the home country in response to the shock as the profitability of operating in the export market for home intermediate-good producers increases. Importantly, through a rise along the extensive margin of trade, this increase in the number of exporters contributes to an increase in trade volumes, generating a 0.9 percent rise in gross exports at the impact of the positive productivity shock. In contrast, in the model without entry and exit, the home country's exports experiences a 0.15 percent fall on impact and gradually rises in the subsequent periods. This is because, with price rigidities, the export growth is delayed as prices and hence demand adjust to the shock over time. The flexible entry margin in my full model partly overcomes this inertia since firms that newly enter the export market are able to set low nominal prices consistent with the observed aggregate productivity and its anticipated evolution in the future.

In addition, notice that the fall in the home export price index is significantly larger in the model with endogenous entry and exit. This too is because new exporters that enter the export market can optimally select their nominal prices. With the positive productivity shock and a large number of new exporters, the average price of home exports is lower than in the model without entry and exit. This contributes to an increase in the foreign demand for home exports, and further increases home exports. The combined effects of the extensive margin arising from the exporter entry and the intensive margin due to the low export prices lead to a larger increase in home net exports in the initial periods after the productivity shock, relative to the model without entry and exit, which explains the higher volatility of net exports that was seen in table 3.

The differing dynamics we find from the comparison of the two sticky-price models are in stark contrast to the results from the models with flexible prices. Figure 5 reveals that, under the flexible-price setting as considered by Alessandria and Choi (2007), the dynamics of trade flows and the export price index are nearly identical across the models with and without entry and exit. This leads them to conclude that export decisions have negligible consequences for international business cycles. However, my results from the settings that allow for time-dependent nominal rigidities suggest that the extensive margin of trade has important implications for business cycle fluctuations in overall trade volumes, export prices, and other aggregate series.

5.3 Exchange rate pass-through

There is extensive evidence reported in empirical studies on international trade that adjustments of import prices to changes in production costs are delayed and incomplete. For example, using aggregate data from 23 OECD countries from 1975 to 2003, Campa and Goldberg (2005) find that the average degree of pass-through is 0.23 in the short run and 0.42 in the long run. In this subsection, I explore the implications of endogenous trade participation for exchange rate pass-through.

In the presence of price rigidities, exchange rate pass-through to a price is zero until the producer adjusts its price. When the price is adjusted, the pass-through is one as the adjusted price accounts for any exchange rate movements. In my model, incomplete exchange rate pass-through arises due to the frictions in price adjustment. With entry and exit, incumbent exporters who adjusted their prices more recently are more likely to continue exporting as their prices are closer to the optimal price consistent with the aggregate state of the economy. Since the probability of exit is higher for those with older prices, one might expect that the mean frequency of price adjustment is higher among the exporters who continue to export, and hence higher pass-through in the presence of exporter entry and exit, overturning the implication of nominal rigidities for incomplete exchange rate pass-through. To explore this possibility, I measure the degree of pass-through from my simulated model, and show that, on the contrary, the pass-through implied by nominal rigidities in my model survives the additional flexibility of endogenous exporter entry and exit.

To measure the degree of exchange rate pass-through, I simulate the models with shocks to both aggregate productivity and the money growth rate for 120 periods, and record the movements of aggregate variables and export prices of a fixed set of incumbent exporters that remain in the export market throughout the entire length of the simulation. I then construct an export price index by averaging the export prices of these surviving exporters. Because new entrants set a new price, their first prices are not responding to exchange rate changes relative to the previous period. Similarly, the last prices of exporters who exit the export market do not respond to exchange rate movements because their prices no longer exist. By including only the prices of continuing incumbent exporters, I ensure that such prices irrelevant to exchange rate pass-through do not influence my measure of pass-through. Using this export price index, I estimate the following pass-through regression used in the empirical study of Gopinath, Itskhoki and Rigobon (2010),

$$\Delta\tilde{p}_t^{X*} = \alpha + \sum_{n=0}^{8}\beta_n\Delta e_{t-n} + \sum_{n=0}^{8}\gamma_n\pi_{t-n}^{*} + \delta\Delta y_t + \epsilon_t, \tag{45}$$

where $\tilde{p}_t^{X*}$ is the constructed foreign export price index, $\pi_t^* \equiv P_t^*/P_{t-1}^*$ is foreign inflation, and y_t is home GDP. From this regression, the degree of pass-through is measured as the sum of coefficients on nominal exchange rate changes, $\beta(n) \equiv \sum_{n=0}^{8}\beta_n$. These coefficients reflect the impact that the current change in the exchange rate has on the price of home imports (i.e. foreign exports) over time.

Figure 6 presents the cumulative pass-through coefficients, $\beta(n)$, from model simulations. For the full model (blue line with circles), we see that the degree of pass-through is 0.48 for concurrent movements in the exchange rate, and gradually rises over time, reaching around 0.90 for exchange rate changes over 2 periods. After 3 periods, the pass-through reaches 1.0, and remains at around this level, meaning that the exchange rate movements from more than 3 periods ago have little effect on the current import price changes. We see, however, that the realistic degree of nominal rigidity calibrated in this model cannot by itself account for the long-run incomplete pass-through observed in the data. This is not surprising considering the degree of nominal rigidity assumed in my model. The calibrated price stickiness in the full model suggests that the firms expect their prices to last 3.89 periods. In fact, in my simulations, during the simulated 120 periods, on average, firms adjust their prices 30.8 times, meaning that the average age of price is 3.89 periods. This is why we see in the figure that the pass-through is complete after 4 periods ($\sum_{n=0}^{3} \beta_n \approx 1$). Goldberg and Hellerstein (2008) and Nakamura and Zeom (2010) evaluate the relative importance of different mechanisms for incomplete pass-through. For example, Goldberg and Hellerstein (2008) find that, on average, local non-traded costs account for 56.1% of the observed incomplete exchange rate pass-through; 33.7% by markup adjustment; and 12.2% by the existence of price rigidity. Nakamura and Zeom (2010) conclude that nominal rigidities are quantitatively successful in explaining the delayed response of prices to costs, but have a negligible effect on long-run pass-through. My simulation result confirms nominal rigidities as a channel for short-run pass-through.

I implement the same exercise using the model with entry and exit but with flexible prices, and the result is plotted with a green line with x's in figure 6. We see that, with flexible prices, concurrent pass-through is nearly complete ($\beta_0 = 0.95$), and the figure remains at around 1 subsequently, indicating that past exchange rate movements have little influence on the current movement of export prices. This is because in this case incumbent exporters can optimally adjust their prices period by period in response to shocks in the economy, and hence exchange rate movements are immediately passed onto the export prices.

I next explore the effect of entry and exit on exchange rate pass-through. In my full model, exporters with newer prices are more likely to continue exporting than those with older prices. Hence, one might expect that the price adjustment frequency among exporters would be higher, and hence higher pass-through, as firms with older prices are more likely to exit and new entrants come into the export market with new prices. However, I find that this is not the case in my model. In figure 6, I also plot the simulation results from the model with sticky prices but without exporter entry and exit (a red line with crosses). Comparing this result to the one with entry and exit (blue line with circles), we see that the degree of pass-through predicted by the two models is almost identical.

To understand this result, I examine the characteristics of exporters in the two models. Figure 7 compares the distributions of price age, current productivity, and the past productivity of exporters in the model without entry and exit (left panels) and those of the exporters that survive through the entire simulation periods in the model with entry and exit (right panels).

I start with the model with entry and exit (panels a, b and c). Panel (a) shows the fractions of exporters for each price-age group. We see that about 26% of exporters adjust their prices in the current period ($j = 0$), about 22% of exporters last adjusted their prices in the previous period ($j = 1$), and so on. This distribution of price age among exporters fluctuates around the steady state distribution throughout the simulation period. For the distribution of current productivity levels (panel b), we see that about half of the exporters have a high productivity level and the remaining half have low productivity. This is due to the assumed symmetry of the transition matrix governing the Markov switching process for the firm-level productivity. For the same reason, the productivity level an exporter had when it last adjusted its price is also equally split among exporters.[12] These results are expected since there is not exporter entry and exit in this model.

Next, I examine the distribution of exporters in the model with entry and exit in the right panels. In panel (d), I find that, throughout the simulation period, the distribution of the age of prices among surviving exporters is almost identical to that in the model without entry and exit (panel a). In fact, on average, the exporters in the full model adjust their prices every 3.89 quarters, the same duration as in the model without entry and exit. Therefore, the aggregate price rigidity among surviving exporters in my full model is the same as that in the model without entry and exit, and hence the degree of pass-through is not affected by endogenous trade participation in my full model.

That we find the same degree of pass-through in the two models may be at odds with the intuition that endogenous trade participation can potentially increase the frequency of price adjustment among exporters and hence higher pass-through, as discussed above. Recall, however, that in the full model, export decisions are influenced not only by the age of a price but also by firms' relative productivity and trade costs. In particular, all else equal, firms with higher productivity are more likely to export than those with lower productivity levels. In my simulations, I find that the effect of productivity differences on export decisions dominates the effect of price age differences. For example, a firm with a 5-period-old price and a high productivity is more likely to export than a firm with a low productivity that adjusts its price in the current period. In panel (e), we see that almost all the surviving exporters have high productivity in the current period, with only a few with low productivity. Consequently, the productivity levels associated with their prices are also dominated by high productivity (panel f). Therefore, the frequency of price adjustment among exporters in the model with entry and exit is nearly identical to that in the model without entry and exit, and hence the degree of exchange rate pass-through is preserved in my full model. This result validates the role of price rigidity as a mechanism to generate short-run incomplete pass-through despite the additional realism of entry and exit in my model.

6 Conclusion

In this paper, I have investigated the implications of endogenous trade participation for international business cycles, the dynamics of trade flows and exchange rate pass-through when firms' price adjustments follow a time-dependent hazard that rises in the age of a price. I developed a two-country dynamic stochastic general equilibrium model wherein individual exporters reset their nominal prices infrequently and the presence of entry and continuation costs induces endogenous

[12] For exporters adjusting their prices in the current period, this productivity associated with its price is the same as the current productivity.

firm entry and exit in the export market. I calibrated my model to be consistent with recent empirical findings in international trade at the firm and sectoral levels. The steady-state predictions of my model are consistent with observations from firm-level data on the fraction of exporters, their relative productivity, entry and continuation rates in export markets, the aggregate import-to-GDP ratio and the average frequency of individual price change.

Relative to existing endogenous trade participation models, my inclusion of micro-level price rigidity reduces the extent to which changes in the prices of existing traded goods absorb changes in the value of exporting following a persistent macroeconomic shock, and instead permits substantial movements in the benefits to firm entry. This induces large and immediate responses in the number of exporters, delivering sharp qualitative differences in the paths of net exports and the export price index over early dates following the shock. I also find that the effect of productivity differences and trade costs on export decisions dominates that of price-age differences, and hence incomplete exchange rate pass-through arising from price rigidity survives the additional realism of exporter entry and exit in my model.

The model and analyses presented in this paper can be extended in several directions. First, it will be a straightforward exercise to replace the time-dependent hazard for price adjustment in the current model with a state-dependent hazard by introducing menu costs of price adjustment. This modification will allow existing exporters to change the timing of their price adjustments in response to aggregate shocks. Another important extension that may be of more immediate concern is a careful analysis of the policy implications of exchange rate pass-through. Pass-through issues have gained renewed attention in recent policy debates as they have broad implications for the effectiveness of monetary policy. For example, low import price pass-through implies that nominal exchange rate fluctuations may lead to lower expenditure-switching effects of domestic monetary policy. This implies that if the insulation is greater, then monetary policy should be more effective in stimulating the domestic economy. This possibility may be examined in my model environment by introducing a policy rule oriented towards stabilization such as a Taylor rule. My general equilibrium model with rich micro-foundations offers flexibility in accommodating large cross-country differences, and thus it is an ideal candidate environment in which to undertake such an analysis.

A Additional variables

a. Home exports

$$EX_t = \int \frac{P_t^X(i) c_t^{H*}(i)}{P_t^X} di = \omega_2^\rho \left(\frac{P_t^X}{P_t^*} \right)^{-\rho} D_t^* \tag{46}$$

b. Home imports

$$IM_t = \int \frac{P_t^{X*}(i) c_t^F(i)}{P_t^{X*}} di = \omega_2^\rho \left(\frac{P_t^{X*}}{P_t} \right)^{-\rho} D_t \tag{47}$$

c. Home net exports

$$NX_t = \frac{e_t P_t^X EX_t - P_t^{X*} IM_t}{P_t^Y Y_t} \tag{48}$$

d. Output

$$Y_t = \frac{\int \left[P_t^D(i) c_t^H(i) + e_t P_t^X(i) c_t^{H*}(i) \right] di}{P_t^Y} \tag{49}$$

where P_t^Y is the GDP deflator

$$P_t^Y = (1 - g_t) P_t^D + g_t e_t P_t^X \tag{50}$$

and g_t is the export-to-output ratio

$$g_t \equiv \frac{e_t P_t^X EX_t}{P_t^Y Y_t} \tag{51}$$

B Data

The data series for the U.S. output, consumption and employment are from the Federal Reserve Bank of St. Louis Economic Data (FRED) and cover 1975:1-2000:4. The series used are real gross domestic product, real personal consumption expenditures, and civilian employment, all quarterly and seasonally adjusted. The investment data is real fixed capital formation from OECD Main Economic Indicators.

The series for the aggregate of European countries (EU15) are from OECD Main Economic Indicators, and include (except for the employment series) Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, Sweden, and United Kingdom. The series used are gross domestic product, private final consumption expenditure, and gross fixed capital formation, all quarterly and seasonally adjusted. The employment series for the EU aggregate is the Civilian Employment Index used in "Financial autarky and international business cycles" (Heathcote and Perri, 2002). This series covers 1975:4-1998:4, and includes Austria, Finland, France, Germany, Italy, Spain, Sweden, and United Kingdom.

The series for net exports is constructed as the U.S. exports of goods to 13 European countries (EU15 excluding Belgium and Luxembourg) minus U.S. imports of goods from these European countries, divided by the U.S. nominal GDP. The trade data are collected from IMF Directions of Trade Statistics, and cover 1975:1-2000:4. The U.S. nominal GDP series is from FRED.

The number of exporters reported in figure (3) is from the Profile of U.S. Importing and Exporting Companies issued annualy by the U.S. Census Bureau.

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Figure 1: Flexible-price model. 1% Home aggregate productivity shock

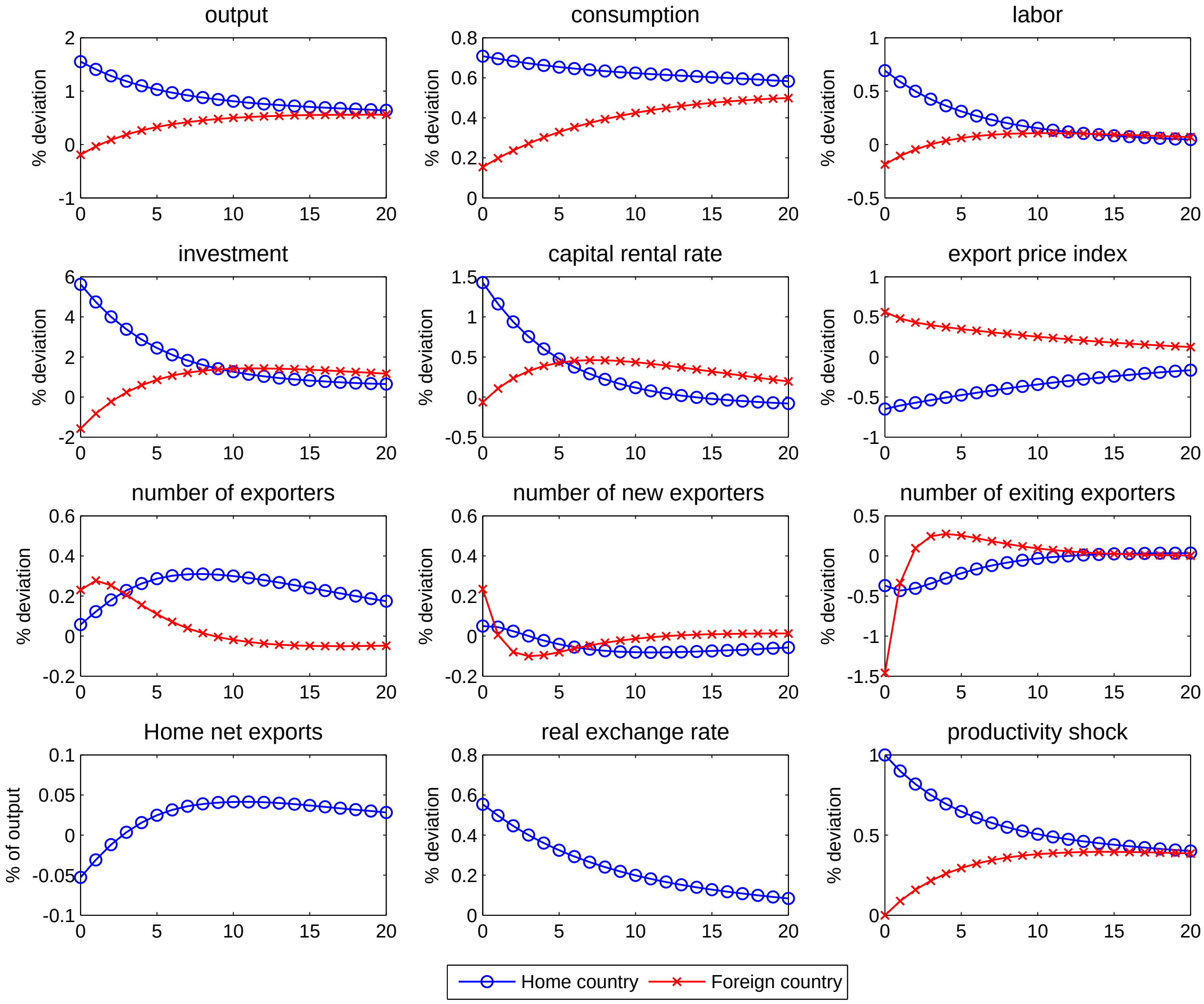


Figure 2: Sticky-price model. 1% Home aggregate productivity shock

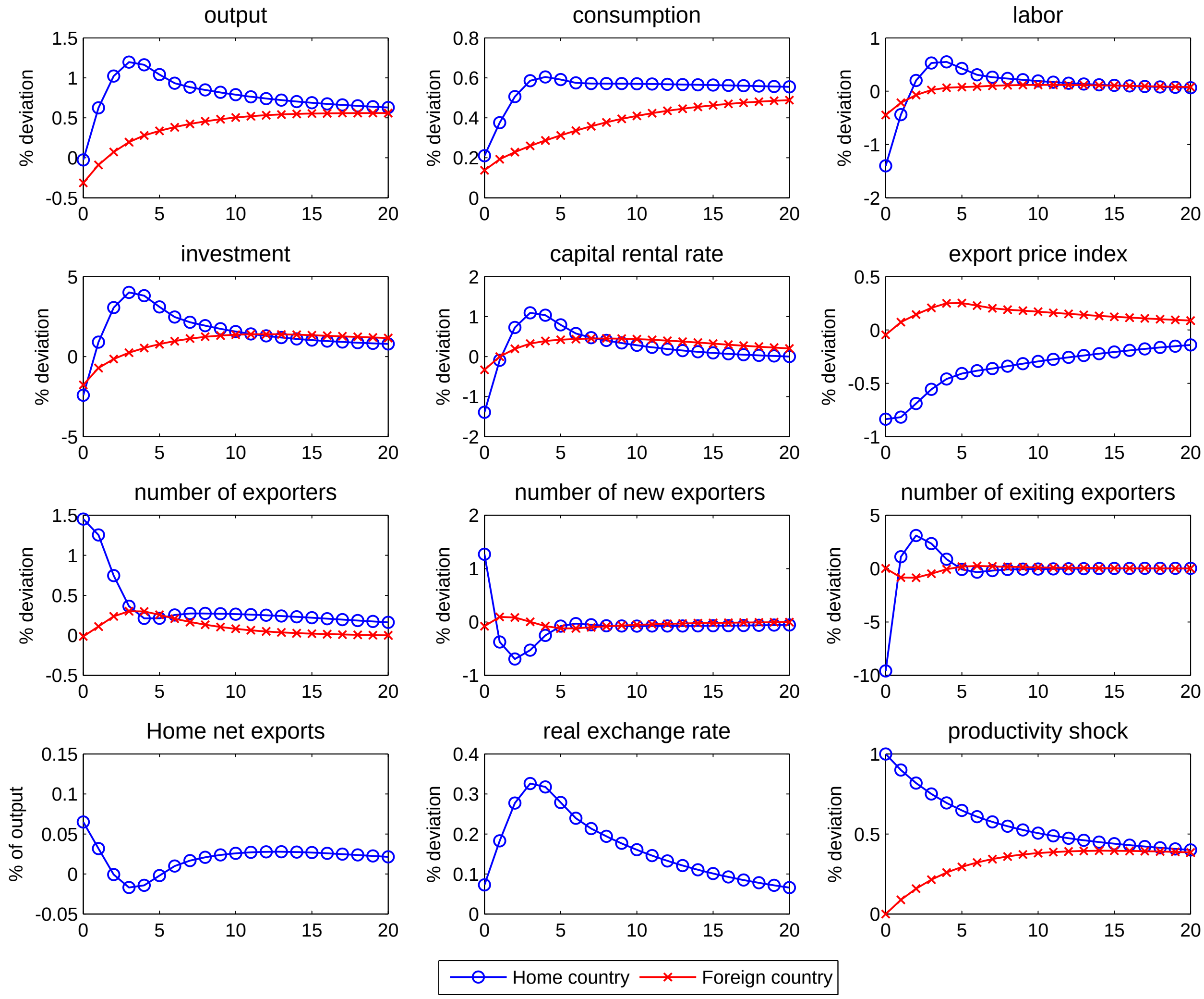
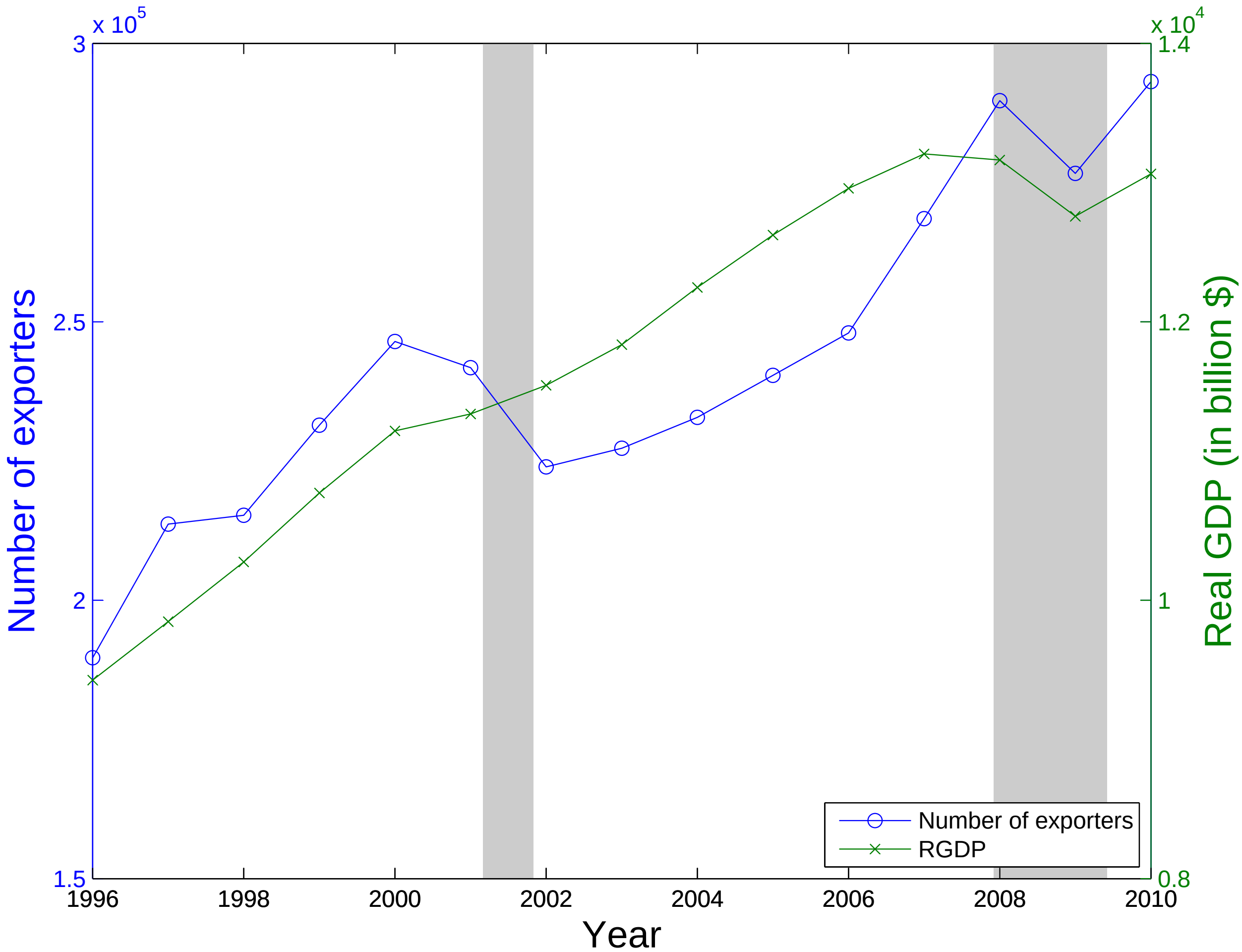


Figure 3: Number of exporters over the business cycle



Notes: Data on the number of exporters is from the *Profile of U.S. Importing and Exporting Companies* issued by the U.S. Census Bureau. The shaded areas indicate U.S. recessions defined by the National Bureau of Economic Research.

Figure 4: Sticky-price models: 1 % Home aggregate productivity shock

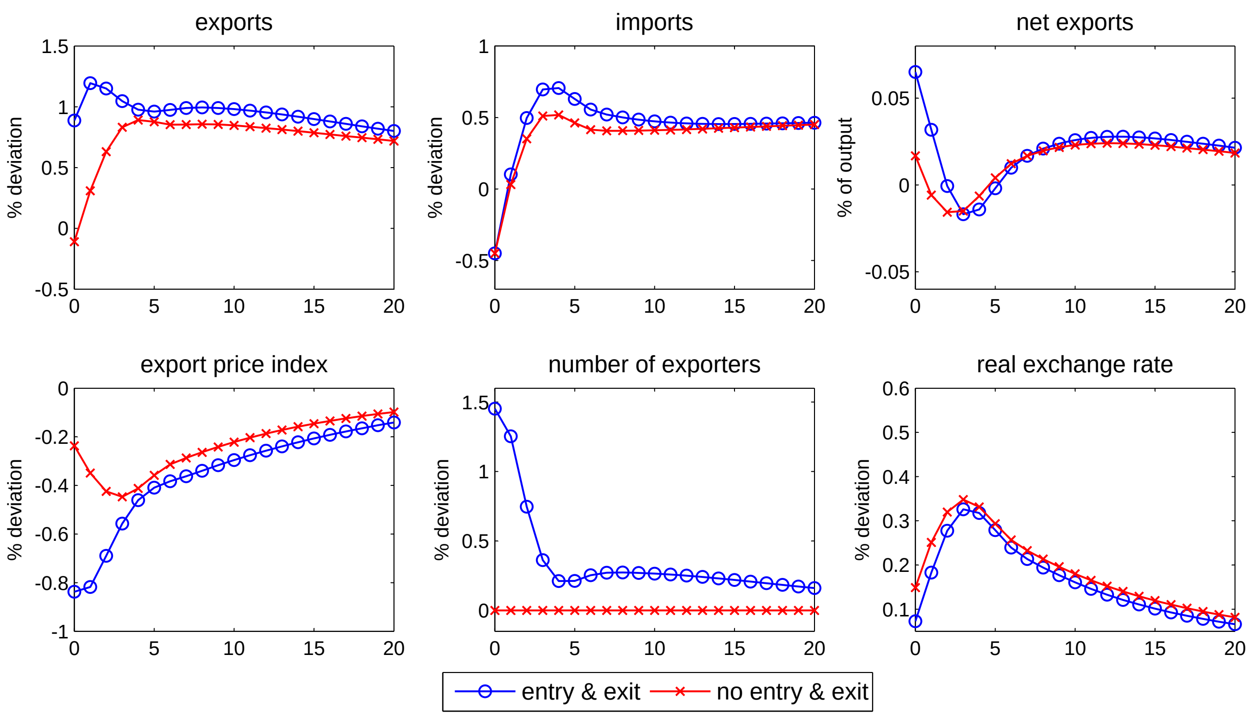


Figure 5: Flexible-price models: 1 % Home aggregate productivity shock

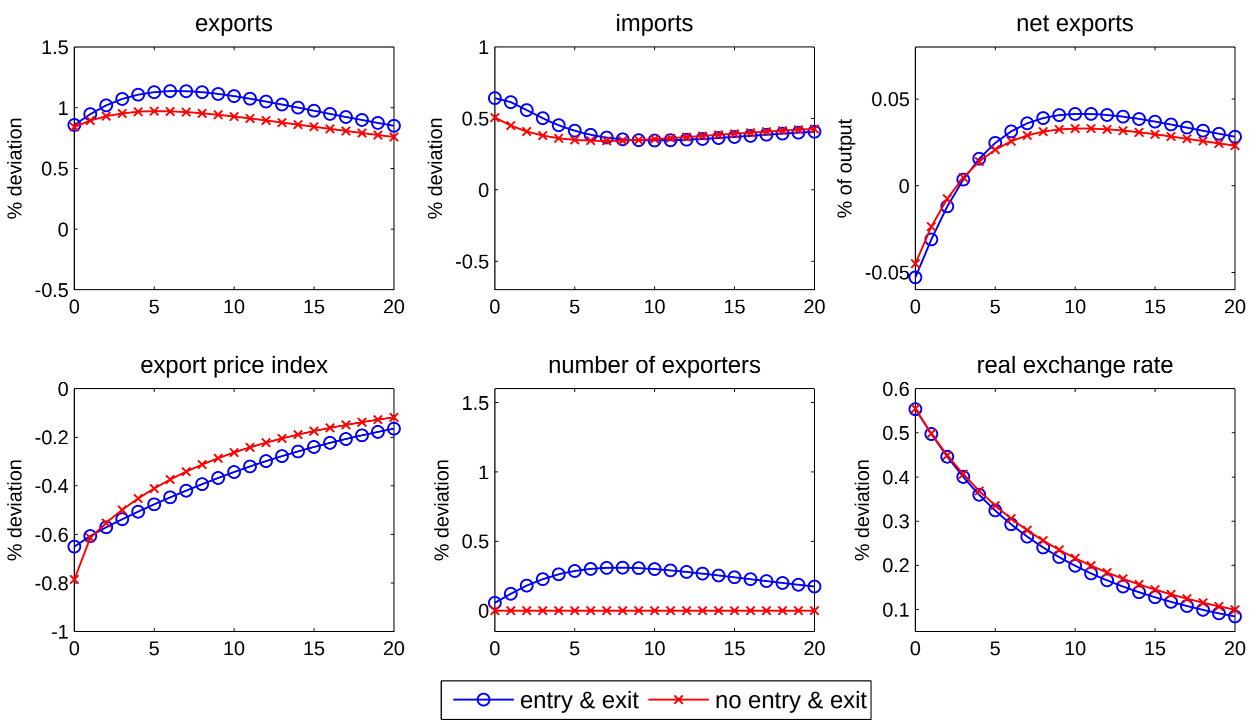
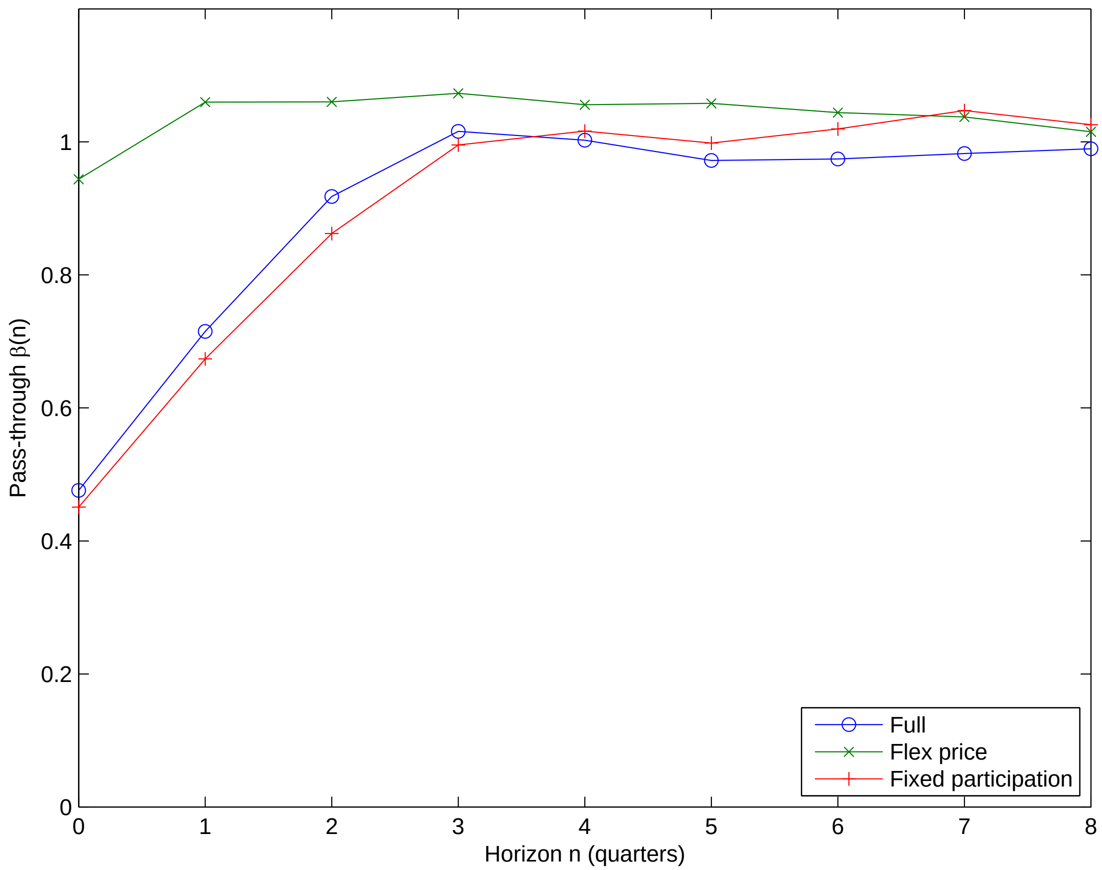
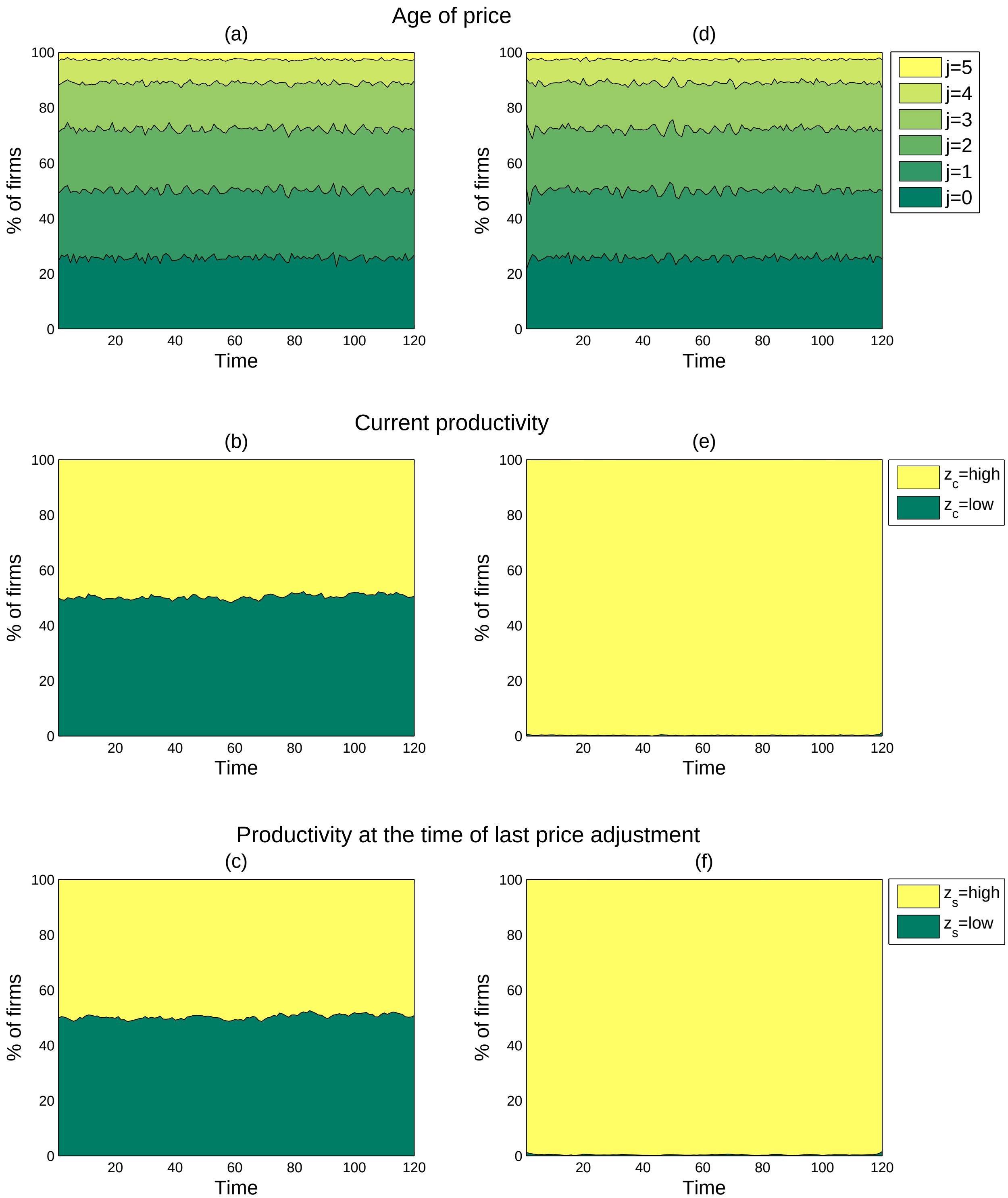


Figure 6: Exchange rate pass-through



This figure plots cumulative effects of exchange rate movements on the export price index, measured by $\Sigma_{n=0}^{8}\beta_n$ from the regression (45). 'Full' is the full model with nominal price rigidities and time-varying export market participation. 'Flex-price' eliminates the micro-price rigidities from the full model. 'Fixed participation' eliminates the time-variation in export participation from the full model. Each model is simulated for 120 periods. The sample size is 2105 firms for 'Full', 1944 firms for 'Flex-price', and 2126 firms for 'Fixed participation'.

Figure 7: Exporter characteristics



Panels (a), (b) and (c) are from the model without entry and exit. Panels (d), (e) and (f) are from the model with entry and exit.

Table 1: Parameter values

Subjective discount factor	β	0.99
Weight on money balance	χ_1	0.0001
Weight on leisure	χ_2	1.9
Domestic elasticity of substitution	γ	3.8
Armington elasticity	ρ	1.5
Home-bias weight	ω	0.762
Capital income share	ν	0.3
Capital depreciation rate	δ	0.025
Capital adjustment cost	κ	2
Trade costs		
Iceberg	τ	1.05
Upper support on entry cost dist.	η_U	2.8
Upper support on continuation cost dist.	k	0.17
Price adj. hazard	$[\alpha_j]$	[0.05, 0.09, 0.25, 0.49, 0.7, 1]
Firm-specific productivity		
Persistence	ρ_z	0.81
Standard deviation of innovation	σ_{ε_z}	0.085

Table 2: Calibration targets

		Entry/Exit		No Entry/Exit	
	Data	Sticky	Flex	Sticky	Flex
Entry and exit					
Fraction exporting	0.21	0.21	0.21	0.21	0.21
Quarterly continuation rate	0.97	0.87	0.86	1	1
Quarterly entry rate	0.04	0.04	0.04	0	0
Avg. relative exporter productivity	1.12-1.18	1.13	1.12	1	1
Imports/GDP ratio	0.12	0.12	0.12	0.11	0.11
Average price duration (quarters)	1.43-4.33	2.66	1	2.66	1

Table 3: Business cycle moments

		Sticky		Flexible	
		(1)	(2)	(3)	(4)
	Data	EE	No EE	EE	No EE
Standard deviations (in %)					
GDP	1.42	1.42	1.49	1.33	1.32
Standard deviations relative to GDP					
Consumption	0.83	0.43	0.41	0.54	0.54
Investment	2.73	4.88	4.85	3.52	3.45
Labor	0.65	1.26	1.40	0.42	0.44
Net exports/GDP	0.15	0.20	0.16	0.15	0.12
Correlations with GDP					
Consumption	0.84	0.87	0.86	0.94	0.93
Investment	0.94	0.94	0.96	0.97	0.97
Labor	0.86	0.79	0.82	0.97	0.96
Net exports/GDP	-0.31	-0.35	-0.38	-0.26	-0.26
Autocorrelations					
GDP	0.86	0.66	0.63	0.69	0.69
Consumption	0.88	0.80	0.80	0.73	0.73
Investment	0.88	0.50	0.50	0.66	0.66
Labor	0.90	0.40	0.40	0.66	0.65
International correlations					
GDP	0.41	0.34	0.39	0.06	0.03
Consumption	0.21	0.56	0.54	0.65	0.64
Investment	0.18	0.17	0.30	-0.32	-0.32
Labor	0.27	0.46	0.54	-0.31	-0.40

Notes: The statistics in the first 13 rows of the data column are calculated from U.S. quarterly data, 1975:1-2000:4. The statistics in the last 4 rows of the data column are calculated from U.S. variables and an aggregate of 15 European countries. The data statistics are logged (except for net exports) and Hodrick-Prescott-filtered with a smoothing parameter of 1,600. The model statistics are computed as the average of 100 simulations, each simulation with 1,000 periods, where the relevant series have been logged and HP-filtered as the data series. See Apendix B for data descriptions.